



CAREER: Fundamental Limits and Practical Algorithms for Complex Cyber-Physical Systems

PI: Sertac Karaman,
Assistant Professor,
Dept. of Aeronautics and Astronautics
Massachusetts Institute of Technology

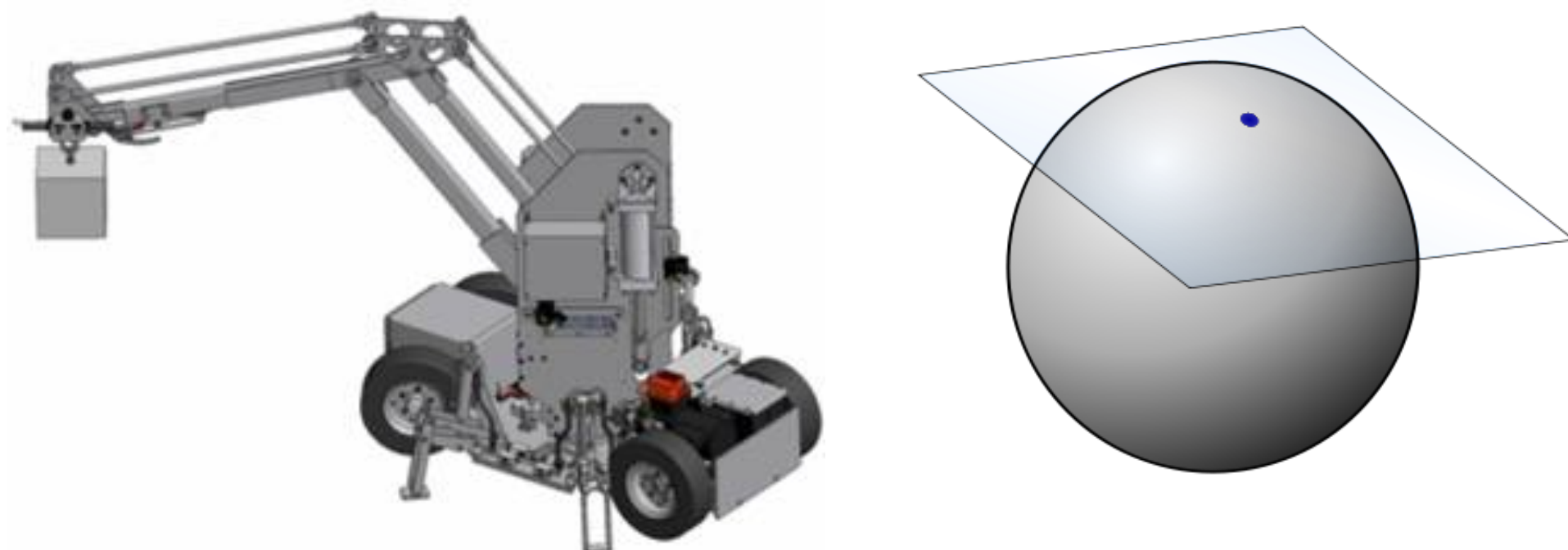
Abstract: Designing software that can properly and safely interact with the physical world is an important cyber-physical systems design challenge. The proposed work includes the development of a novel approach to designing planning and control algorithms for high-performance cyber physical systems. The new approach was inspired by statistical mechanics and stochastic geometry. It will (i) identify behavior such as phase transitions in cyber-physical systems and (ii) capitalize this behavior in order to design practical algorithms with provable correctness and performance guarantees. The algorithms developed through this research effort hold the potential for immediate industrial impact, particularly in the development of real-time robotic systems. These algorithms may strengthen the rapidly developing U.S. robotics industry. The proposed research activity will also vitalize the PI's educational plans. Undergraduate and graduate courses that make substantial contributions to the embedded systems education at MIT will be developed. The classes will focus on provably-correct controller synthesis for cyber-physical systems, which is currently not thought at MIT. Undergraduate students will be involved in research activities.

Goal: Exploiting connections with statistical mechanics for analysis and design of complex CPS:

- **Geometric Complexity:** Inherent in many robotics applications and beyond. For instance, configuration space describing the geometry of a robot and its environment.
- **Differential Constraints:** Nonlinear, non-holonomic differential equations describing the physics.
- **Stochastic Constructs:** May arise in stochastic environments or in randomized algorithms.

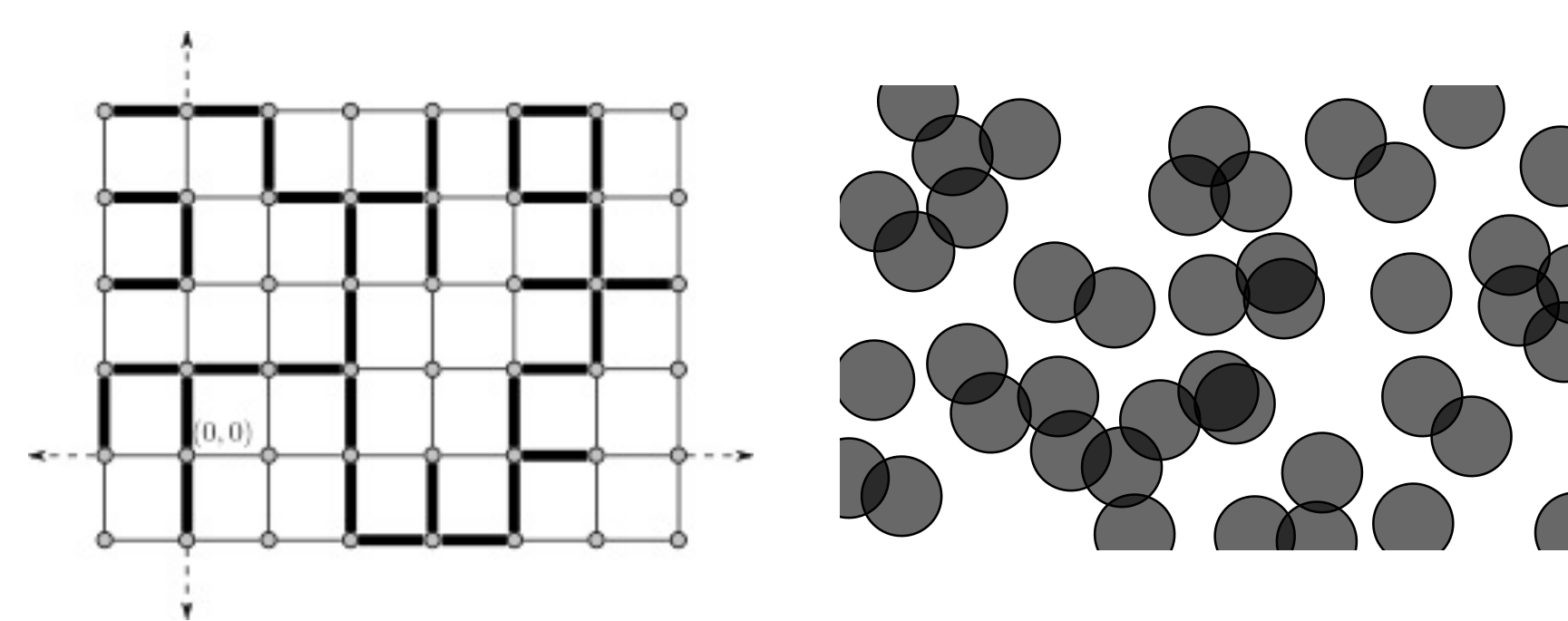
Core Research: An Excursion into Differential and Stochastic Geometry

1. Differential geometry, in particular sub-Riemannian geometry can characterize small-time reachable sets of complex dynamical systems.



The analysis reveals the asymptotic shape of the small-time reachable set, and algorithmic methods to constructs its approximations.

2. Stochastic geometry, in particular percolation theory, characterizes the topology of random geometric shapes in the Euclidean space.



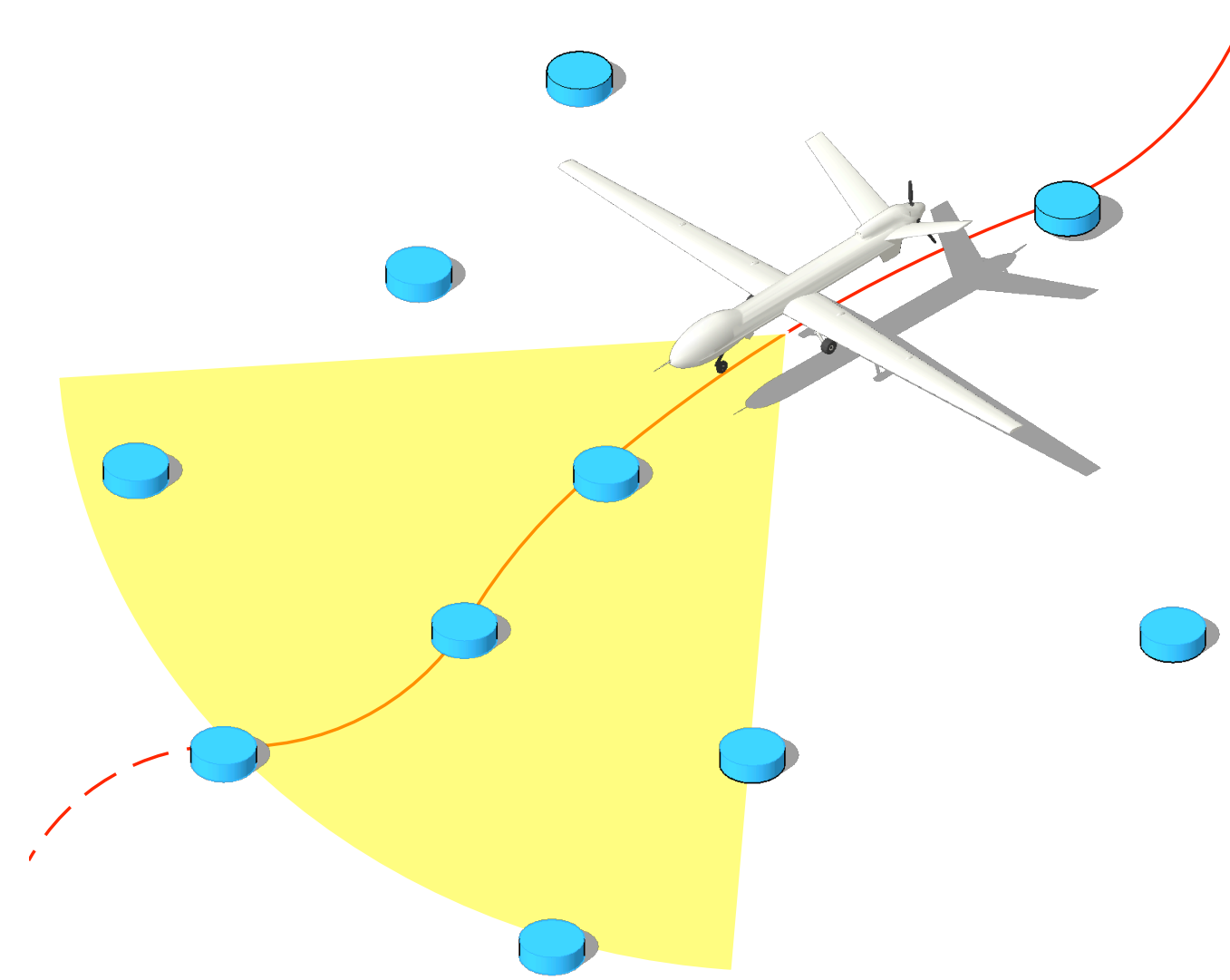
The analysis reveals phase transitions and describes a number of natural and engineered emergent behavior.

An overarching goal of the proposed research effort is to develop *differential and stochastic geometry*.

In particular, we aim to investigate:

- Percolation processes on sub-Riemannian manifolds.
- Dynamic stochastic growth processes on sub-Riemannian manifolds.

Design of UAVs for Data Gathering Tasks: Scaling wrt Perception, Actuation, and Computation



Problem Setup:

- Vehicle navigating in a sensor field.
- Measurement locations are available from a distance – perception range. Vehicle has limited perception range.
- Vehicle has limited agility and limited computation power.
- *How does sensing performance scale with the perception, actuation, and computation capabilities of the vehicle?*

Major results: Perception Range

- Performance increases *exponentially* with increasing perception range, when the reward distribution is bounded.
- Equivalently, only log perception range is enough to perform optimally, i.e., as if the vehicle has infinite perception range.
- We conjecture this result extends to when the distribution is light tailed.

Theorem 5. Suppose the reward locations are generated by a Poisson point process with intensity λ on \mathbb{R}^2 . Suppose that these rewards $r(p_i)$ are uniformly almost-surely bounded random variables, i.e., there exists some b such that

$$\mathbb{P}(|r(p_i)| \leq b) = 1 \text{ for all } i \in \mathbb{N}$$

and that R_2^* is finite. The robot dynamics satisfies the following ordinary differential equation:

$$\dot{x}_1(t) = v, \quad \dot{x}_2(t) = u(t),$$

where $|u(t)| \leq v$ (i.e., robot agility is 1). Then, for any $\delta > 0$, there exists some constant c such that

$$\lim_{m \rightarrow \infty} \mathbb{P}\left(\left|\frac{Q(L(m), m)}{L(m)} - R_2^*\right| \geq \delta\right) = 0.$$

where $L(m) = e^{cm}$ for some constant c that is independent of m (but depends on δ).

Corollary 3. Suppose the assumptions of Theorem 5 hold. Then, for any $\delta > 0$, there exists some constant c such that

$$\lim_{L \rightarrow \infty} \mathbb{P}\left(\left|\frac{Q(L, c \log L)}{L} - R_2^*\right| \geq \delta\right) = 0.$$

- However, the perception range required to navigate optimally is almost linear, when the reward distribution is Pareto.
- We conjecture this generalizes to all heavy-tailed distributions

Theorem 6. Suppose the reward locations are generated by a Poisson point process with intensity λ on \mathbb{R}^d . Suppose that these rewards $r(p_i)$ follow a Pareto distribution with parameter $\alpha \in (1, 2)$. The robot dynamics satisfies the following ordinary differential equation:

$$\dot{x}_1(t) = v, \quad \dot{x}_2(t) = u(t),$$

where $|u(t)| \leq v$ (i.e., robot agility is 1). Then there exists a probability space (Ω, \mathcal{F}, P) such that as m goes to infinity,

$$\mathbb{E}\left[\frac{Q(L; m)}{L}\right] = c \cdot m^{(2/\alpha)-1}, \quad \forall L > m,$$

for some positive constant c .

Major results: Agility

- The performance curve with respect to agility can be characterized exactly:

Theorem 7. Suppose the reward locations are generated by a Poisson point process with intensity λ on \mathbb{R}^d . The robot dynamics satisfies the following ordinary differential equation:

$$\dot{x}_1(t) = v, \quad \dot{x}_2(t) = u(t),$$

where $|u(t)| \leq w$. Then for any finite $L > 0$, there exists a constant $c > 0$ such that

$$\mathbb{E}[\mathcal{R}(L)] = c\sqrt{\alpha} = c\sqrt{w/v}.$$

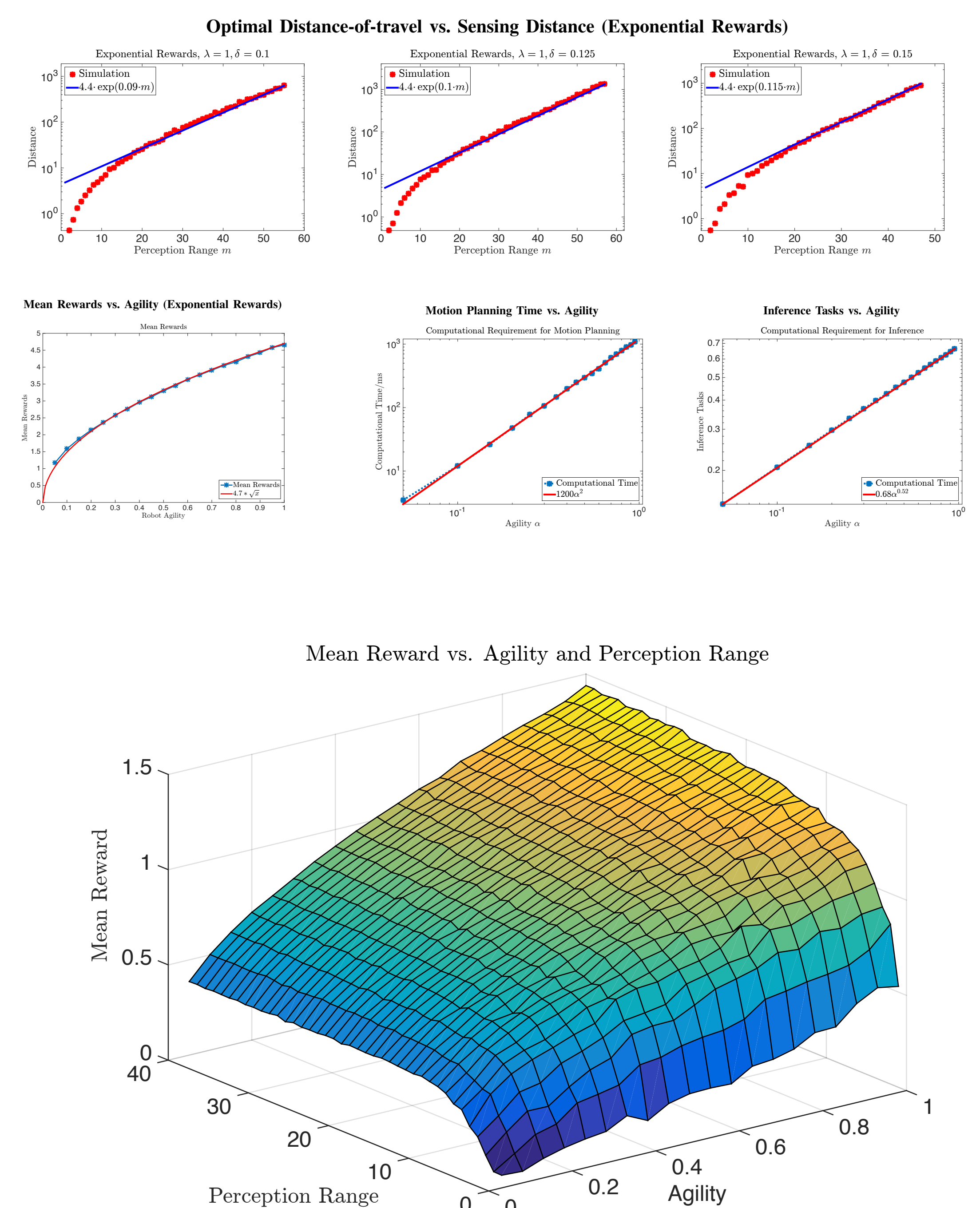
Major results: Computation

- The scaling of computation can also be characterized exactly:

$$T_{\text{planning}} = O(\alpha^2 m^4). \quad T_{\text{inference}} = O(\alpha^{1/2} m^0) = O(\sqrt{\alpha}).$$

Design of UAVs for Data Gathering Tasks: Scaling wrt Perception, Actuation, and Computation

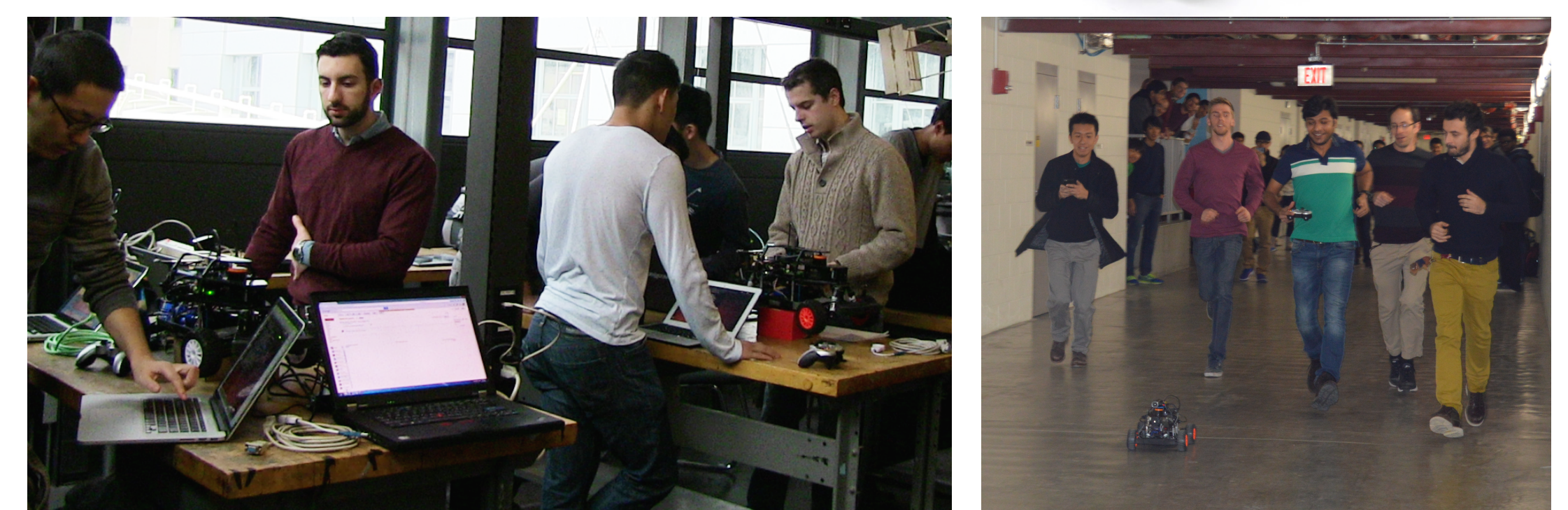
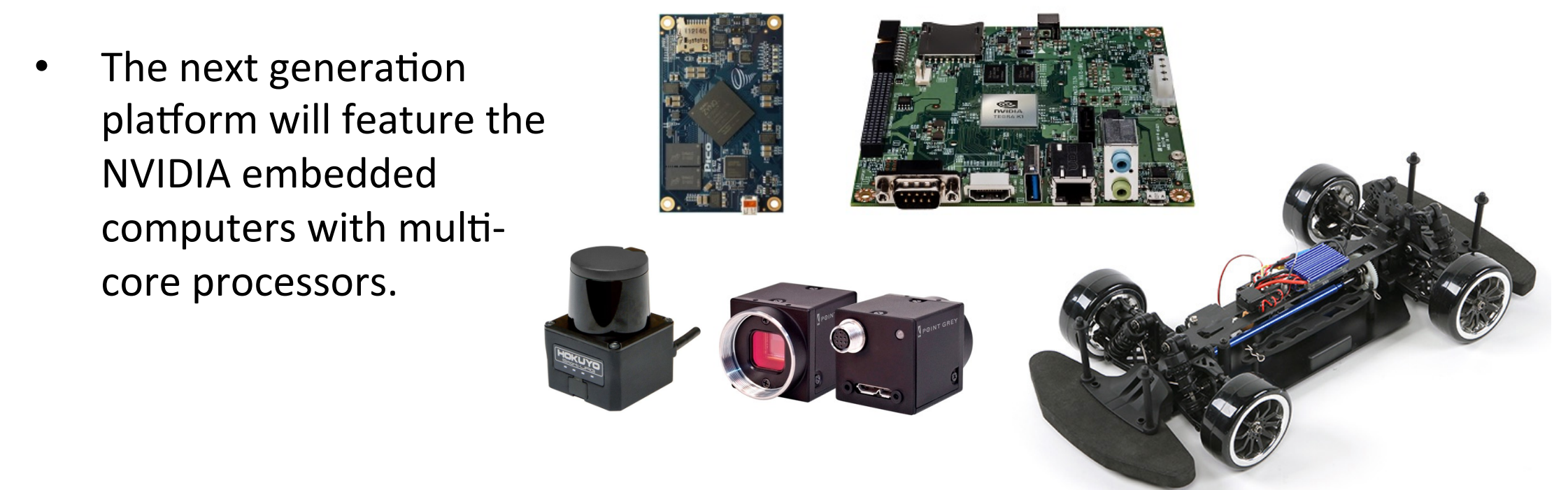
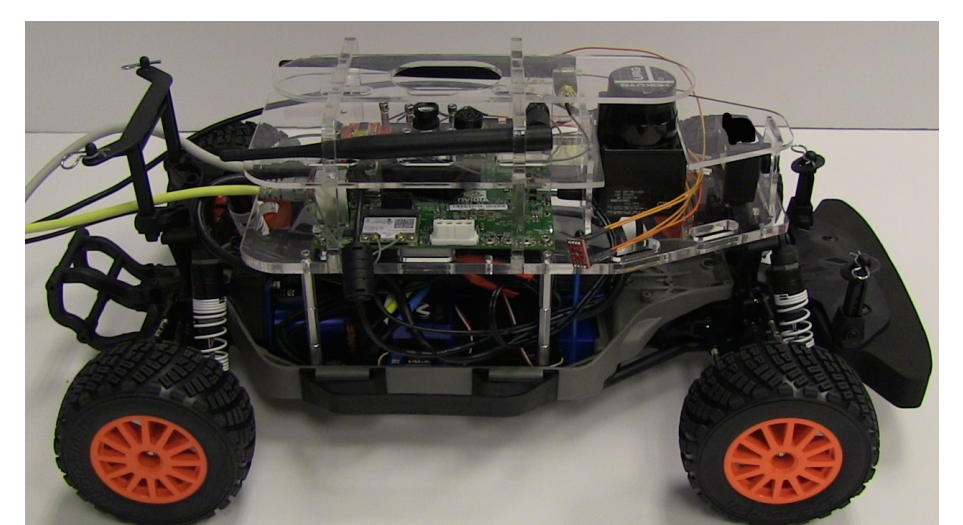
Simulation Results:



Current Educational Activities

1. MIT Robotics Course and Hackathon: Autonomous mini race cars in MIT's tunnels

- The legendary MIT tunnels will serve as the race tracks with foam obstacles.
- Students will use embedded computational platforms, gain experience working with CPS.
- Students will design algorithms and compete to race through the tunnels.



2. MIT Aerospace Feedback Control Systems: Teaching with Mini Drones

- MIT's Feedback Control Systems course is giving one mini drone to each student enrolled. The students can do the labs at home.
- The students can design feedback control systems, using our Matlab toolbox.
- The toolbox generates C code, which is compiled and uploaded to the drone.

