

## Background

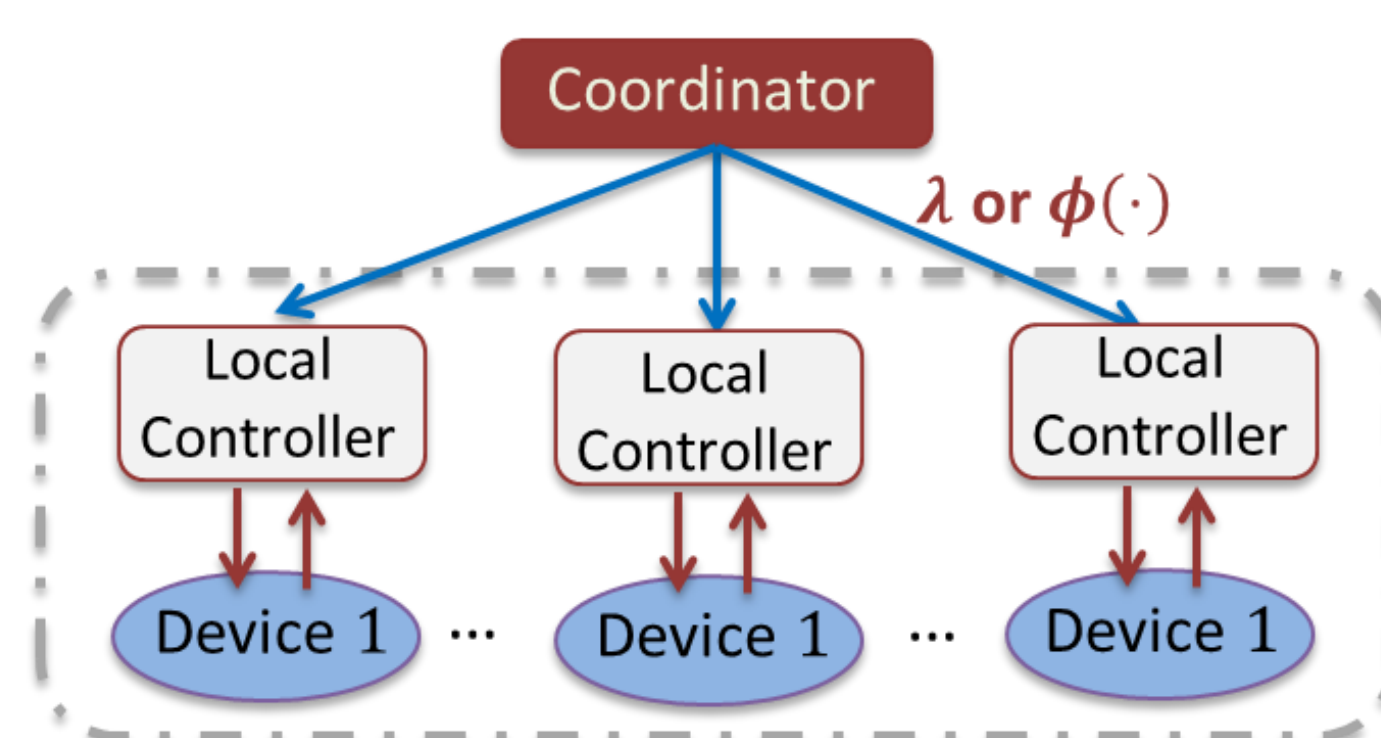


- Large population of agents
- Decoupled static or dynamic models
- Coupled decision due to shared environment/resources
- Often involve human decision makers

### Fundamental Question

How to design control strategies to achieve a desired population objective while respecting local preferences and constraints for individual agents?

## Hierarchical Control Architecture



### Research Challenges

- information structure
- rationality assumption
- agent dynamics
- population dynamics

### Individual Agents

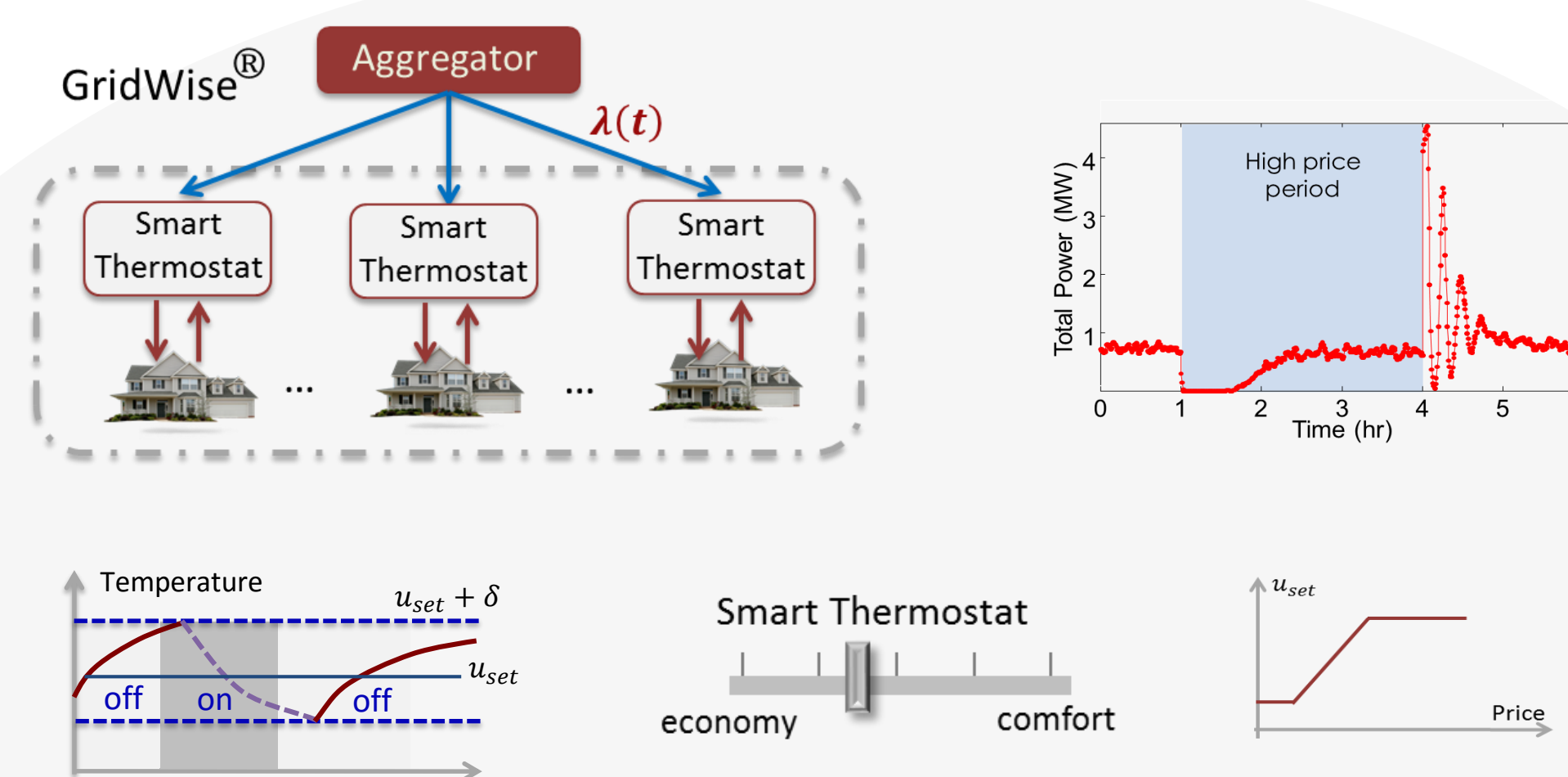
- Local objectives depend on coordinator control signal
- Local control law respects local preferences/constraints

### Coordinator's objective

- Design macroscopic control signal/rules to achieved desired aggregate population behavior

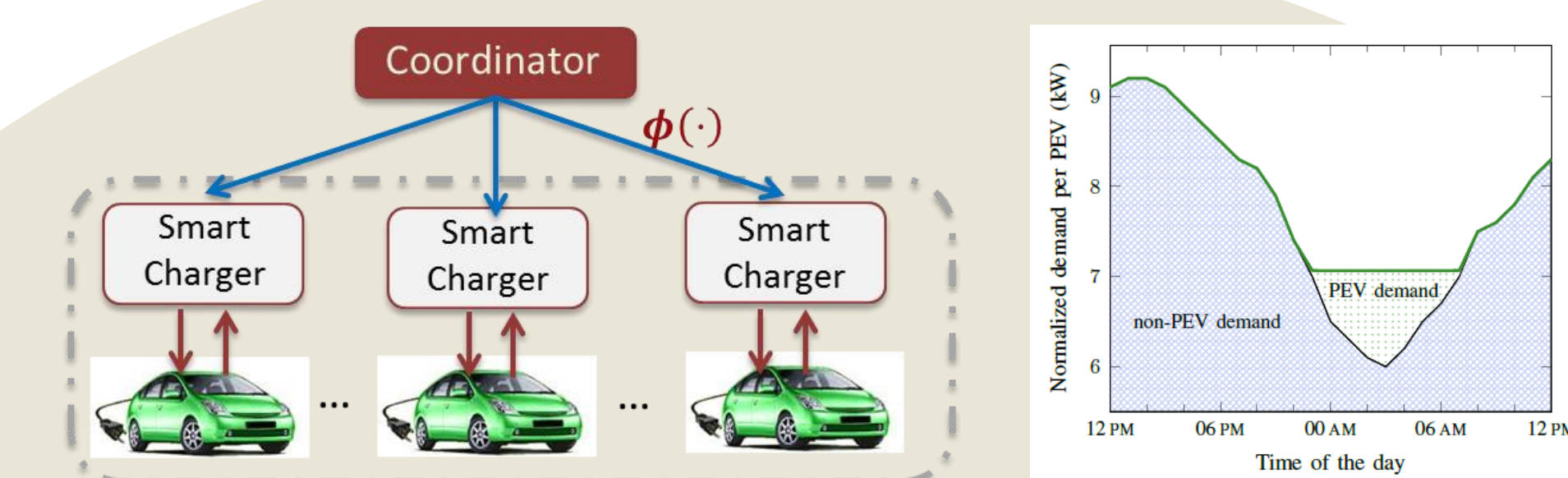
## Solution: Proposed Two Paradigms for Hierarchical Population Control

**Paradigm I: Hierarchical population control with nonstrategic agents (HPCN)**  
Predefined local control law with user specified parameters



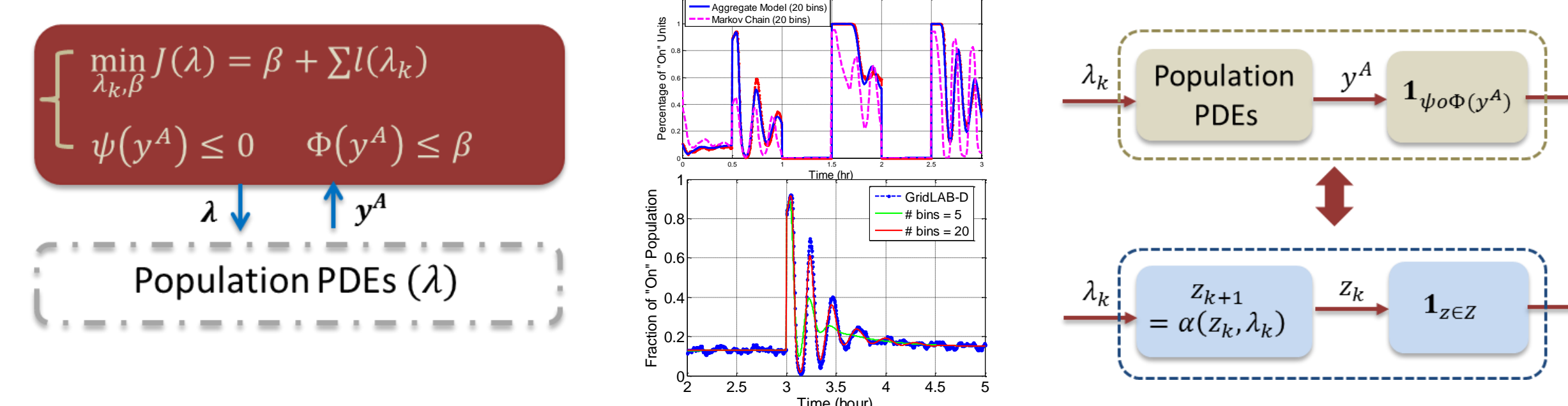
- Simple agent dynamics and local response logics
- But complex population dynamics

**Paradigm II: Hierarchical population control with strategic agents (HPCS)**  
Agents strategically optimizes utility functions coupled through coordinator's rule



- Electricity price depends on total demand
- The coupling price induces a game among all strategic agents
- Most works focus on analysis of the mean field equilibrium for a given game without evaluating the performance of equilibrium

## Current Results and Future Work



- Developed a unified hybrid system model for individual agent dynamics under local response rules
- Developed a stochastic hybrid system model to capture population dynamics
- This project will further study general HPCN Problem: **Optimization problem with PDE Constraints**
  - Coupled PDEs & complex boundary conditions
  - Develop novel abstraction based framework

## Scientific Impact

- Advance scientific and engineering foundations for large scale CPS
- Abstraction for hybrid systems, stochastic hybrid systems theory, mean-field game theory, mechanism design, Stackelberg games

## Current Results and Future Work

**Our contribution:** Identified an important class of mean-field games for which

- mean field equilibrium is socially optimal
- social optimal solution is  $\epsilon$ -Nash
- Computing mean-field Nash is a convex optimization
- This project will further study general HPCS Problem: **Uniform price mechanism design for large population**
  - Uniform price mechanism design for a large population of *dynamic* agents via  $\epsilon$ -Nash equilibrium
  - Theoretically sound + numerically tractable

## Broader Impact:

- Provide design principles and algorithms for real world CPS
- Smart power grid: DER coordination for renewable integration (collaboration with Pacific Northwest National Laboratory)
- Intelligent transportation (collaboration with the Smart City initiative at Columbus)