CIF: Small: Best wiretap codes for real-world informationtheoretic security



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https://icelab.byu.edu/research-projects



Decoder $\stackrel{\hat{M}}{\longrightarrow}$ Bob Alice $\stackrel{M}{\longrightarrow}$ Encoder Main Channel Eavesdropper's Channel

Figure 1: Wiretap channel model.

Coset M' = 0 M' = 1M' = 2 M' = 3 $\overline{M=0}$ 0011 1101 1110 0000 \mathcal{C}_1 M=11000 1011 0101 0110 \mathcal{C}_2 M=21001 1010 0100 0111 M=30001 0010 1100 1111

Table 1: Code table for n = 4, n - k = 2 secrecy code.

Problem Description:

 This project seeks to find/design best wiretap codes that provide reliable communications between Alice and Bob, and keep secrets as measured by the equivocation

$$\mathbb{H}(M|Z^n) = \sum_{z^n \in \mathcal{Z}^n} p(z^n) \mathbb{H}(M|Z^n = z^n) \cdot$$

- Encoder function: $x = \begin{bmatrix} m' & m \end{bmatrix} \begin{bmatrix} G \\ G' \end{bmatrix}$. Decoder: recover the message.

· Equivocation is given by

$$\begin{split} \mathbb{H}(M|Z^n) &= \sum_{z^n \in \mathcal{Z}^n} p(z^n) \mathbb{H}(M|Z^n = z^n) \\ &= \sum_{z^n \in \mathcal{Z}^n} p(z^n) \left[\mathbb{H}(M) - \mu_z + \operatorname{rank}(G_{\mu_z}) \right] \\ &= \sum_{\mu = 0}^n \sum_{i^n \in \mathcal{I}_\mu} (1 - \varepsilon)^\mu \varepsilon^{n - \mu} \left[k - \mu + \operatorname{rank}(G_{\mu_i}) \right]. \end{split}$$

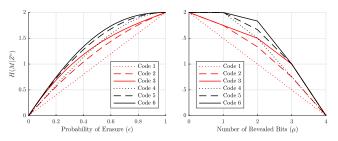
Thus, we are looking for encoders that maximize

$$\sum_{i\in\mathcal{I}_\mu}\operatorname{rank}(G_{\mu_i})$$

for all μ .

Approach:

- Identify/prove characteristics of best codes.
- Classify families of codes.
- Improve code design to enhance security in wireless communication systems.
- Start with simple channel models and progress to more complex/real-world cases.



Broader Impact (technical):

- Wiretap coding could enhance secrecy in new telecommunications standards.
- Need to understand finite blocklength code limitations and benefits.
- Knowledge of best codes informs engineering design.

Broader impact (outreach):

- **BYU IMMERSE**
- Women in Engineering at BYU (WE@BYU)
- · BYU Engineering Together (BE Together)
- Subgroup approach to including undergrads in information-theoretic research (REU).

