CPS: Breakthrough: A Dynamic Optimization Framework For Connected Automated Vehicles In Urban Environments

CENTER FOR INFORMATION & SYSTEMS ENGINEERING

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Traffic Assignment Problems

User-Centric. Self-interested optimal traffic flows as a variational inequality (VI) problem:

$$\mathbf{t}(\mathbf{x}^*)'(\mathbf{x}-\mathbf{x}^*) \geq \mathbf{0} \quad \forall \mathbf{x} \in \mathcal{F} \qquad (1)$$

Travel Time function; Equilibrium Traffic Flows; Set of Feasible Flow Vectors

System-Centric. Socially optimum traffic flows.

$$\min_{\mathbf{x}\in\mathcal{F}} \sum_{a\in\mathcal{A}} x_a t(x_a) \qquad (2)$$

Inverse Problem. Find travel latency cost functions given data flows.

$$\min_{\mathbf{t},\boldsymbol{\epsilon}} \ \|\boldsymbol{\epsilon}\| \qquad (3)$$

$$\text{s.t.} \quad \mathbf{t}(\mathbf{x}^{(k)})'(\mathbf{x}-\mathbf{x}^{(k)}) \geq -\epsilon_k, \quad \forall \mathbf{x}\in\mathcal{F}^{(k)}, k\in[\![\mathcal{K}]\!],$$

$$\epsilon_k > 0, \quad \forall k \in [\![\mathcal{K}]\!],$$

Mobility-on-Demand Optimization

System-Centric Routing and Rebalancing. Jointly select routes and rebalancing policies of an intermodal (vehicle, subway, walk, micromobility) Autonomous Mobility-on-Demand (AMoD) service.

$$\min_{\mathbf{x}^{\mathcal{W}},\mathbf{x}^r} \underbrace{\sum_{(i,j)\in\mathcal{A}} t_{ij}(x_{ij})x_{ij}^u}_{\text{User travel time}} + \underbrace{\sum_{(i,j)\in\mathcal{A}_R} c_{ij}x_{ij}^r}_{\text{Rebalancing cost}} \qquad (4)$$

s.t.
1. Demand is met
2. All nodes have available vehicles (network is balanced)

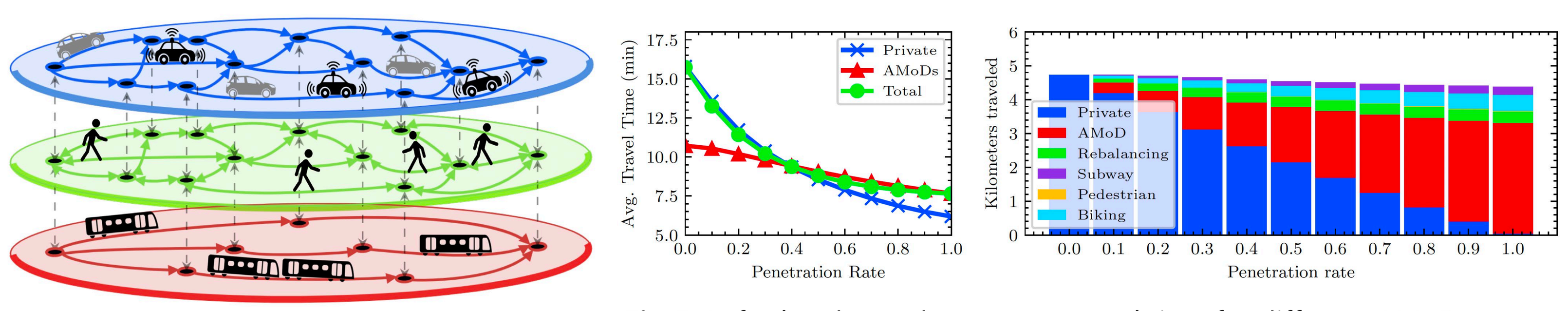


Fig 5. AMoD network consisting of three layers. Dashed arrows represent switching arcs.

Fig 5. Left plot shows the average travel time for different AMoD penetration rates in NYC transportation network. Improvements are up to 50% in travel times. Right plot depicts the miles traveled per mode of transportation for each AMoD penetration rate.

Pricing and Rebalancing. Select surge prices and rebalancing policies jointly to maximize AMoD profit or social welfare.

$$\max_{\mathbf{u},\mathbf{r}} \sum_{ij} \underbrace{\lambda_{ij}(u_{ij})u_{ij}p_{ij}^0}_{\text{Revenue}} - \underbrace{c_{ij}^o\lambda_{ij}(u_{ij})}_{\text{Operational cost}} - \underbrace{c^r(r_{ij}T_{ij})}_{\text{Rebalancing cost}} \qquad (5)$$

Willingness-to-pay function

s.t. All nodes have available vehicles (network is balanced)

Policy	AM	MD	PM	NT
Only pricing	29.8%	8.8%	6.6%	26.0%
Only rebalancing	33.3%	28.7%	29.2%	40.7%
Rebalancing, then pricing	13.7%	9.4%	10.9%	15.8%
Jointly but a single surge price per origin	5.3%	5.3%	5.1%	7.0%

Table 1. Relative deviation in percentage of each policy compared to the joint pricing (considering origin and destination) and rebalancing policy obtained when solving (5) for different time slots.

Datasets

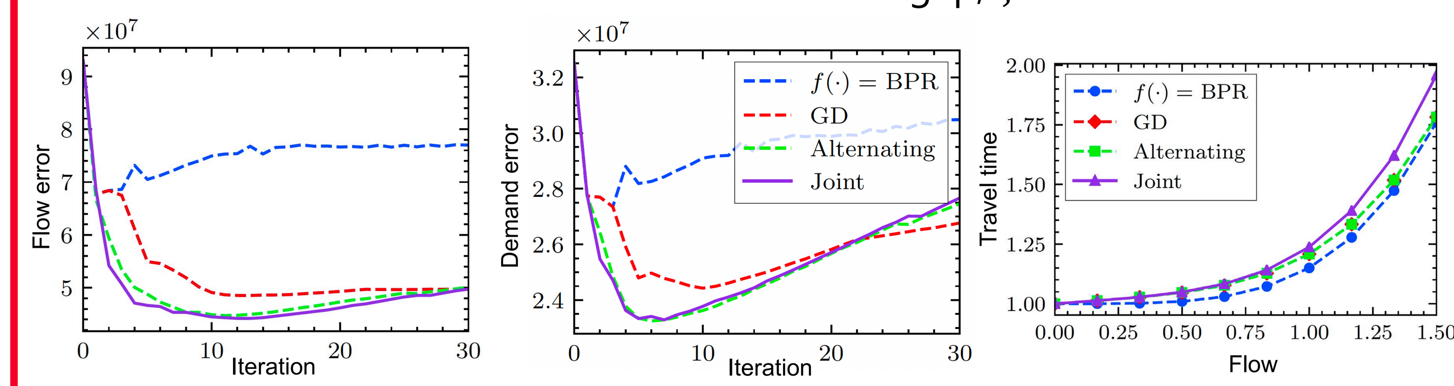
- **Eastern Massachusetts Area (EMA):**
  - **Roads and topology**: Provided by the Boston Region Metropolitan Planning Organization (MPO). Includes flow capacity (veh/hr) for more than 100,000 road segments in Eastern Massachusetts
  - **Speed**: Provided by the MPO, Includes avg. speeds (mph) per-minute for major roadways and arterial streets in Eastern Massachusetts for 2012 and 2015
- **New York City (NYC)**
  - **Speed:** Uber Movement
  - **Demand:** Taxi data records
  - **Roads and Topology:** OpenStreetMaps (OSM) database
- **Here Maps BU Platform**
  - **Speed:** Avg. speed and free flow data (5 min) on any city on the US and many around the world
  - **Roads and Topology:** Based on Here Maps API

Joint Estimation of OD Demand and t(·)

Estimate the parameters of the Traffic Assignment Problem from flow data to enable further optimization and control. Modeled as a Bi-level Program. Solved using a *feasible-direction* iterative algorithm.

$$\min_{\boldsymbol{\beta},\mathbf{y},\boldsymbol{\epsilon},\mathbf{g},\boldsymbol{\nu},\xi} \sum_{\tilde{u}=1}^{|\tilde{\mathcal{U}}|}\sum_{i=1}^{|\tilde{\mathcal{A}}|}(x_{i\tilde{u}}(\boldsymbol{\beta},\mathbf{g})-x_{i\tilde{u}}^*)^2+\lambda\xi \qquad (7)$$

s.t
1. Primal Inverse feasibility of (3)
2. Dual Inverse feasibility
3. Primal-Dual relaxed gap, ξ



(a). Flow error (b). Demand error (c). Travel Latency Cost Function

Fig 5. Estimation results for a model validation example on the EMA network.

Transportation Networks for Analysis

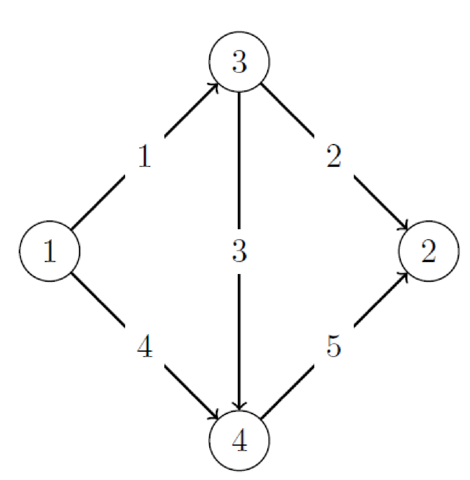


Fig 6. Braess network, for validation and interpretability purposes

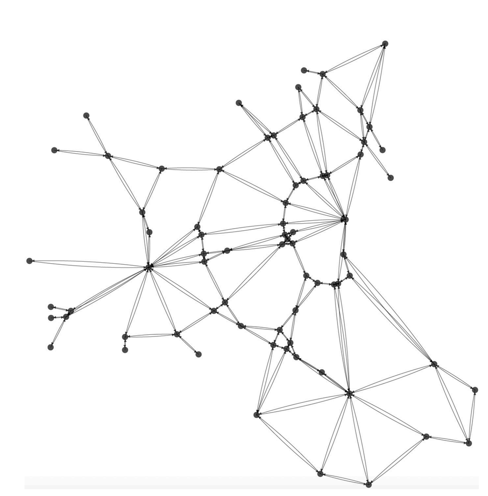


Fig 7. EMA network, for emulating highway conditions

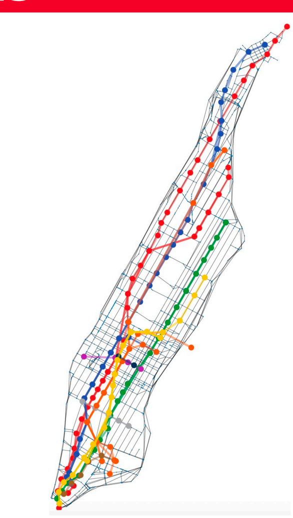


Fig 8. NYC network, for emulating urban conditions

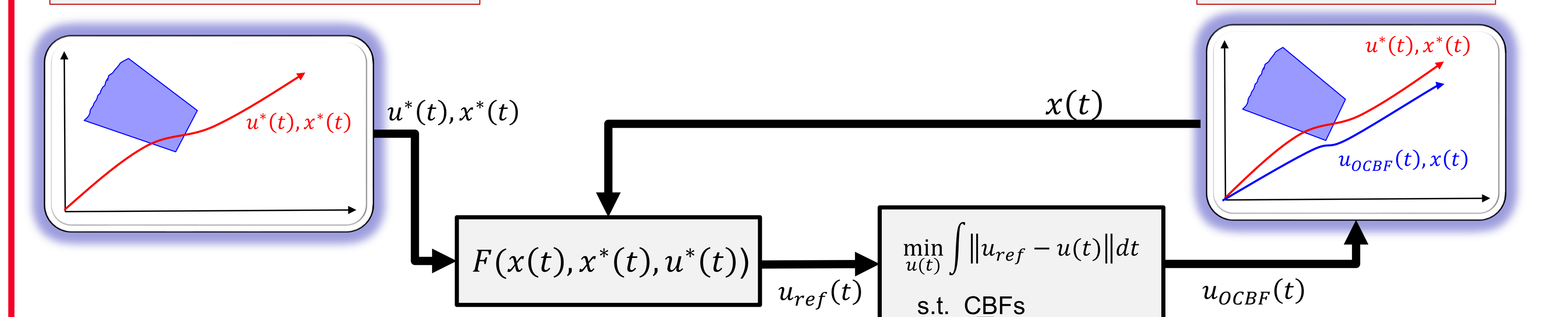
OCBF control: Optimal Control (OC) + Control Barrier Functions (CBF)

Optimal Control problem: $J(\boldsymbol{u}(t)) = \int_0^T C(\|\boldsymbol{u}(t)\|)dt + p\|\boldsymbol{x}(T)-\boldsymbol{K}\|^2$

Subject to: $\dot{\boldsymbol{x}} = f(\boldsymbol{x}) + g(\boldsymbol{x})\boldsymbol{u} + \boldsymbol{w}$ $\quad \boldsymbol{u}_{min} \leq \boldsymbol{u} \leq \boldsymbol{u}_{max}$ $\quad \boldsymbol{x}_{\min} \leq \boldsymbol{x}(t) \leq \boldsymbol{x}_{\max}$

Hard SAFETY constraints: $b_j(\boldsymbol{x}(t)) \geq 0$

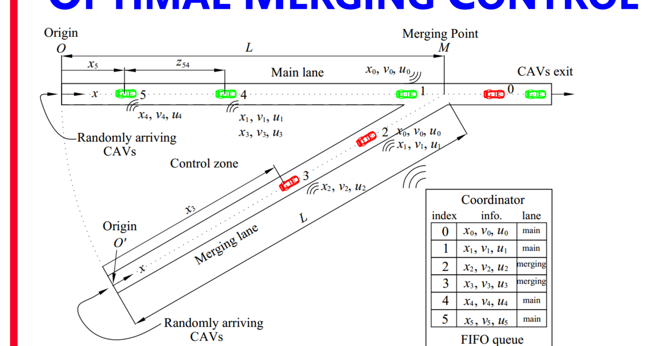
GET BEST POSSIBLE OC SOLUTION WITHIN REAL-TIME CONSTRAINTS → GET FEEDBACK → OPTIMALLY TRACK WITH GUARANTEED SAFETY

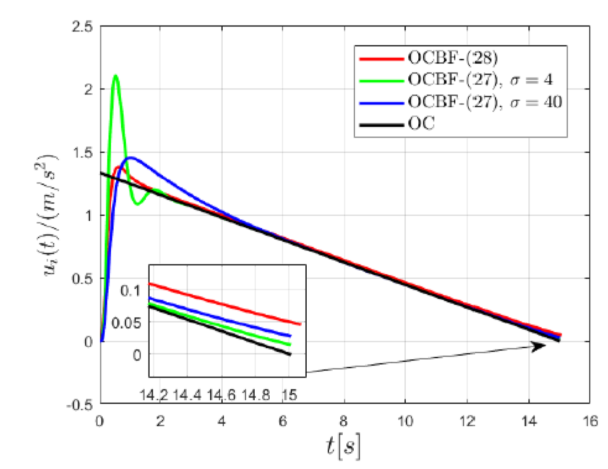


Simplest case: $u_{ref}(t) = u^*(t)$

CBF constraints for State/Safety constraints
$$L_f b(\boldsymbol{x}) + L_g b(\boldsymbol{x})\boldsymbol{u} + \alpha(b(\boldsymbol{x})) \geq 0$$
CLF constraint for terminal state cost:
$$L_f V(\boldsymbol{x}) + L_g V(\boldsymbol{x})\boldsymbol{u} + \varepsilon V(\boldsymbol{x}) \leq \delta_i(t)$$

Optimal Merging Control





$$\min_{u_i(t)} \ \beta(t_i^m - t_i^0) + \int_{t_i^0}^{t_i^m} \frac{1}{2}u_i^2(t)dt$$

1. CAV linear dynamics,
2. Speed/Acceleration constraints,
3. Safety: $z_{i,i_p}(t) \geq \varphi v_i(t) + \delta, \forall t \in [t_i^0, t_i^m]$

$x_i(t_i^0) = 0, \quad x_i(t_i^m) = L, \quad$ given $t_i^0, \ v_i^0$

Table 3: Comparison (data in average) of OC, CBF and OCBF (with noise)

Method	α	Noi.	Time(s)	$\frac{1}{2}u_i^2(t)$	Obj.
CBF	N/A	no	14.6978	26.9178	N/A
OC	0.01	no	**25.4291**	**0.1725**	**2.1288**
OCBF		no	25.6879	1.0582	3.0256
		yes	25.7494	2.2373	4.1976
OC	0.25	no	**17.0472**	**4.9069**	**36.4909**
OCBF		no	17.1176	5.5569	37.1139
		yes	17.1396	6.8959	38.1605
OC	0.40	no	**15.1713**	**10.6508**	**53.1120**
OCBF		no	15.2286	11.3629	53.7157
		yes	15.2527	12.7671	54.6325
OC	0.60	no	**13.1035**	**24.4079**	**70.2922**
OCBF		no	13.1560	25.2468	70.8720
		yes	13.1692	26.6534	71.4938

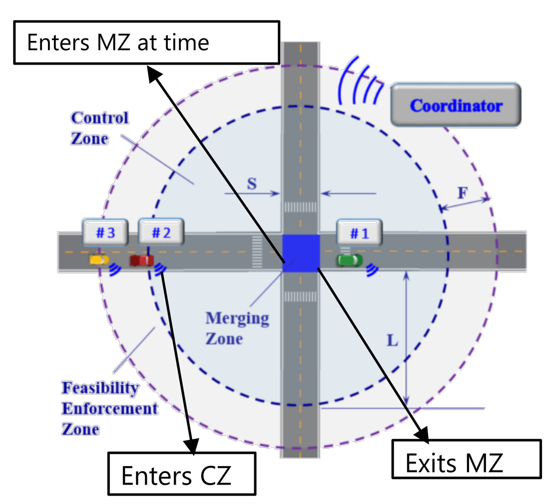
Optimal Control solution is analytically tractable and guarantees safety constraints. Unconstrained: <<1sec. Constrained: 1-30sec

OCBF solution (<<1 sec) nearly recovers OC solution and is analytically tractable, guarantees safety constraints, allows noise

Joint Energy and Travel Time Optimal Control + Control Barrier Functions Framework

Optimal Signal-Free Intersection Control

A decentralized framework to optimally control CAVs at signal-free intersections including comfort-constrained turns and safety guarantees.



CAVs crossing urban intersection

CAV dynamics
Constraints

$$\dot{p}_i = v_i(t), \ p_i(t_i^0) = 0; \ \dot{v}_i = u_i(t), \ v_i(t_i^0) \text{ given}$$
$$u_{i,min} \leq u_i(t) \leq u_{i,max}, \quad \text{and}$$
$$0 \leq v_{min} \leq v_i(t) \leq v_{max}, \quad \forall t \in [t_i^0, t_i^f]$$

Four subsets (including turns)

- $\mathcal{E}_i(t)$: rear-end collision at the end of MZ
- $\mathcal{S}_i(t)$: rear-end collision at the beginning of MZ
- $\mathcal{L}_i(t)$: lateral collision in MZ
- $\mathcal{O}_i(t)$: no collision in MZ

Read-end safety: $s_i(t) = p_k(t) - p_i(t) \geq \delta, \quad \forall t \in [t_i^0, t_i^f]$

Lateral safety: $\Gamma_i \cap \Gamma_j = \varnothing, \ \forall t \in [t_i^m, t_i^f], \ j \in \mathcal{L}_i(t)$ where $\Gamma_i \triangleq \{t \mid t \in [t_i^m, t_i^f]\}$

Order constraint: $t_i^f \geq t_{i-1}^f$ (can be relaxed by dynamic resequencing)

Joint Optimization Problem

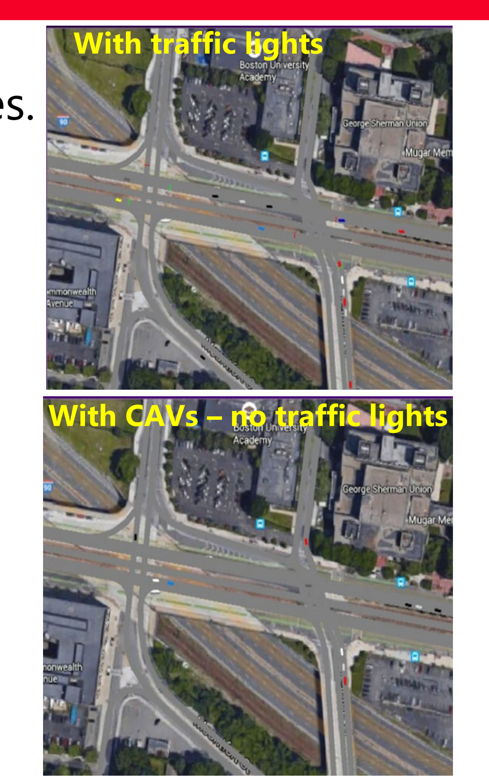
Control Zone
$$\min_{u_i \in U_i} \int_{t_i^0}^{t_i^m} [\gamma_1 + \gamma_2 u_i^2(t)] \ dt$$
Travel time; Energy consumption

Merging Zone
$$\min_{J_i} \frac{1}{2}\int_{t_i^m}^{t_i^f} [\rho_1 u_i^2(t) + \rho_2 J_i^2(t)]dt$$
Passenger discomfort

Constrained Optimal Control Analysis

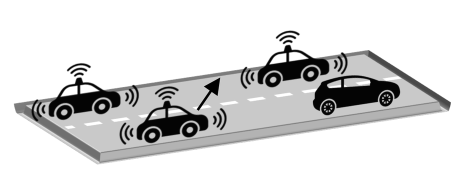
$$u_i^*(t) = \begin{cases} a_i t + b_i & t \in [t_i^0, \tau] \\ u_k^*(t) & t \in (\tau, t_i^m] \end{cases}$$

Optimal solution is analytically tractable



Simulation of real intersection in Boston

Lane Change Maneuvers For Cooperating CAVs



The basic lane change maneuver process.

$$\min_{u_i(t)} \int_0^{t_f} [w_t + [w_{1,u}u_1^2(t) + w_{2,u}u_2^2(t) + w_{C,u}u_C^2(t)]]dt \qquad (12)$$

1. CAVs dynamics
2. Speed/Acceleration limits
3. Safety Constraints

Optimal solution is analytically tractable and guarantees safety constraints

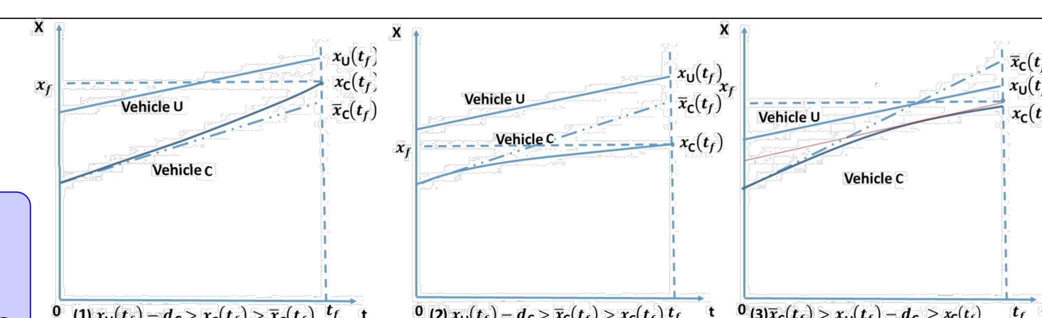


Fig 13. Three feasible cases for the optimal maneuver of vehicle *C*

Case	Energy		
	CAVs	Human	Improvement
1	6.8	16.4	59%
2	23.0	46.0	50%
3	59.5	103.5	43%

Cases:
1. *C* accelerates to merge. Safety constraint inactive
2. *C* deaccelerates to merge. Safety constraint inactive
3. *C* accelerates to merge. Safety constraint active

Selected Publications

- Cassandras C.G., "Automating Mobility in Smart Cities", *Annual Reviews in Control*, Vol. 44, pp. 1-8, 2017.
- Zhang Y., Cassandras C.G., "Decentralized Optimal Control of Connected Automated Vehicles at Signal-Free Intersections Including Comfort-Constrained Turns and Safety Guarantees", *Automatica*, Vol. 109, 2019.
- Wollenstein-Betech S., Sun C., Zhang J., and Paschalidis I. Ch., "Joint Estimation of OD Demands and Cost Functions in Transportation Networks from Data". *Proc 58th IEEE Conference on Decision and Control*, 2019.
- Wollenstein-Betech S., Paschalidis I. Ch, and Cassandras C.G., "Joint pricing and rebalancing of autonomous mobility-on-demand systems", *Proc 59th IEEE Conference on Decision and Control*, 2020.
- Wollenstein-Betech S., et.al.,, "Congestion-aware routing and rebalancing of autonomous mobility-on-demand systems in mixed traffic", *Proc IEEE Intelligent Transportation Systems Conference 2020.*
- Chen R., Cassandras C.G., and Tahmasbi-Sarvestani A., "Cooperative Time and Energy-Optimal Lane Change Maneuvers for Connected Automated Vehicles", *IEEE Transactions on Intelligent Transportation Systems*, 2020.
- Xiao W and Cassandras C.G., "Decentralized optimal merging control for Connected and Automated Vehicles with safety constraint guarantees", *Automatica*, Vol. 123, 2021.
- Xiao W., Belta C., and Cassandras C.G., "Bridging the Gap between Optimal Trajectory Planning and Safety-Critical Control with Applications to Autonomous Vehicles", *Automatica*, Vol. 129, 2021.