CNS-1329619: Robust team-triggered coordination for real-time control of networked cyber-physical systems

Event-Triggered Interactive Gradient Descent for Real-Time Multiobjective Optimization

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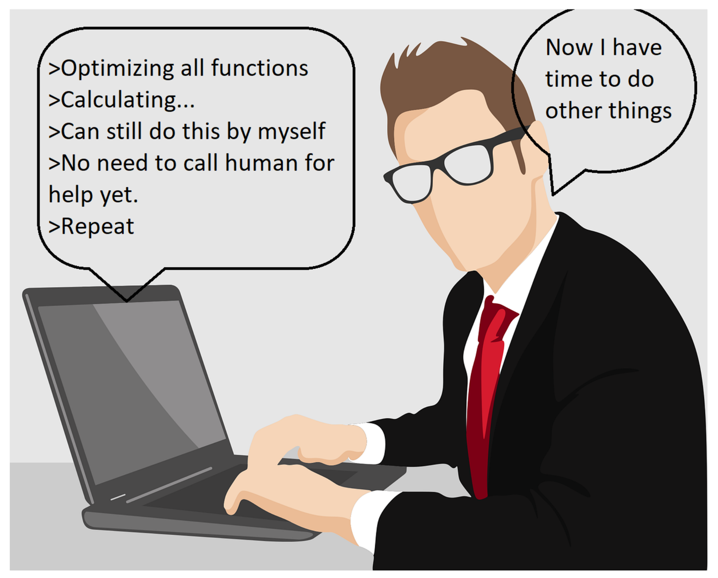
Overview

- ***Scenario:*** human and robot interact to solve multi-objective optimization.
- ***Overarching project goal:*** Enable multi-objective continuous gradient descent with opportunistic queries for human inputs.
- In real-time applications, humans have a ***response time*** and need a ***rest time***.



Contributions

- ***Design*** of event-triggering condition to query for human inputs for with accommodations to the rest time and the response time.
- ***Analysis*** of the proposed event-triggered human input update policy:
  - ***Global asymptotic stability*** of the optimizer.
  - ***Exponential stability*** for strongly convex implicit human cost function
  - Uniform lower bound on the inter-event times (***no Zeno*** behavior)
  - ***Explicit expressions*** for design variables, convergence rate, and inter-event times.
- Characterization of the ***trade-off*** between human involvement, convergence speed, and robustness to human error.

Opportunistic Human Queries

Consider the continuous human-robot multi-objective gradient descent dynamics of the human implicit cost function $c \circ f$,

$$\dot{x}(t) = \underbrace{\nabla c(f(x(t)))}_{\text{human}} \underbrace{J_f(x(t))}_{\text{robot}}.$$

Problem: Can we remove the continuity in the human part to get the following

$$\dot{x}(t) = \underbrace{\nabla c(f(x(t_k)))}_{\text{human}} \underbrace{J_f(x(t))}_{\text{robot}}, \quad t \in [t_k + D_k, t_{k+1} + D_{k+1})$$

with t_{k+1} to be determined iteratively with a designed condition?

Assumptions:

- There exists a human implicit cost function $c \circ f$, which is continuously differentiable, strictly convex, time invariant and radially unbounded.
- The human portion of the gradient, $(\nabla c) \circ f$, is locally Lipschitz with a constant L_c, which is assumed to be known.
- There is an upper bound to the response time $D > D_k$ for all k.

Design Objective

Design $\{t_k\}_{k=0}^{\infty}$ such that

1. ***Global asymptotic stability:*** the states converge to the optimizer.
2. ***Autonomous operation:*** each t_{k+1} can be determined without the human during the time period $[t_k, t_{k+1})$.
3. ***No Zeno behavior:*** $\lim_{k \to \infty} t_k = \infty$.

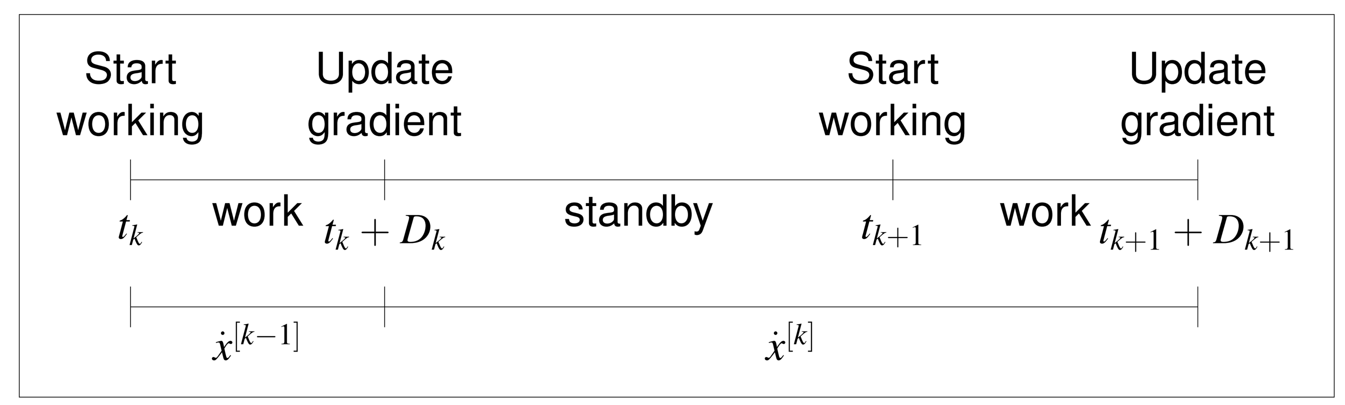
Event-Triggered Design

We propose that the human only provides her gradient information when asked by the robot, and that the robot determine the time, t_{k+1}, to query the human for new information as follows,

$$t_{k+1} = \min_{t} \left\{ t \geq t_k \mid \|x(t) - x(t_k)\| = \sigma \frac{\|\nabla c(f(x(t_k))) J_f(x(t))\|}{L_c J_{\max}} \right\}.$$

with $\sigma \in (0,1)$ as a design parameter.

Response Time



The human, when queried, needs some time D_k before she can provide the gradient information to the robot.

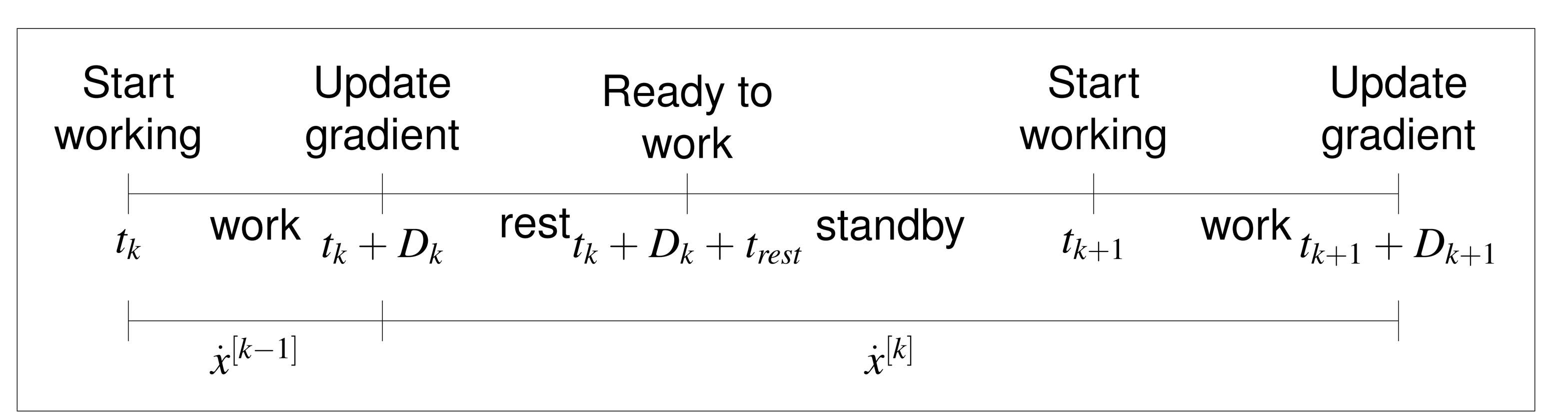
"I'll Get Back to You" Human

To accommodate for the maximum ***response time*** D, the design parameter σ must satisfy

$$\frac{L_c J_{\max}}{\beta} \left(\frac{1+\gamma}{1-\gamma} \right)^2 (e^{\beta D} - 1) < \sigma \leq \frac{L_c J_{\max}}{\beta} (e^{\beta(\tau_\gamma - D)} - 1)$$

where $\gamma \in (0,1)$ is a parameter that will affect the convergence rate.

Response and Rest Time



The human will take some time, t_{rest}, to "rest" after a supervision period, with a length of D_k, with the robot. We propose a more restrictive bound on the design parameter.

"I'll Need a Break After Work" Human

In addition to accommodating for ***response time***, the triggering event will not occur before the ***rest time*** t_{rest} has elapsed after the human has just responded with a gradient if the design parameter satisfies

$$\frac{L_c J_{\max}}{\beta} \left(\left(\frac{1+\gamma}{1-\gamma} \right)^2 (e^{\beta D} - 1) e^{\beta t_{rest}} + (e^{\beta t_{rest}} - 1) \right) < \sigma \leq \frac{L_c J_{\max}}{\beta} (e^{\beta(\tau_\gamma - D)} - 1).$$

Analysis: Satisfaction of Design Goals

1. ***Global asymptotic stability:***

Lyapunov Stability

Let $V(x) = c \circ f(x)$ be the Lyapunov function. Then,

$$\frac{d}{dt} V(x(t)) \leq -\frac{1-\gamma}{(1+\gamma)^2} \|\nabla c(f(x(t))) J_f(x(t))\|^2 < 0.$$

Moreover, if $c \circ f$ is strongly convex with a parameter μ, the optimizer is **exponentially stable** with the following bound,

$$V(x(t)) \leq V(x_0) e^{-\frac{2\mu(1-\gamma)}{(1+\gamma)^2} t}.$$

2. ***Autonomous operation:***

Human independent triggering condition

The only human related term is $\nabla c(f(x(t_k)))$, which is known to the robot.

3. ***No Zeno behavior:***

Uniform Lower Bound for the Inter-Event Times

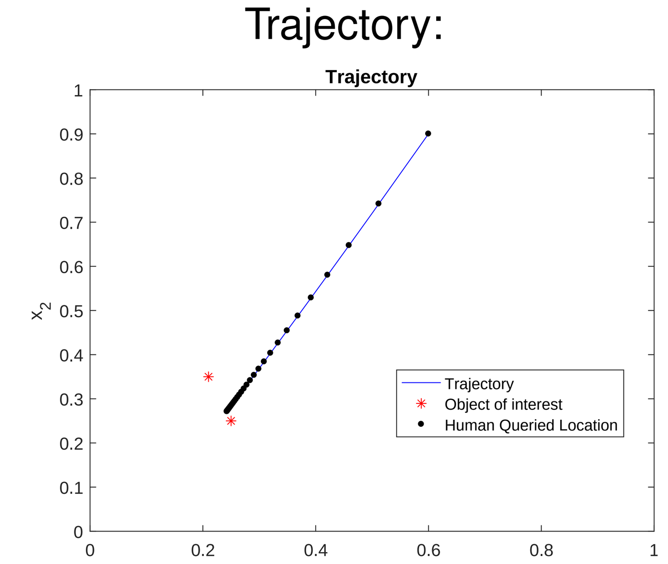
$\boldsymbol{t_{k+1} - t_k \geq \tau_\sigma}$ for all $k \in \mathbb{N} \cup \{0\}$ where τ_σ is a constant given by

$$\tau_\sigma = \frac{1}{\beta} \ln \left(\frac{1 + \beta \frac{\sigma}{L_c J_{\max}}}{1 + \left(\frac{1+\gamma}{1-\gamma} \right)^2 (e^{\beta D} - 1)} \right) + D$$

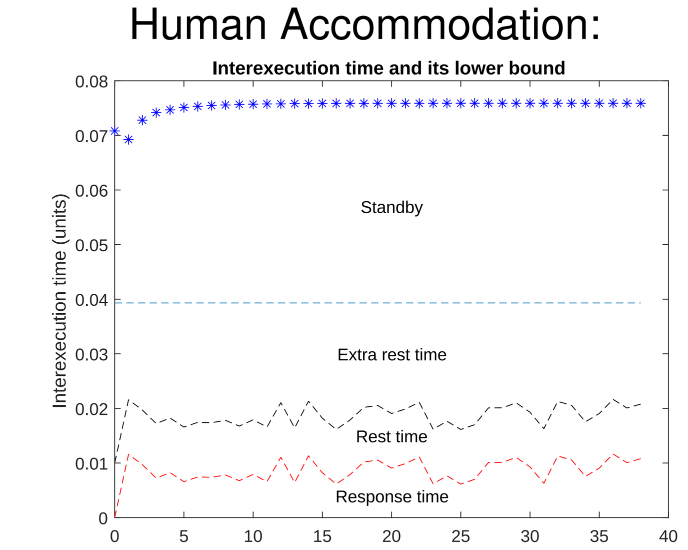
Simulation

A robot with objectives of getting close to two objects, consults human for help

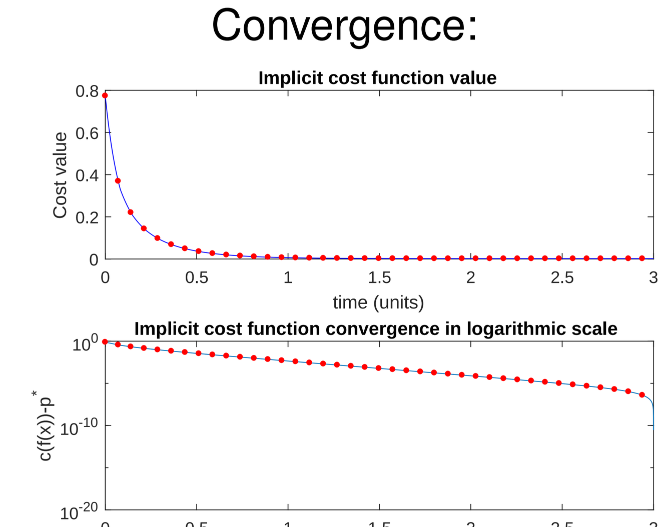
Trajectory:



Human Accommodation:



Convergence:



Trade-off: