

# Symbolic and Numerical Techniques for Verification and Synthesis of Cyber-Physical Systems



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This project is aimed at improving the scalability of verification and synthesis techniques applied in Cyber-Physical Systems. In the prior work, the PI proposed and improved the representation called **generalized star sets** that are useful in computing reachable sets not only Cyber-Physical Systems, but also for Neural Networks.

## Challenges in verification of Cyber-Physical Systems:

- Curse of dimensionality.
- Trajectories are nonlinear functions of states and involves matrix exponentials:  $e^{At} = I + \frac{At}{1!} + \frac{(At)^2}{2!} + \dots$
- Verification tools often ignore model uncertainties and return only one counter-example for system designer.

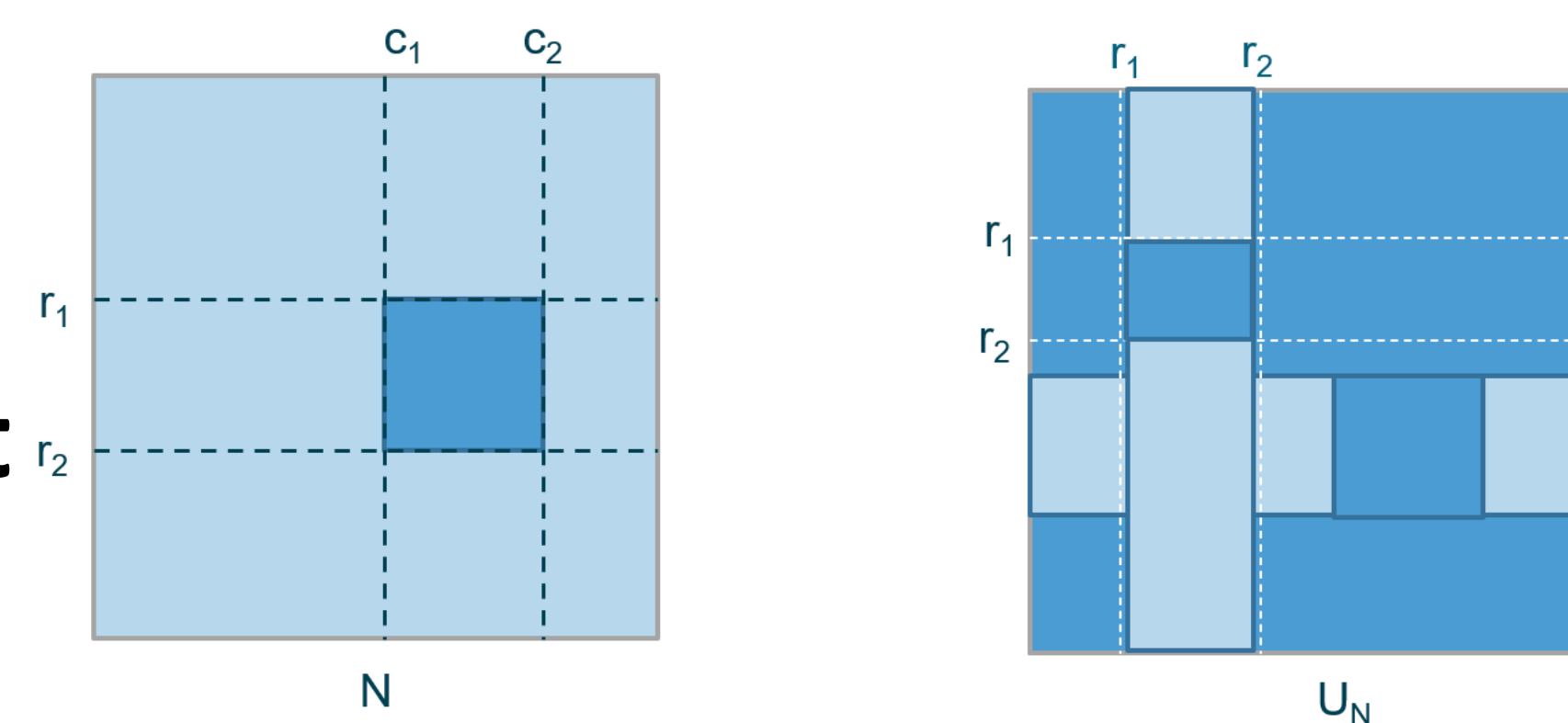
## Contributions to other Cyber-Physical Systems projects:

- The reachable set representation called **generalized star** proposed by the PI is also useful in analyzing safety of neural network guided control systems.
- The techniques for generating counter-examples can be extended to other disciplines like software and hardware verification.

**Technical contributions:** (over the last year): Robust Reachable Set and Counterexample Search.

Q: When can we compute efficient reachable sets efficiently?

A: If a structural property of the system holds.



Q: If property doesn't hold?

A: Compute **statistically robust reachable set**.

**Broader Impacts for Society:** These techniques help in designing and analyzing robust Cyber-Physical Systems

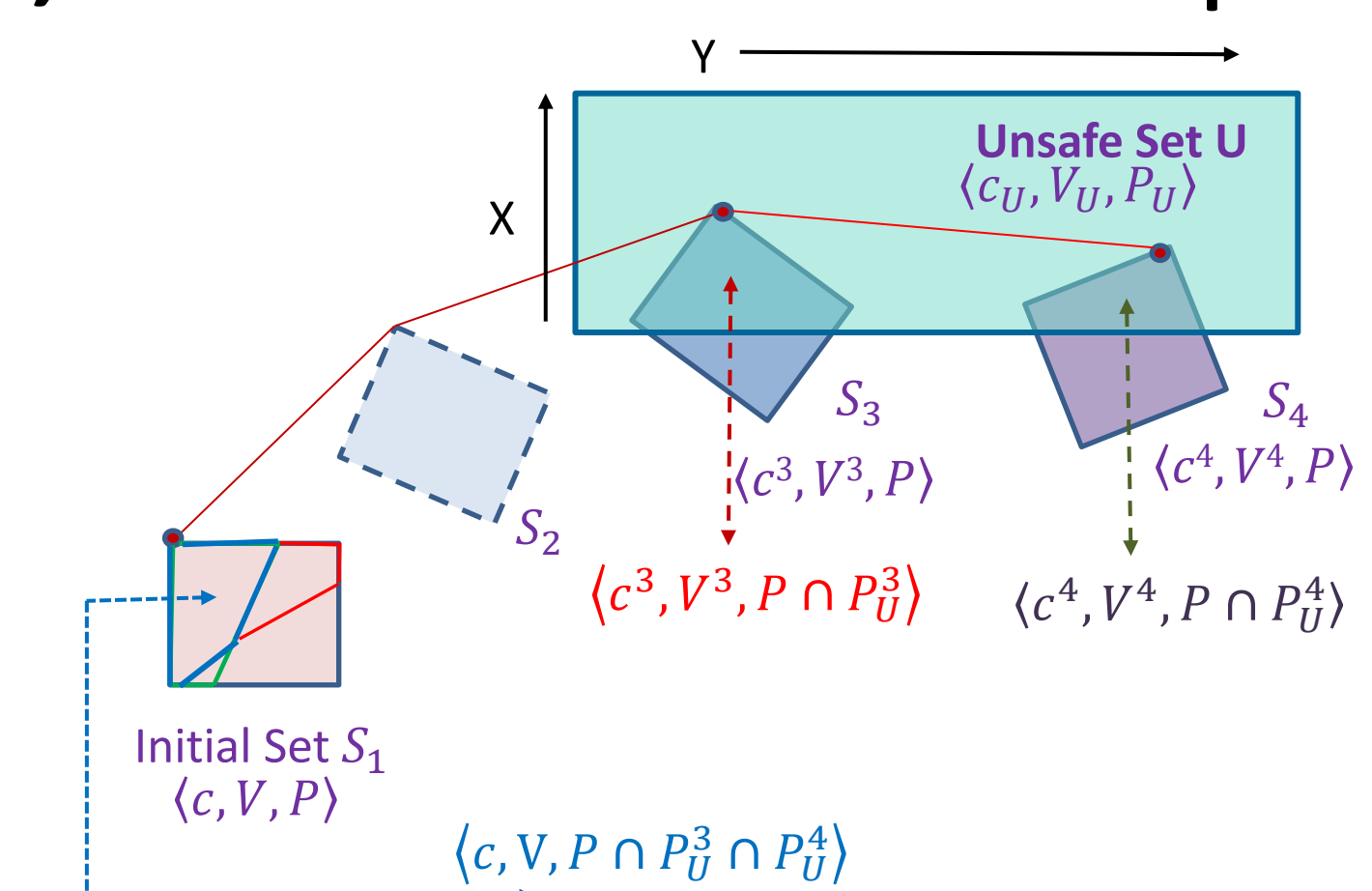
**Broader Impacts for Industry:** 1) Proved safety of satellite rendezvous systems. 2) Counter-examples help system designer diagnose the faults in CPS.

Q: What are the various types of counter-examples useful for control designers?

A: Taxonomy – **longest, deepest, robust** counter-examples.

Q: How to generate them?

A: Use generalized star representation, formulate it as MILP or SMT-Optimization.



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