CPS: Synergy: Collaborative Research:

Efficient Traffic Management: A Formal Methods Approach

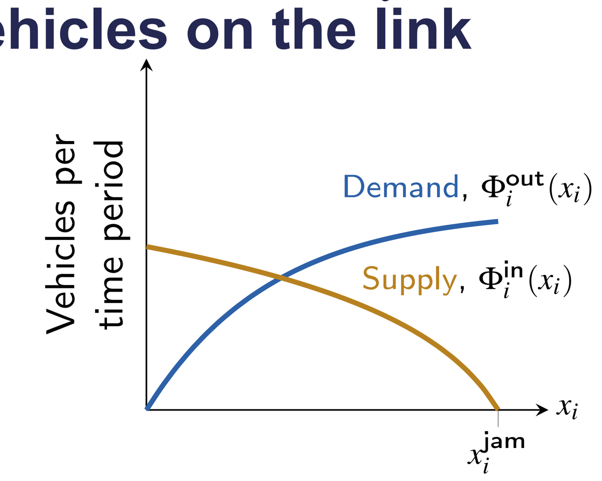
Murat Arcak (UC Berkeley), Calin Belta (Boston U.), Roberto Horowitz (UC Berkeley)

Overview

This project is bringing tools from formal methods to traffic management to meet control objectives expressed in temporal logic. This approach is being applied to signal timing and ramp metering strategies for signalized intersections and freeway traffic control. In addition to meeting temporal logic specifications we aim to incorporate optimality criteria, such as total travel time, throughput, and vehicle miles traveled.

Traffic Network Model

- For each link $\ell \in \mathcal{L}$, the state $x_\ell[t] \in [0, x_\ell^{\mathrm{cap}}]$ represents the number of vehicles on the link
- Each link has:
  - Demand $\Phi_\ell^{\mathrm{out}}(x_\ell)$ to move downstream
  - Supply $\Phi_\ell^{\mathrm{in}}(x_\ell)$ to accept upstream flow

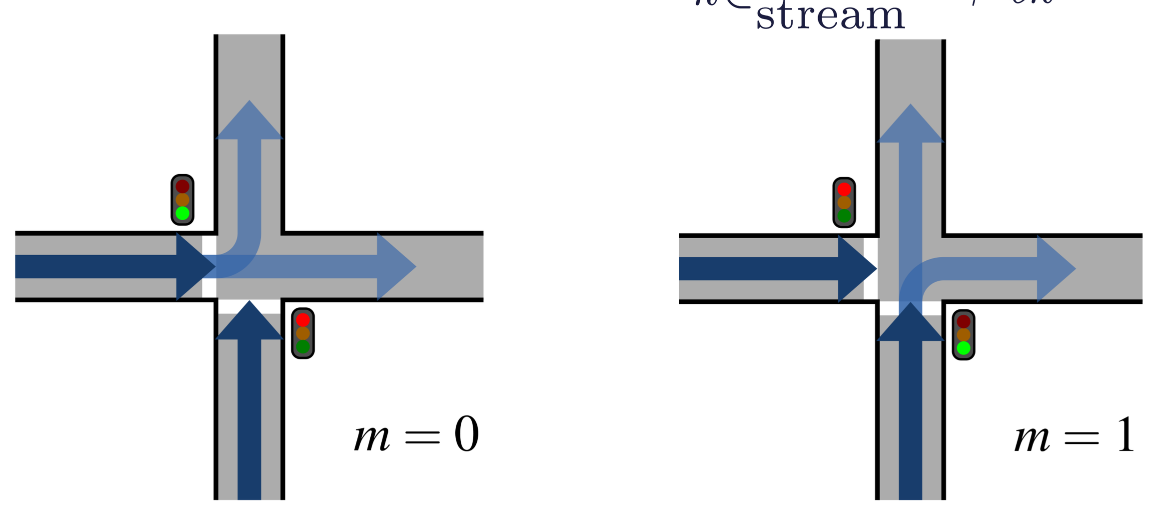


Dynamics:

$$x_\ell^+ = x_\ell + f_\ell^{\mathrm{in}}(x) - f_\ell^{\mathrm{out}}(x) =: F_\ell(x)$$

- Turn ratios $\beta_{\ell k}$ divide demand among downstream links and supply ratios $\alpha_{\ell k}$ divide supply among upstream links
- Signal variable $s_\ell \in \{0,1\}$ indicates if link ℓ is active

$$f_\ell^{\mathrm{out}}(x) = s_\ell \cdot \min\left\{\Phi_\ell^{\mathrm{out}}(x_\ell), \min_{k \in \text{Down-stream}} \left\{\frac{\alpha_{\ell k}}{\beta_{\ell k}} \Phi_k^{\mathrm{in}}(x_k)\right\}\right\}$$



Mixed Monotonicity

- Traffic networks are *mixed monotone* systems:

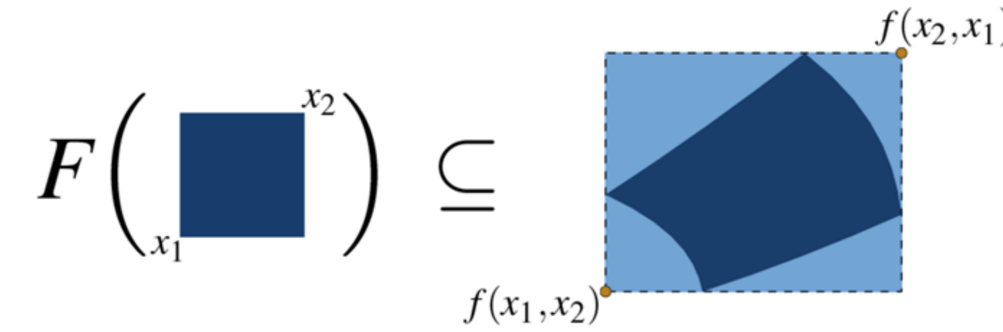
$$\exists \delta_{\ell k} \in \{-1, 1\} \quad \text{s.t.} \quad \delta_{\ell k} \frac{\partial F_\ell(x,d)}{\partial x_k} \geq 0 \quad \forall \ell, k$$

- Increasing and decreasing components
- Decomposition function $f(x, y, d)$
- Congestion causes non-monotone behavior

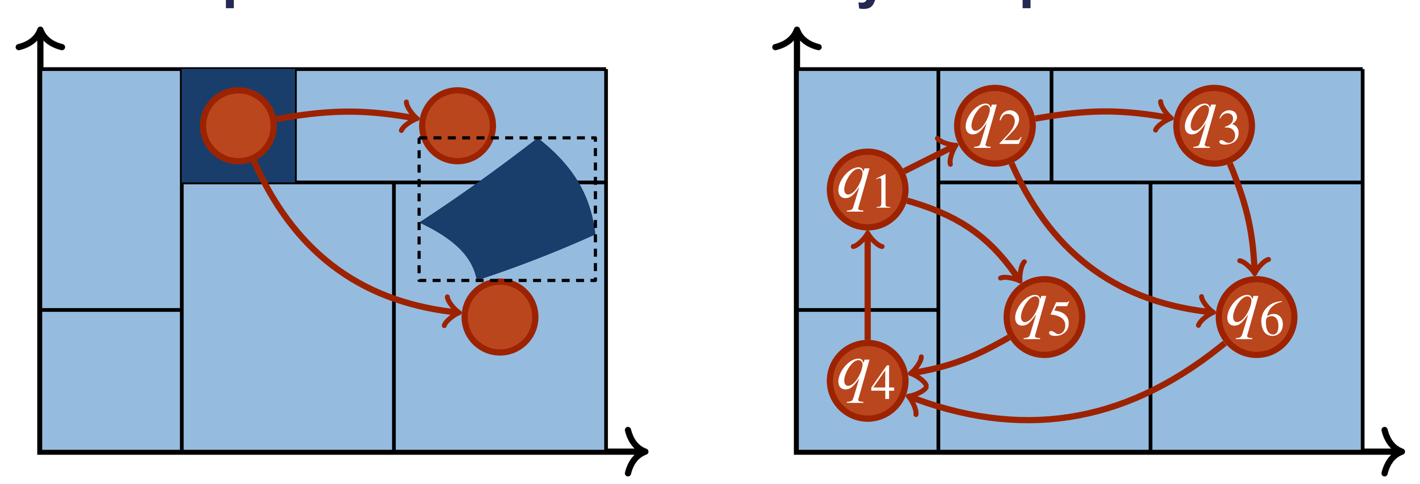
Reachable Set Over Approximations

- Mixed Monotonicity leads to efficient abstraction:

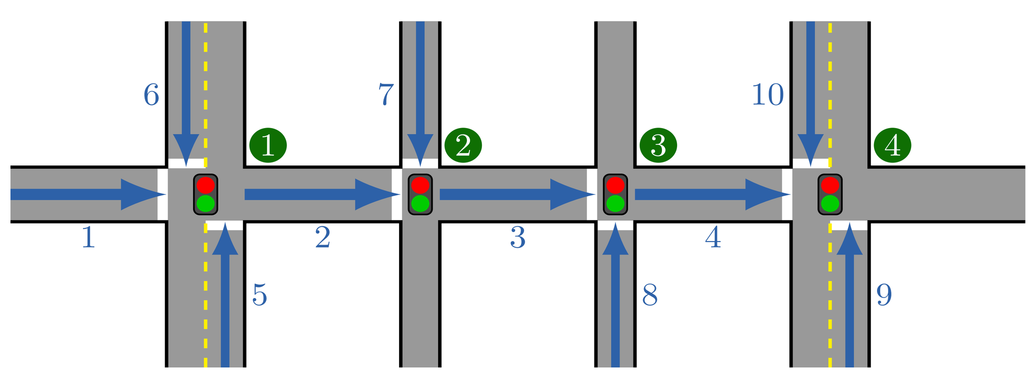
$$\left(\underline{x} \leq x \leq \overline{x}\right) \implies \left(f(\underline{x}, \overline{x}, d) \leq F(x, d) \leq f(\overline{x}, \underline{x}, d)\right)$$



- The one-step reachable set from a box of initial conditions is tightly over-approximated by computing the decomposition function at only two points

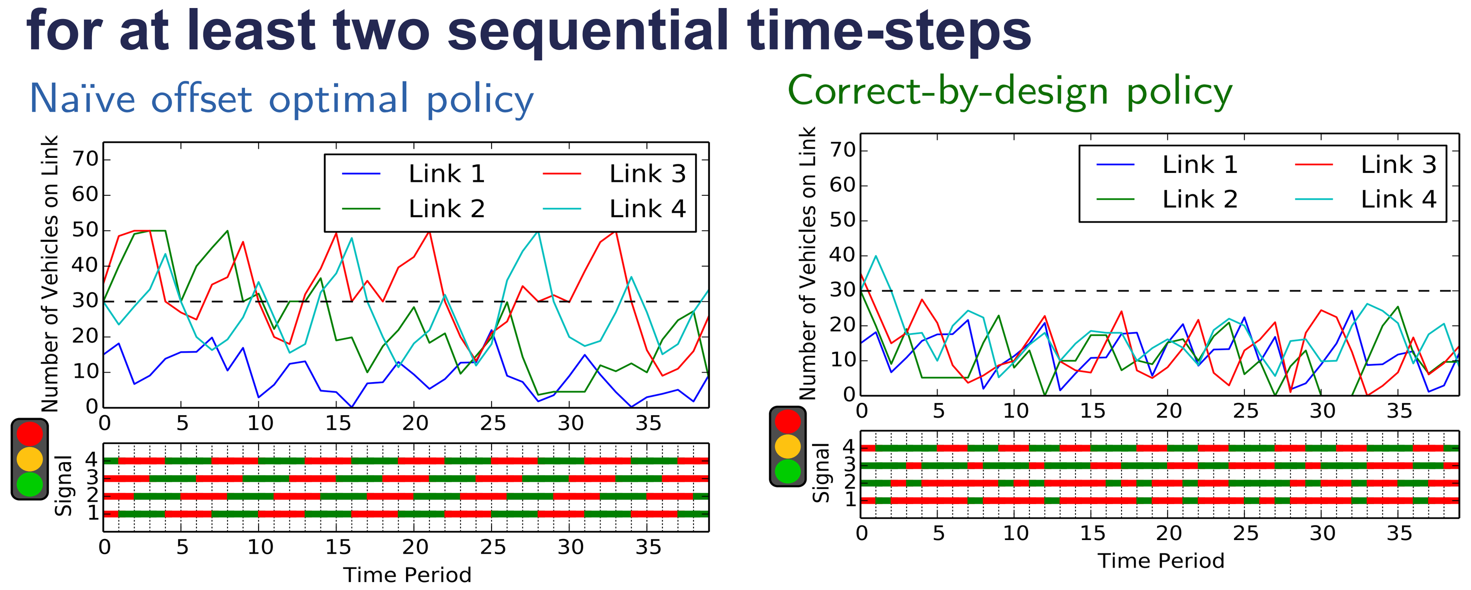


Example



LTL Specification:

- Each signal actuates cross street traffic infinitely often
- Eventually, links 1, 2, 3, and 4 have fewer than 30 vehicles on each link and this remains true for all time
- The signal at junction 4 must actuate cross street traffic for at least two sequential time-steps

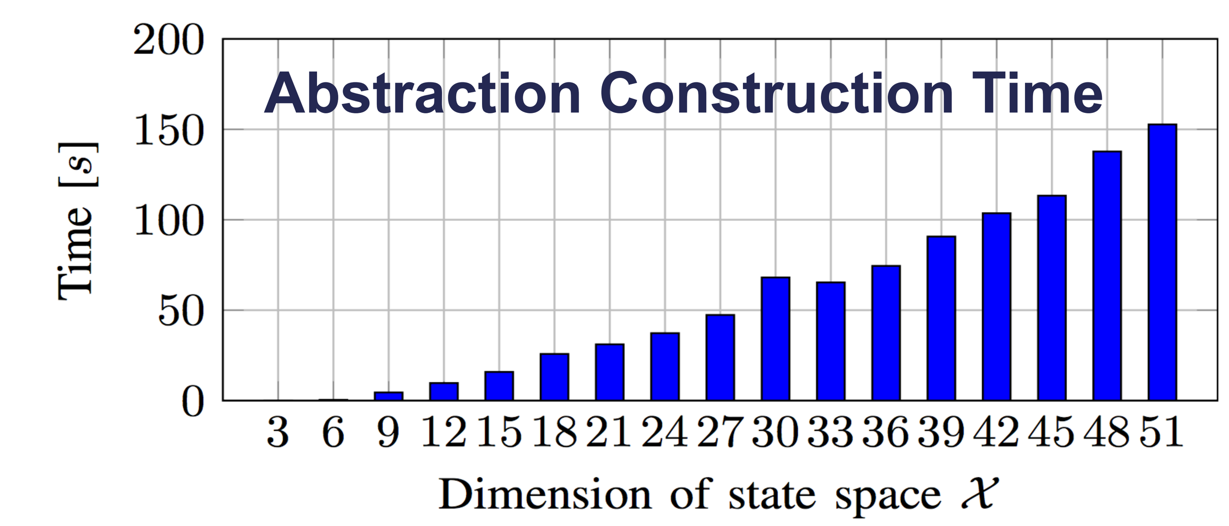


Dynamic Contracts

Distributed controllers designed with assume-guarantee contracts that change in response to runtime conditions and reduce cumulative delays. Dynamic contracts overcome the conservatism of fixed contracts.

Sparsity-Aware Abstraction

Many real life networks exhibit a sparsity property because geographically distant links do not interact. A sparsity-aware abstraction procedure leverages sparsity by selectively computing reachable sets from low dimensional subspaces, and is able to construct abstractions of high dimensional systems with 10^{51} discrete states.



Publications

- Coogan, Arcak, Belta, “Formal methods for control of traffic flow”, IEEE Control Systems Magazine, 2017
- Sadraddini, Rudan, Belta, “Formal synthesis of distributed optimal traffic control policies”, ICCPS 2017
- Kim, Arcak, Seshia “A small gain theorem for parametric assume-guarantee contracts” HSCC 2017.
- Kim, Sadraddini, Belta, Arcak, Seshia “Dynamic contracts for distributed temporal logic control of traffic networks ” CDC 2017.
- Gruber, Kim, Arcak “Sparsity-Aware Finite Abstraction” CDC 2017
- Kim, Arcak, Seshia, “Symbolic control design for monotone systems with direceted specifications”, Automatica, 2017
- Coogan, Gol, Arcak, Belta, “Traffic network control from temporal logic specifications” IEEE Transactions on Control of Network Systems, 2016.
- Sadraddini, Belta, “A provably correct MPC approach to safety control of urban traffic networks” ACC 2016.
- Coogan, Arcak, Belta, “Finite state abstraction and formal methods for traffic flow networks” ACC 2016.
- Kim, Arcak, Seshia, “Directed specifications and assumption mining for monotone dynamical systems” HSCC 2016.
- Kim, Arcak, Seshia “Compositional controller synthesis for vehicular traffic networks” CDC 2015.
- Coogan and Arcak, “Efficient finite abstraction of mixed monotone systems” HSCC 2015.