

Distributionally Robust Stochastic Control of IoT-Enabled Systems

CPS: CRII: Information-Constrained Cyber-Physical Systems for Supermarket Refrigerator Energy and Inventory Management

CNS 1657100

Insoon Yang

Electrical Engineering Department, University of Southern California

insoonya@usc.edu



Background

Abstract

Various critical decision-making and control problems associated with engineering and socio-technical systems are subject to uncertainties. Large-scale data collected from the Internet-of-Things and cyber-physical systems can provide information about the probability distribution of these uncertainties. Such distributional information can be used to dramatically improve the performance of closed-loop systems if they adopt appropriate controllers, which reduce the conservativeness of classical techniques, such as robust control. Several concerns have been raised about how best to incorporate the collected data into critical control and decision-making problems. These concerns center on robustness, safety, risk and reliability because the data and statistical models extracted from the data often result in inaccurate distributional information. The use of poor distributional information in constructing a stochastic optimal controller does not guarantee optimality and can even cause catastrophic system behaviors. The proposed research aims to establish a control-theoretic foundation to resolve these issues by allowing distributional errors in statistical models and by providing control strategies that are robust against these errors.

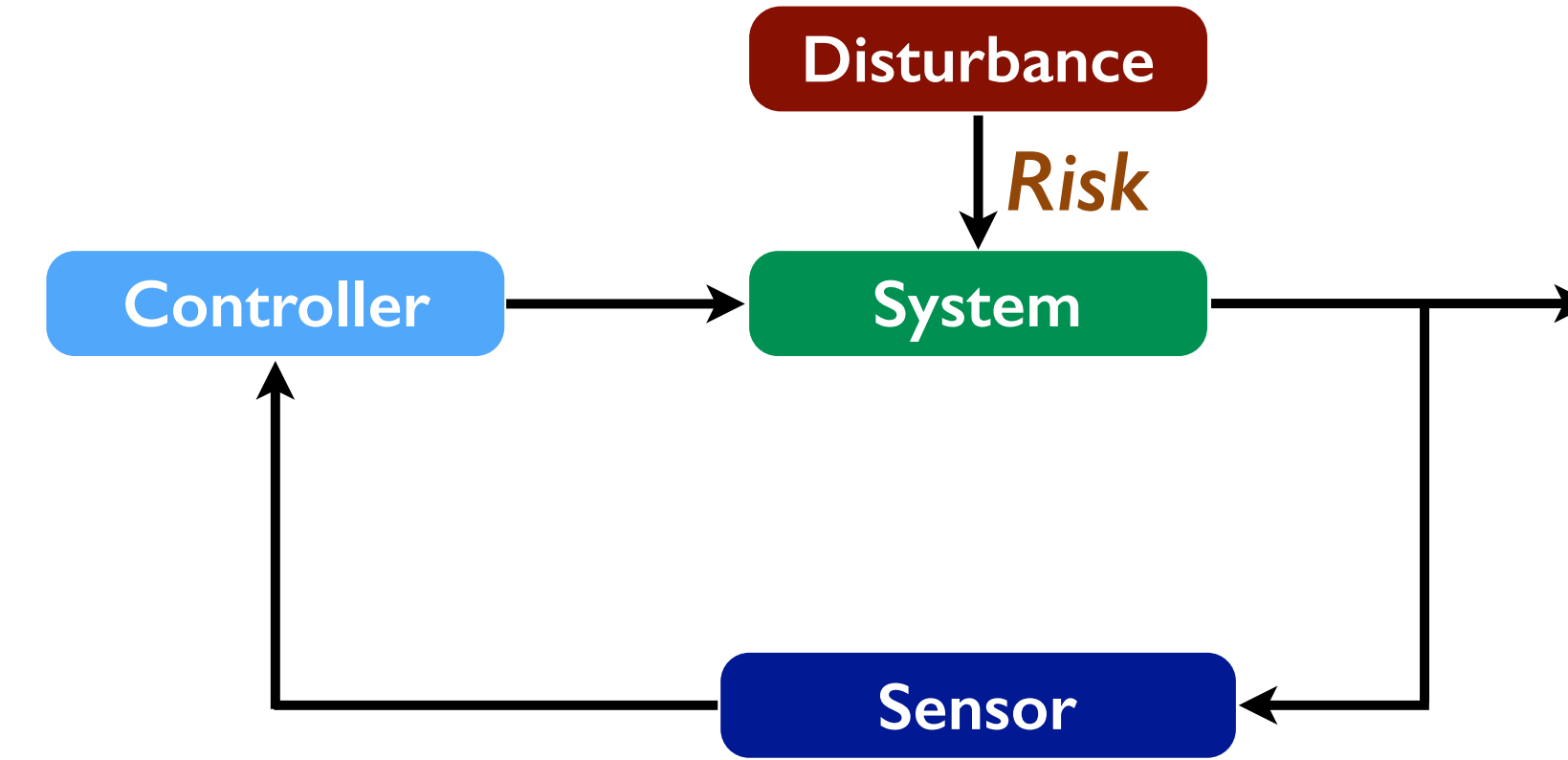
Motivation: supermarket refrigerator energy and inventory management



- Supermarket refrigeration systems account for 7% of the total commercial energy consumption in US. [DOE]
- Food safety risk: perishable food products, such as meat and dairy, must be stored at proper temperatures.
- CPS provide new opportunities to improve energy efficiency and food safety:
 - Real-time monitoring of inventory level through RFID sensors and food temperatures.
 - This information can be used to **jointly optimize refrigerator and inventory control** signals for minimizing operation costs and guaranteeing food safety.
- Issue: difficult to have reliable future demand distribution.**

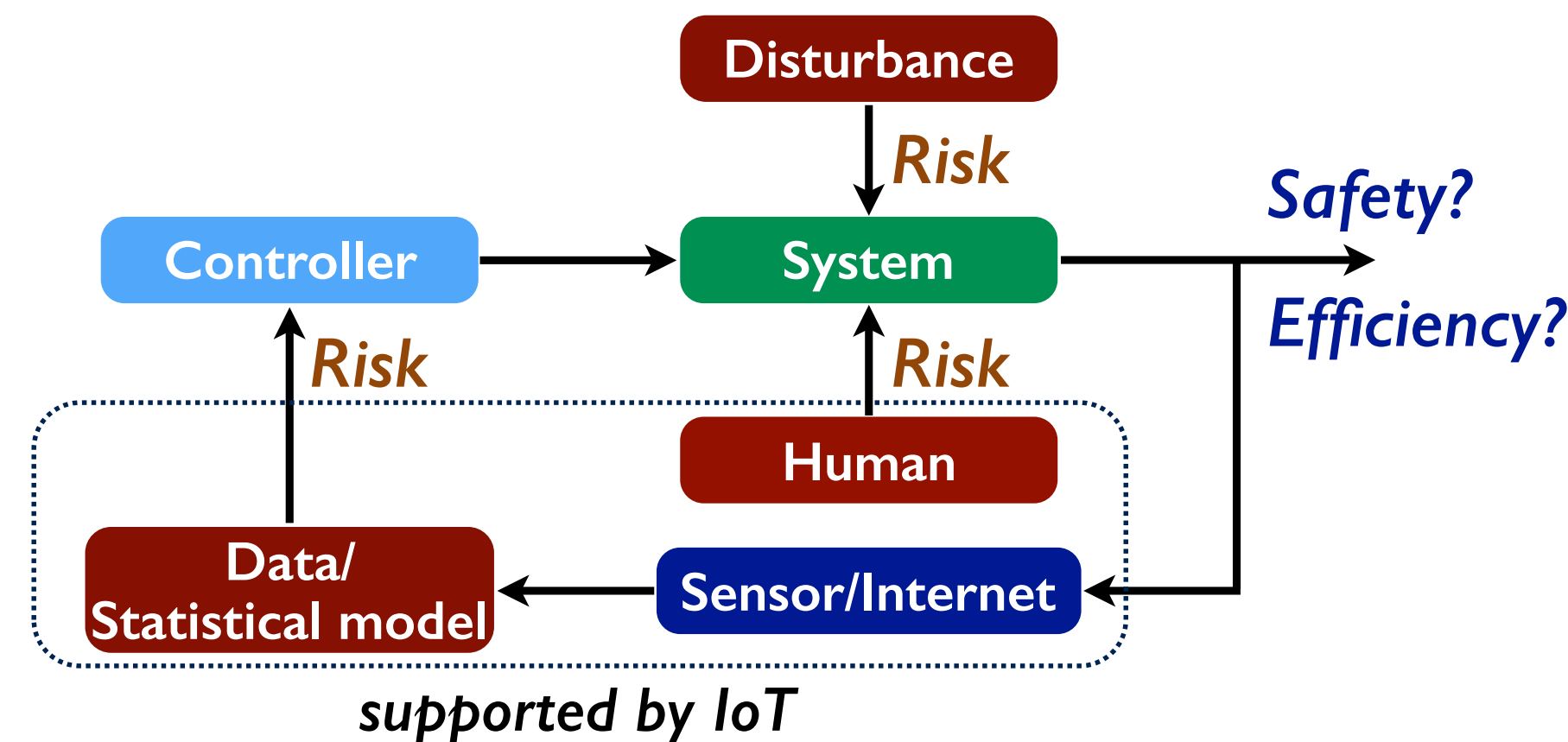
Risk from data, statistics, human

Classical control systems



- Stability
- Robustness w.r.t. disturbances
- Optimality (given information)

IoT-enabled control systems



- Data: inaccuracy, insufficient samples (e.g., customers' demand of products)
- Statistical model: local optimality, wrong prior info. (e.g., demand distribution estimator)
- Human behavior: difficult to predict (e.g., customers' buying behaviors)

Research questions

- Distributional robustness:** How can we design a control strategy that is robust against errors in estimated or empirical distribution obtained from data?
- Risk management:** How can we mitigate the risk (e.g., safety) generated by the new components in IoT-enabled control systems?

Distributionally Robust Control

Stochastic systems with distributional ambiguity

$$\underbrace{x_t}_{\text{state}} = f(\underbrace{x_t}_{\text{state}}, \underbrace{u_t}_{\text{control}}, \underbrace{w_t}_{\text{uncertainty}}) \quad w_t \sim \mu_t$$

- The prob. distribution μ_t of w_t is not fully known.
- Assume that it is contained in an **ambiguity** set of probability distributions: we allow errors in the prob. distribution obtained from data.

Distributionally robust stochastic control

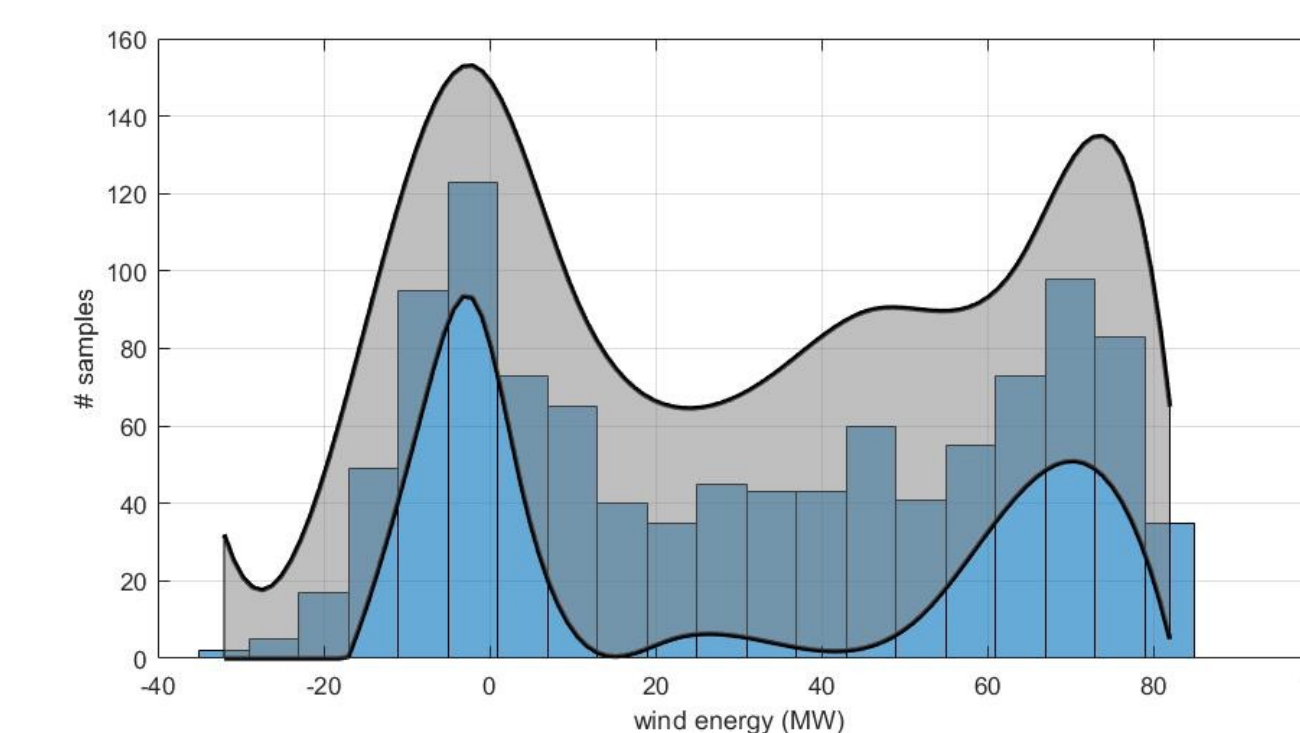
$$\min_{\pi \in \Pi} \max_{\gamma \in \Gamma} \mathbb{E}^{\pi, \gamma} \left[\sum_{t=0}^{T-1} r(x_t, u_t) + q(x_T) \right]$$

s.t. $\pi(x_t) = u_t$
 $\gamma(x_t) = \mu_t \in \mathbb{D}_t$ (ambiguity set)

- Provides a **performance guarantee**: An optimal control strategy is robust against distributional errors characterized by the ambiguity set.
- Non-randomized Markov policy is optimal under semi-continuity assumptions. [Yang, *Automatica*, provisionally accepted]

Modeling ambiguity sets

- Moment-based:** Constraints on the first- and second-moments [Scarf et al., 1958], [Delage & Ye, 2010], [Samuleson, Yang, 2017], etc.
- Statistical ball:** Phi-divergence [Ben-Tal et al., 2013], Wasserstein distance [Esfahani & Kuhn, 2015], [Gao & Kleywegt, 2016], [Yang, 2017], etc.
- Confidence sets:** [Xu & Mannor, 2012], [Wiesemann et al., 2014], [Yang, 2017], etc.



Dual Dynamic Programming

Duality-based reformulation of Bellman equations

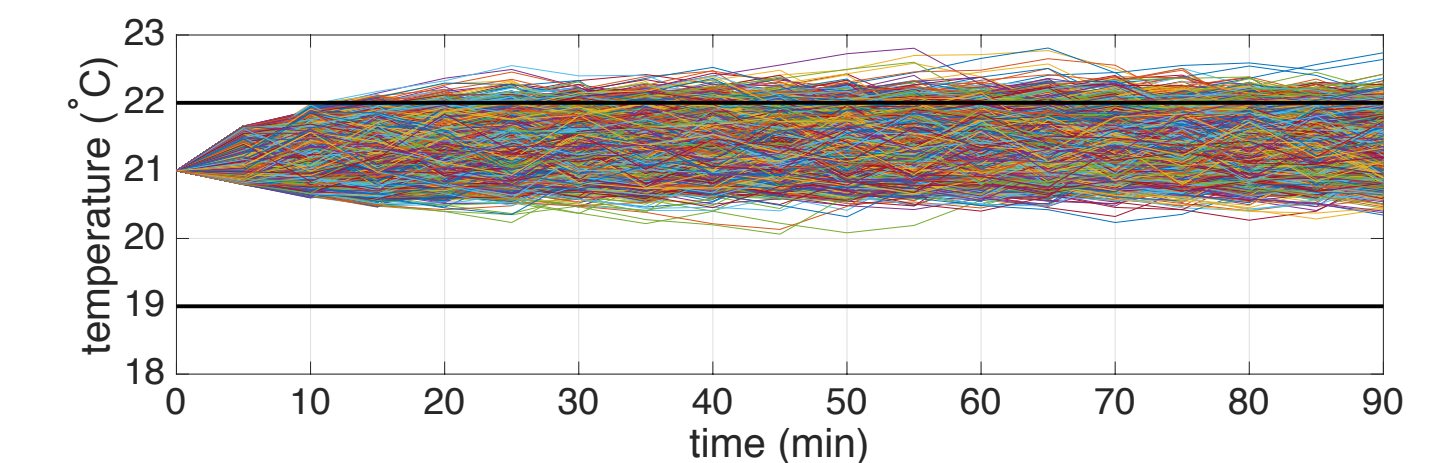
$$v_t(x) = \inf_{u, \underline{\lambda}, \bar{\lambda}, \nu} r(x, u) + (b_t - \mathbf{m}_t)^\top \underline{\lambda} + (b_t + \mathbf{m}_t)^\top \bar{\lambda} + c_t \text{Tr}(\Sigma_t \Lambda) + \nu$$

s.t. $w^\top (\bar{\lambda} - \underline{\lambda}) + (w - \mathbf{m}_t)^\top \Lambda (w - \mathbf{m}_t) + \nu \geq v_{t+1}(f(x, u, w)) \quad \forall w \in W_t$
 $u \in \mathcal{U}_t, \underline{\lambda}, \bar{\lambda} \geq 0, \Lambda \geq 0$

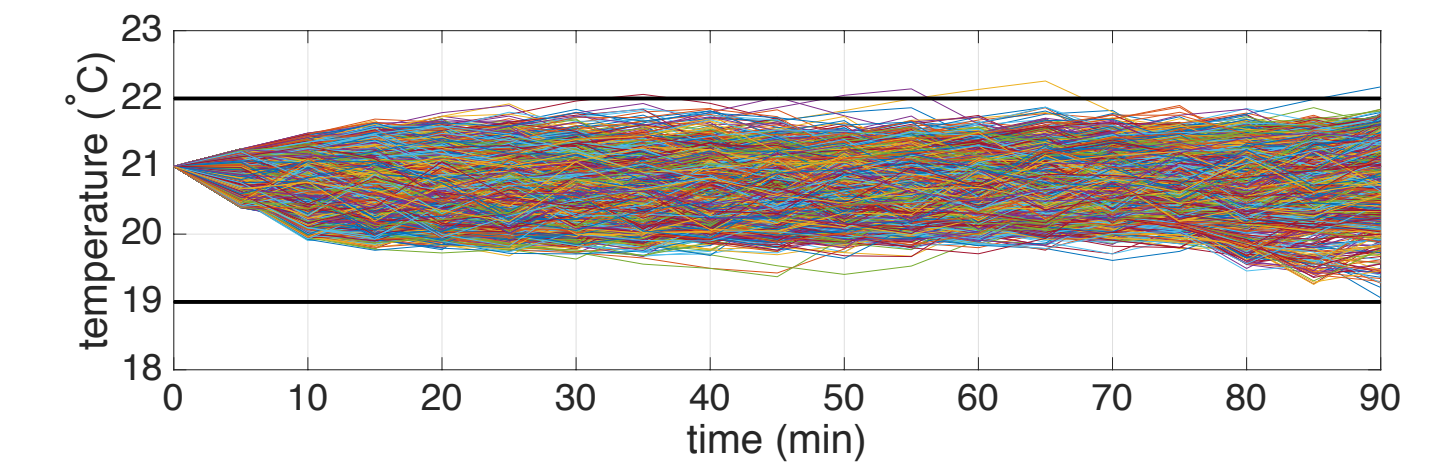
- Infinite-dimensional minimax** optimization -> **semi-infinite** program (solved by convergent algorithms).
- Strong duality** holds: no loss of optimality.
- Moment uncertainty [Yang, *Automatica*, provisionally accepted], Wasserstein distance [Yang, *IEEE Cont. Sys. Lett.*, 2017], conic confidence sets [Xu & Mannor, *Math. Oper. Res.*, 2012], [Yang, *CDC*, 2017].
- Risk-constraints can be added [Miller & Yang, *SIAM Contr. Opt.*, 2017].

Control of thermostatically controlled loads (constraint: Prob (temp. in desired range) ≥ 0.95)

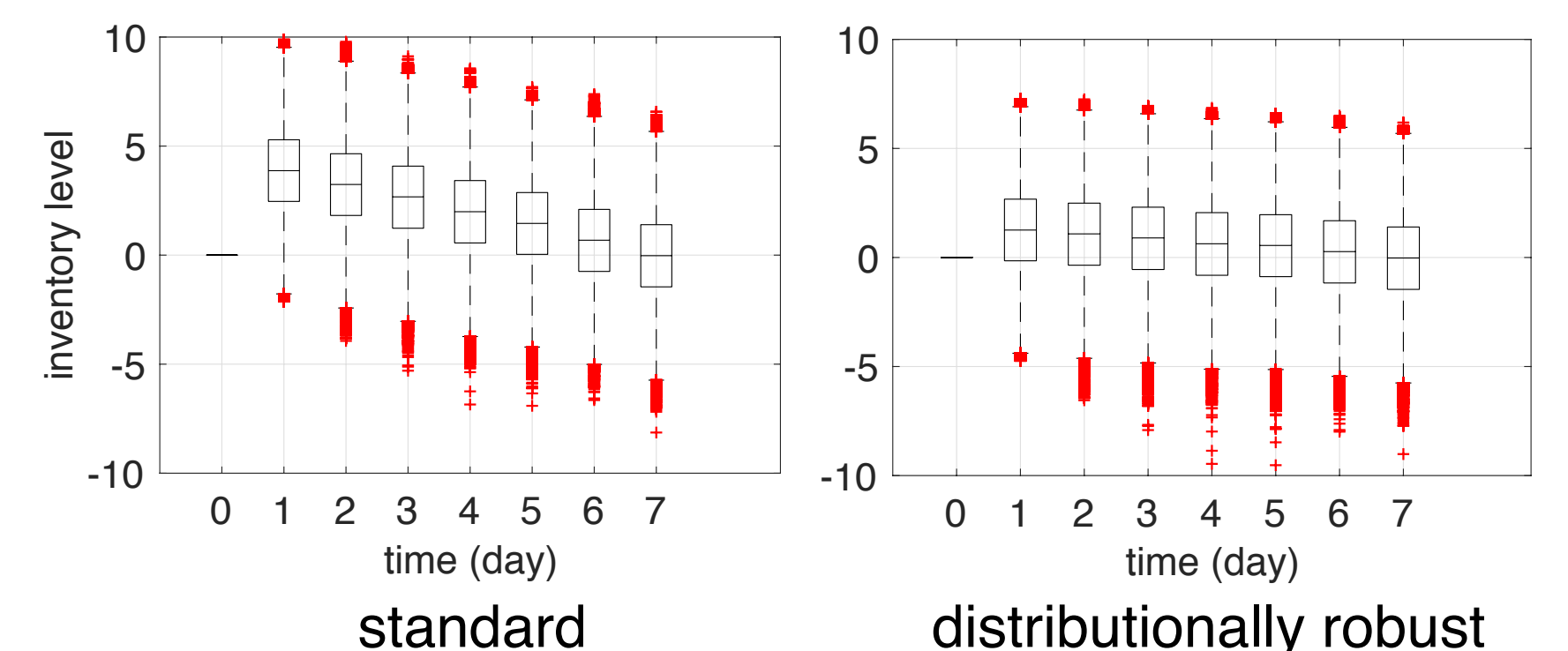
- Standard safety-preserving control (prob. of safety: 0.86)



- Distributionally robust control (prob. of safety: 0.995)



Inventory control



- Distributionally robust control reduces the cost by 29.6%, by maintaining the inventory level close to 0.