Compositional Synthesis of Multi-Robot Motion Plans via SMT Solving

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Keywords: Multi-robot systems, motion planning, LTL specification, SMT solving

1 BackGround

Traditionally, the objective of motion planning has been to move a robot from point A to point B while avoiding obstacles. However, when we deal with multi-robot systems, there may be many other complex requirements as part of the specification, that go far beyond the simple reachability requirement. One may wish to maintain a set of invariant properties during the movement of the robots, for example, a particular formation among the group of robots, a precedence relationship between the robots, or a minimum distance between the robots. One may also wish a group of robots to perform a task repetitively. For example, the group of robots may be required to monitor the state of an object and report it to some base station in a periodic manner [1]. These properties can be formally specified using some temporal logic, for example, Linear Temporal Logic (LTL) [2].

The motion planning problem where the specification is given in terms of some temporal logic has been addressed in a number of recent works [3, 4, 5, 6, 7, 8, 1]. In these works, a finite model for the robot dynamics is first generated using an abstraction process based on discretization of the configuration space [9], and then game theoretic synthesis techniques [3, 8] are used to generate high level motion plans and low level control policies on the abstract model. However, the abstraction algorithm and the synthesis algorithm both scale exponentially with the dimension of the configuration space, thus limiting the application of this approach to simple systems with lower dimensions. In this project, we would like to explore a different avenue to solve the multi-robot motion planning problem in a scalable way and enhance the application of multi-robot systems in many different CPS domains.

2 Proposed Research

In this proposed research, we plan to address the LTL motion planning problem for multi-robot systems, where the robots have complex dynamics, and complex runtime specification of the system is given in terms of a set of LTL properties [10]. The problem of motion planning for multi-robot system is challenging as the set of controls lies in an infinite dimensional space, and the computational complexity grows exponentially with the number of robots [11]. Due to these reasons, the temporal logic motion planning for multi-robot systems has been limited to a system containing only a few robots [4, 1], even though the dynamics of the targeted robots are relatively simple. In our approach, we aim to deal with the complexity of the problem by decomposing it into two subproblems. First, we will design a set of controllers to control different aspects of the members of the multi-robot system. For example, for a UAV one may have different controllers for moving it to different positions and orientations. A controller together with the corresponding closed loop trajectory and the associated cost to run the controller is termed a *motion primitive*. Second, we will utilize these motion primitives to build a system of constraints where the decision variables encode the choice of motion primitives used at any discrete-time point on the trajectory. The system of constraints involves Boolean combination of linear constraints. Our strategy is to leverage the power of an off-the-shelf SMT (Satisfiability Modulo Theories) solver [12] to solve the system of constraints. SMT solvers have been widely used in program synthesis with significant success (for example, see [13] and the references there). However, composing a set of motion primitives to synthesize trajectories for a group of robots using an SMT solver has not been attempted before.

One auxiliary goal of the motion planning problem is to generate a trajectory that is optimal according to a cost criteria. Though SMT solvers can produce a solution satisfying a set of constraints, they cannot natively solve an optimization problem. In this project, we would like to investigate how an SMT solver can be adapted to solve the optimal motion planning problem for a multi-robot system.

In summary, we would like to make the following contributions:

- We aim to build a compositional LTL multi-robot motion planning framework that reduces the planning problem to a constraint solving problem. Unlike previous work (see, for example, [4, 1]), our method would not require over-approximations or discretization of continuous dynamics and would be able to handle second and higher order dynamics.

- We would employ an SMT solver to synthesize trajectories for a multi-robot system. We would automatically generate the constraints in the input language of a state-of-the-art SMT solver to generate a solution very efficiently.

- We would develop a mechanism to adapt an SMT solver to synthesize an optimal trajectory for each robot.

3 Potential Impact

A multi-robot system can be seen as a canonical example of a cyber-physical system that is expected to be coordinated, distributed and connected, secured, robust, and responsive. Multi-robot systems are useful in many applications such as monitoring, surveillance and disaster response, where there are tasks that are performed better by a team of robots rather than a single robot. Motion planning is the most fundamental problem to be solved efficiently for widespread applications of multi-robot systems. If our research becomes successful, we will be able to provide an automated and scalable mechanism to solve multi-robot planning problem for complex specification.

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