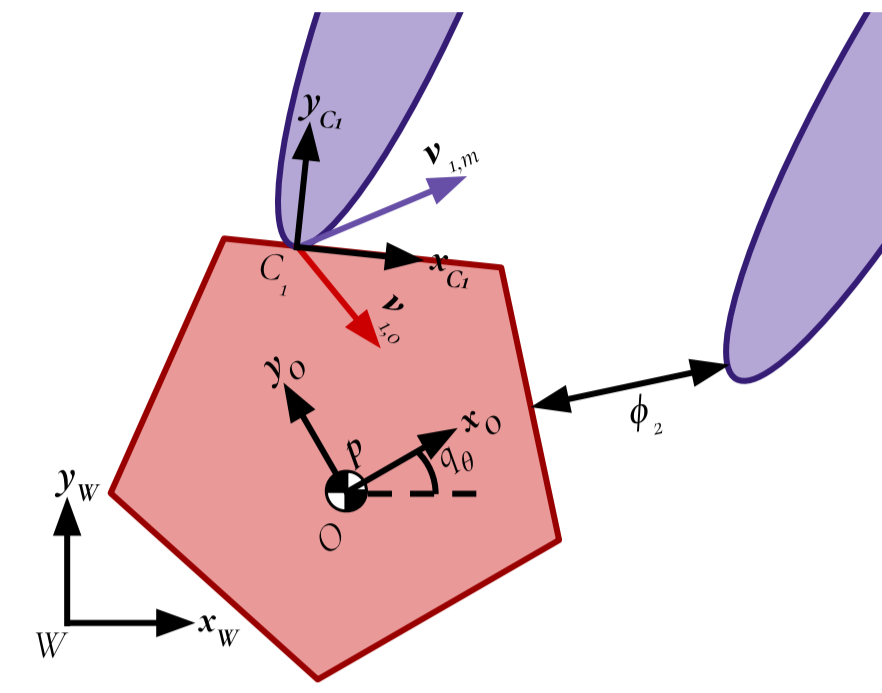


## INTRODUCTION

- Frictional contact is the **fundamental behavior** of robot locomotion and manipulation.
- However, in uncertain environments, robots move slowly and cautiously, often avoiding, rather than embracing, contact.
- This project aims to enable robots to intelligently make and break contact while manipulating complex and uncertain objects.

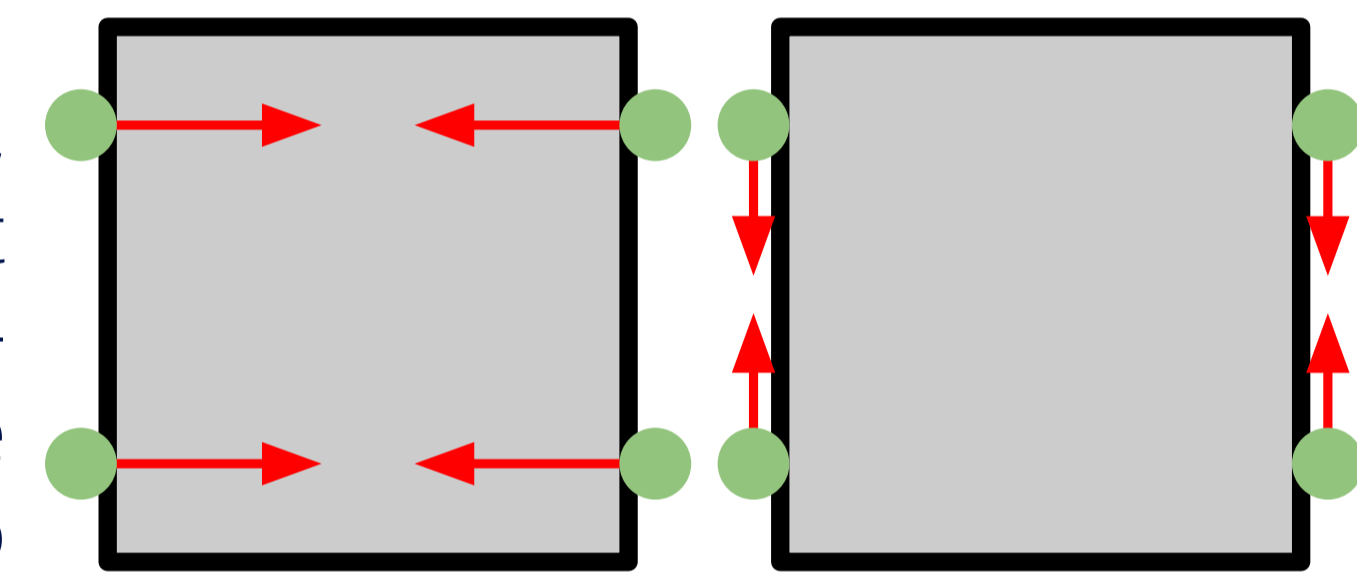


For this, we propose two hypotheses:

- Formal, computational algorithms can **find** and **verify** simple, non-combinatoric, approaches to robotic grasping and manipulation.
- Explicit consideration of the dynamics of manipulation can lead to more robust and more capable approaches.

## QUASISTATIC MODELING [1]

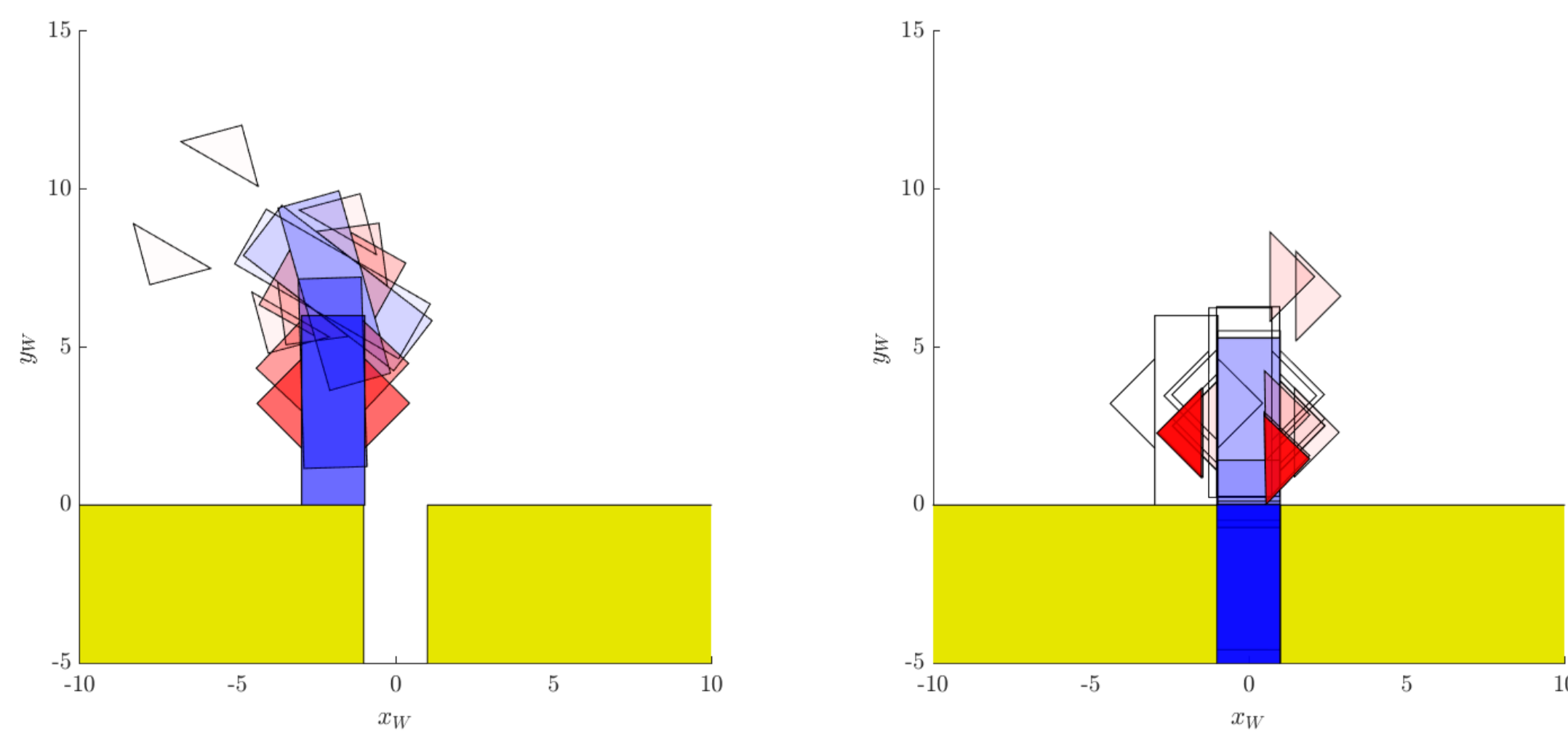
Prior, quasistatic approaches, while popular, assume direct control of velocity, but are fundamentally unable to capture grasping and jamming. To resolve these issues, we have developed a **comprehensive model for quasistatic manipulation**.



A square object contacted by four fingers, with commanded velocities shown. With traditional methods, (left) has no possible solutions and (right) yields ambiguous sticking or sliding behavior.

By replacing pure velocity control with a more realistic force law, we derive a theoretically sound model of quasistatic manipulation.

- Computationally efficient:** a linear complementarity problem (LCP).
- Provable existence:** the LCP is guaranteed to have solutions for all commands, including pushing, grasping, and jamming.
- Limiting behavior:** captures the reality of feedback-based velocity control, realizing pure velocity commands when dynamically feasible.



A peg-in-hole trajectory. (Left) grasping and orientation against the wall. Right: sliding and insertion.

## CONTACT-AWARE CONTROL SYNTHESIS [2]

- The challenge in contact-rich manipulation lies in the discontinuous dynamics, due to frictional forces and impacts.
- We design **provably stable control policies that leverage tactile feedback**.
- Modeling dynamics as a Linear Complementarity System (LCS)

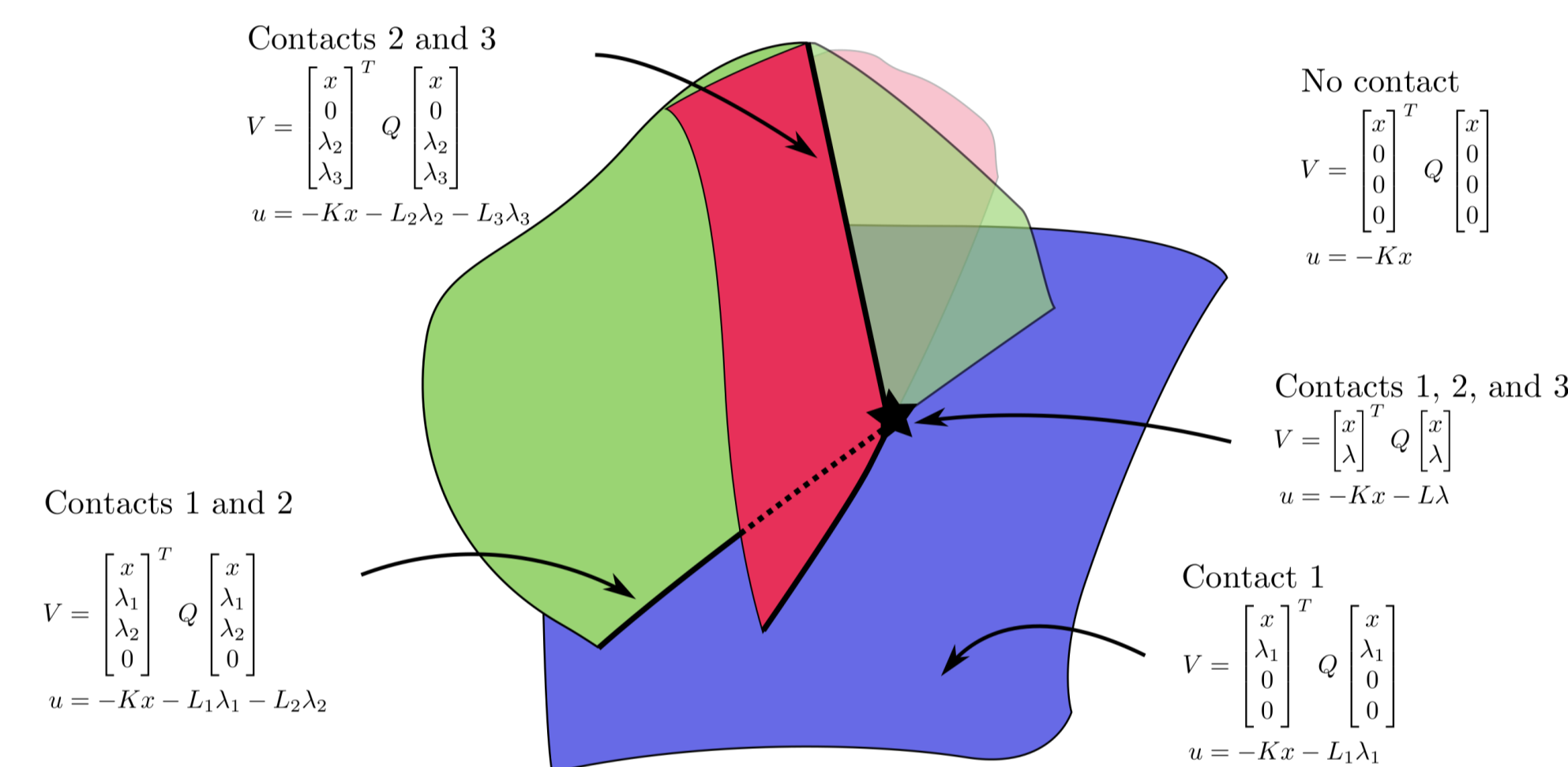
$$\dot{x} = Ax + Bu + C\lambda, \quad 0 \leq \lambda \perp Dx \geq 0.$$

- Mirror this structure in the controller and Lyapunov function

$$u = -Kx - L\lambda, \quad V = \begin{bmatrix} x \\ \lambda \end{bmatrix}^T Q \begin{bmatrix} x \\ \lambda \end{bmatrix}.$$

Controllers and certificates utilize tactile feedback, are piecewise-differentiable, but are **non-combinatoric** and scalable.

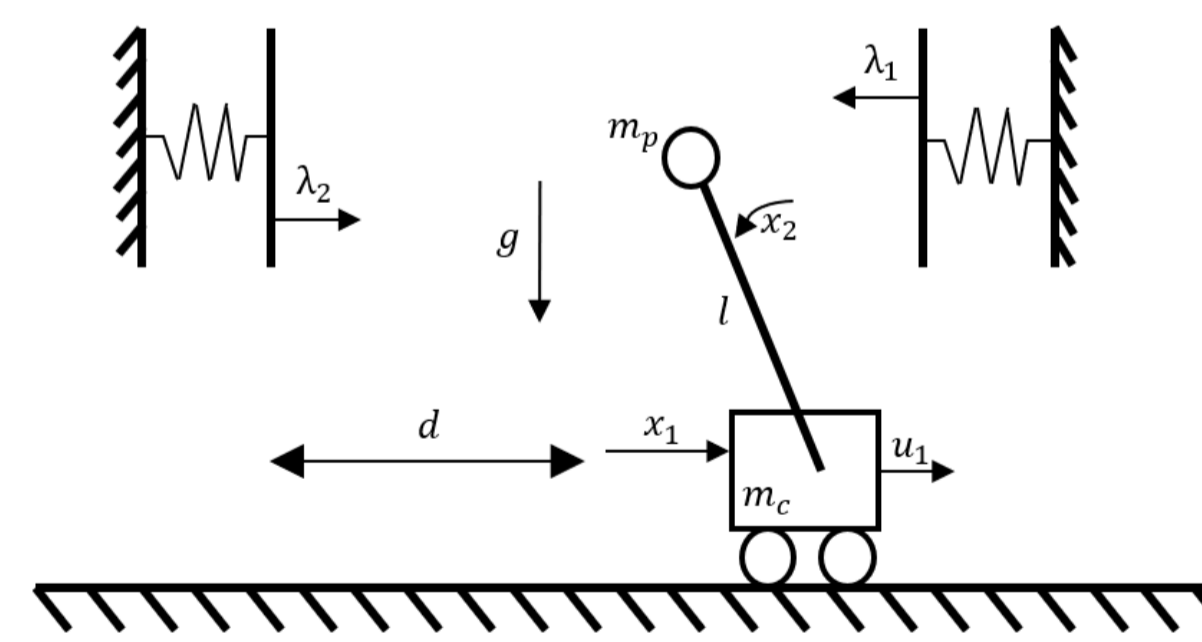
- Synthesis of a stabilizing controller solved as a **bilinear matrix inequality**.



For three potential contacts, there are  $2^3$  modes. Rather than define a separate policy per mode, we design and verify a more structured and tractable controller.

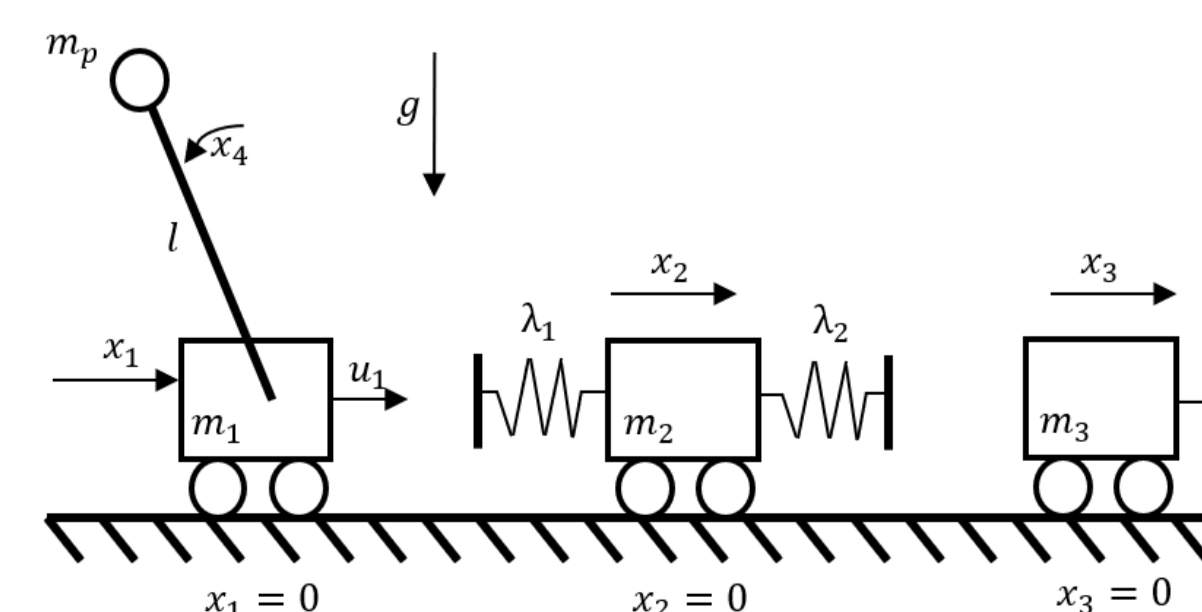
## EXAMPLE: CART-POLE WITH SOFT WALLS

- Two stiff spring-based walls that interact with the pole
- Linearization-based methods cannot reason about the non-smooth dynamics, but linearizing the *smooth* aspects of the dynamics and kinematics gives a LCS and stabilizing controller.



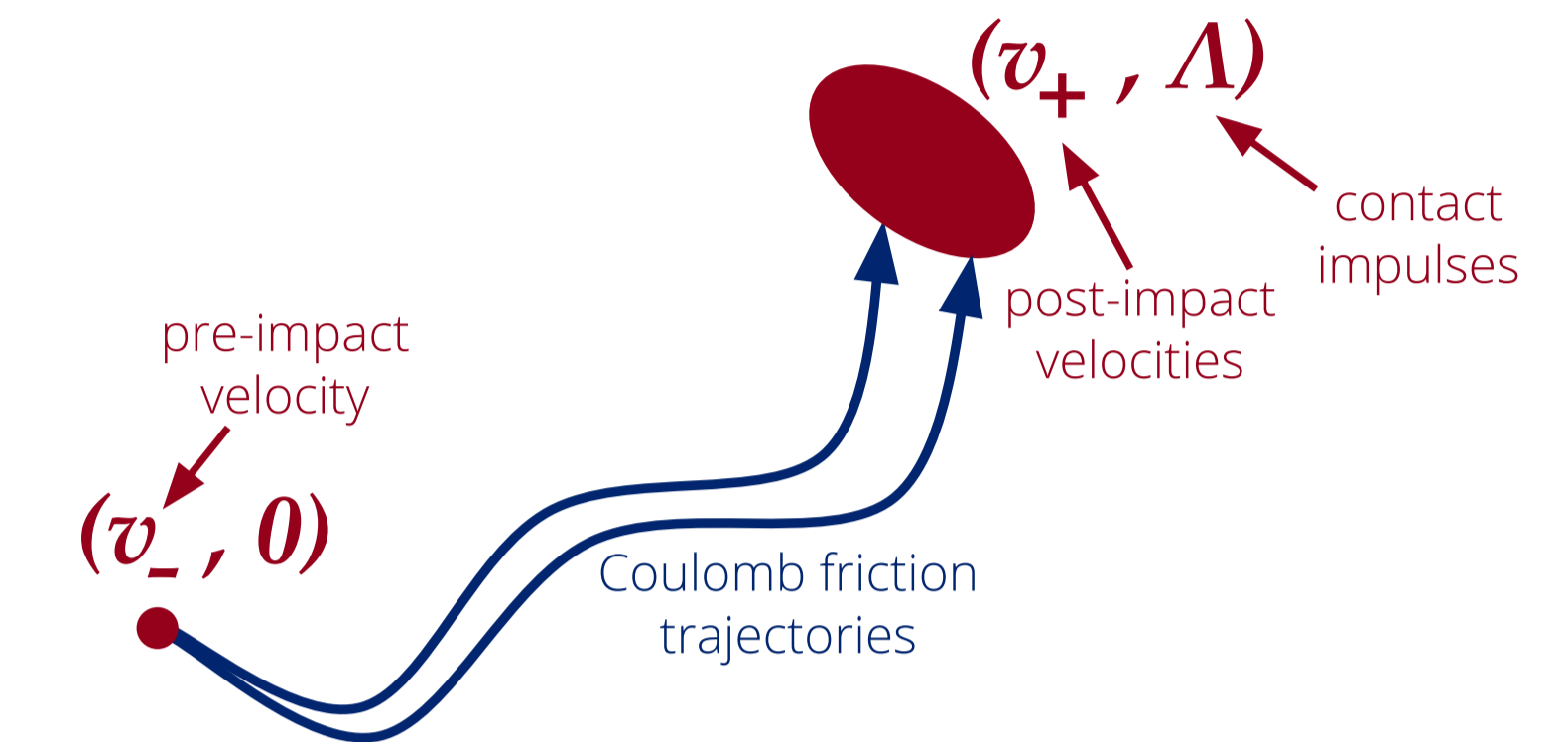
## EXAMPLE: PARTIAL STATE FEEDBACK

- Tactile feedback can be used when state is unknown.
- The middle underactuated cart is not sensed.
- Imposing sparsity constraints on control policy, we still synthesize a provably stabilizing strategy.



## SIMULTANEOUS IMPACT EVENTS [3]

- Simultaneous frictional impacts between rigid bodies are **pervasive, extremely sensitive, and poorly understood**.
- We developed a continuous-time rigid body dynamics model that enables reasoning over impact ambiguity.



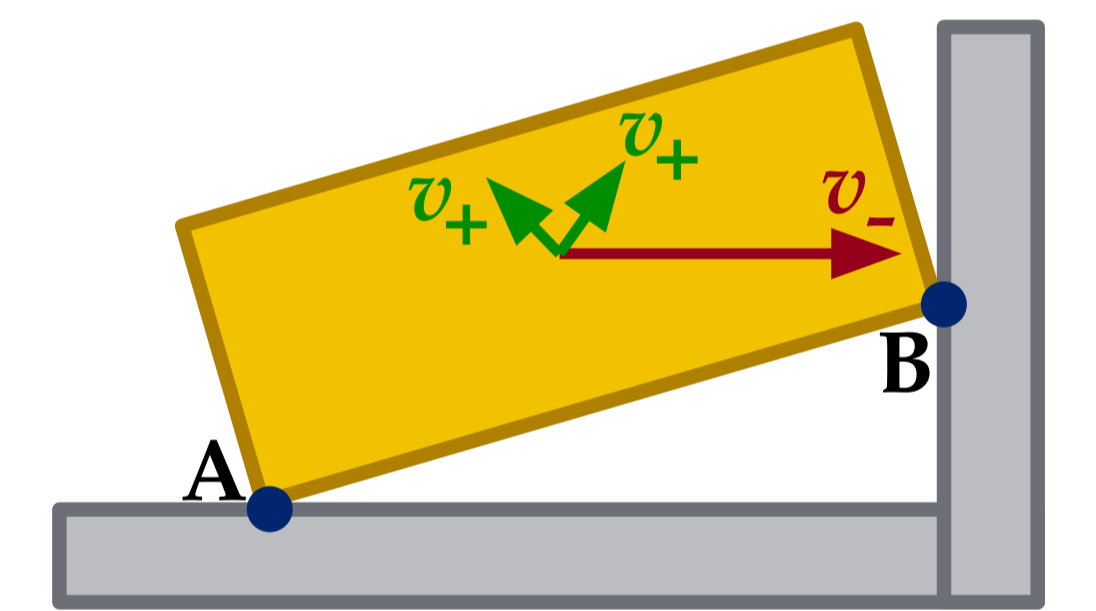
- Finds change in velocity  $v$  and net impulse  $\Lambda$  under Coulomb friction
- Formulated as a differential inclusion, permits **arbitrary resolution of contact orderings**:

$$v'(s) \in F(v(s)).$$

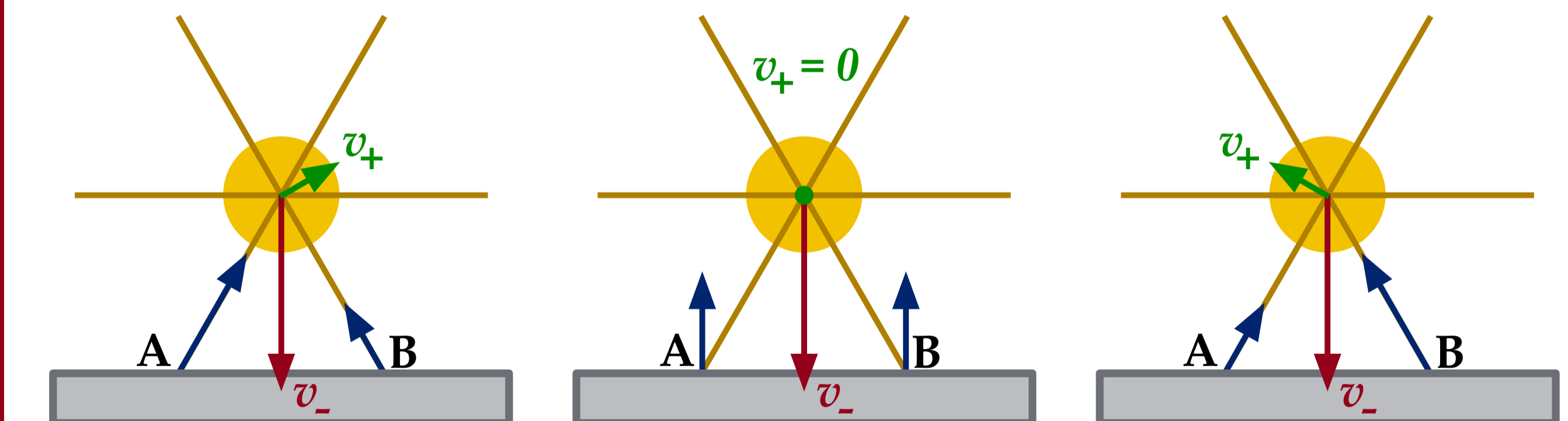
- Simulated, along with continuous dynamics, as an LCP with guaranteed existence of solutions.
- Enables reachability analysis via simulation or Lyapunov analysis.

## EXAMPLE: MANIPULATION

**Subtle non-uniqueness** emerges even without simultaneous impact. A block sliding into a wall (right) will have sensitive behaviors due to propagation of impact events.



## EXAMPLE: RIMLESS WHEEL



Impact model not only gives each of the three first-principles results, but also returns every reasonable **intermediate result**.

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