Control of Spreading Processes in Arbitrary Networks

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Question – how to control outbreak of an epidemic?

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H1N1 virus reaches epidemic levels in the US

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Six weeks into the flu season, the H1N1 virus is still killing young adults and middle-aged Americans at epidemic levels, according to the Centers for Disease Control and Prevention.

Tags
Health, Medicine, Security, Swine flu, USA

Although the death toll isn't nearly as bad as the 2009 influenza epidemic that swept across the United States, that same H1N1 virus has seen a resurgence this year, infecting more than 6,600

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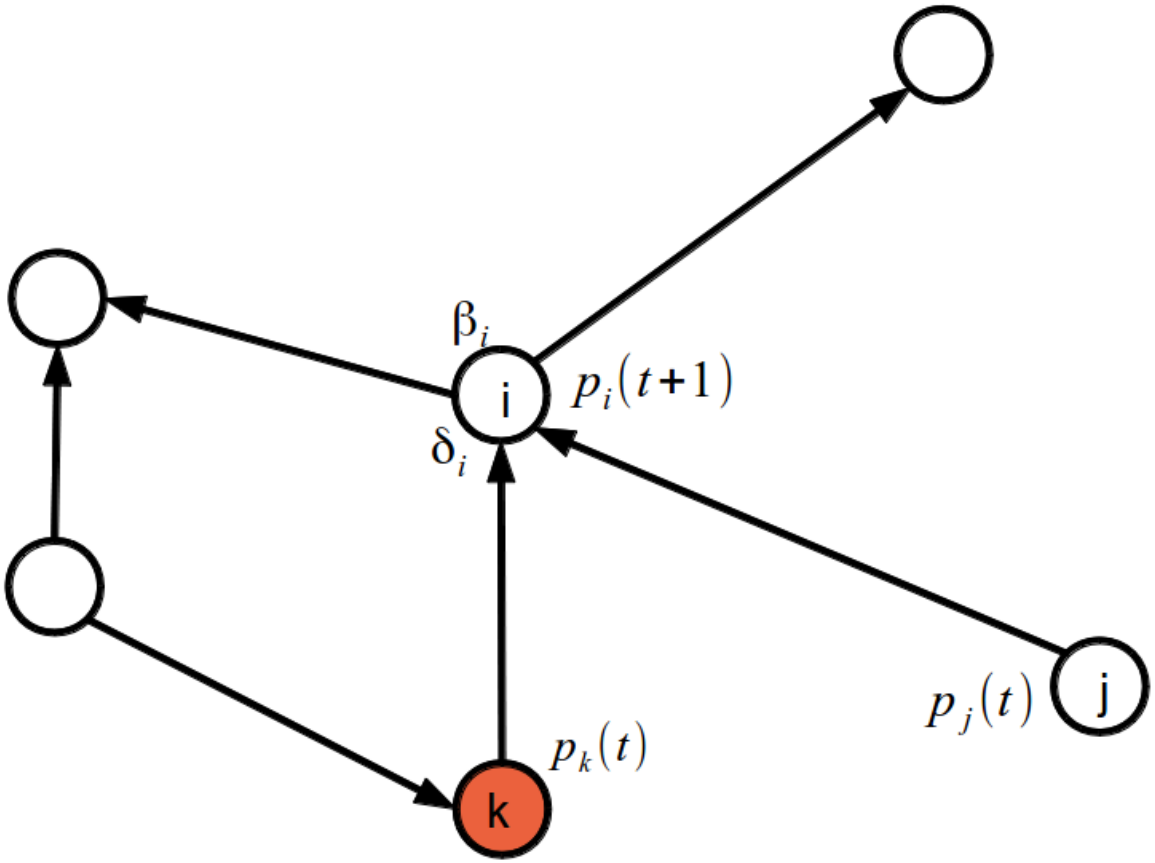
Propagation model

Red node k is infected; others nodes are susceptible.



Propagation model and control

After linearizing around infection-free equilibrium, probability of infection is:

$$\frac{dp_i(t)}{dt} = (1 - p_i(t))\,\beta_i \sum_{j=1}^{n} a_{ij} p_j(t) - \delta_i p_i(t)$$

$$\frac{d\mathbf{p}(t)}{dt} = (BA_{\mathcal{G}} - D)\,\mathbf{p}(t) - P(t)\,BA_{\mathcal{G}}\mathbf{p}(t) \leqslant (BA_{\mathcal{G}} - D)\mathbf{p}(t) \qquad (1)$$

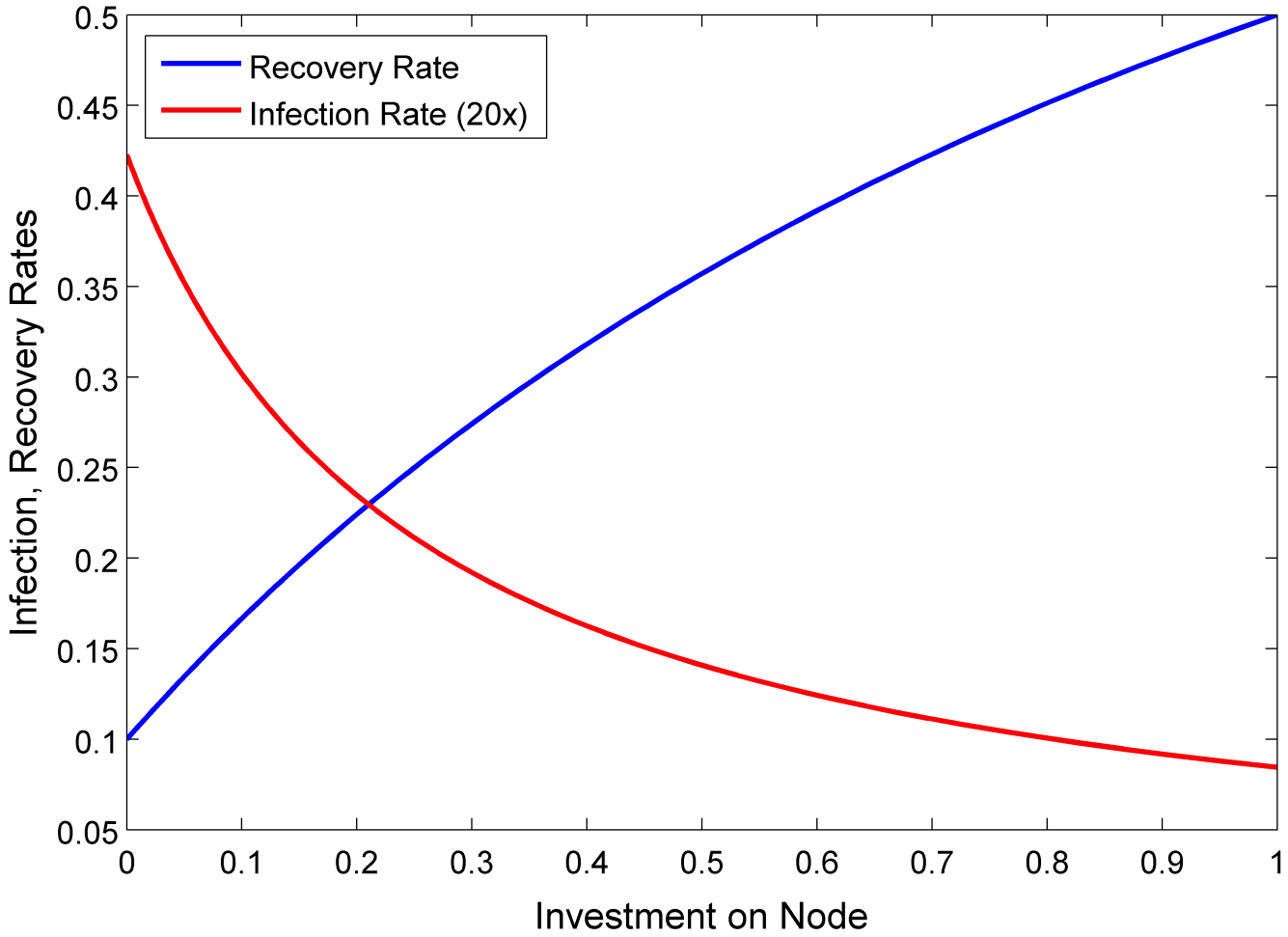
Control an outbreak if

$$\Re\left[\lambda_1\left(BA_{\mathcal{G}} - D\right)\right] \leqslant -\varepsilon, \qquad (2)$$

for some $\varepsilon > 0$, the disease-free equilibrium $(\mathbf{p}^* = 0)$ is globally exponentially stable., i.e., $\|\mathbf{p}(t)\| \leqslant \|\mathbf{p}(0)\|\, K \exp(-\varepsilon t)$, for some $K > 0$.

Cost functions

At node i, $f_i(\beta_i)$ is cost to lower infection rate; $g_i(\delta_i)$ is cost to to increase recovery rate.



Resource Allocation problems

Rate-constrained problem

$$\underset{\{\beta_i,\delta_i\}_{i=1}^n}{\text{minimize}} \sum_{i=1}^{n} f_i(\beta_i) + g_i(\delta_i) \tag{3}$$

$$\text{subject to } \Re\left[\lambda_1\left(\text{diag}(\beta_i) A_{\mathcal{G}} - \text{diag}(\delta_i)\right)\right] \leqslant -\overline{\varepsilon}, \tag{4}$$

$$\underline{\beta}_i \leqslant \beta_i \leqslant \overline{\beta}_i, \tag{5}$$

$$\underline{\delta}_i \leqslant \delta_i \leqslant \overline{\delta}_i, \ i = 1, \ldots, n, \tag{6}$$

What do solutions look like? Is centrality sufficient? Budget-constrained problem?

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What do solutions look like? Is centrality sufficient? Budget-constrained problem?

$$\underset{\varepsilon,\{\beta_i,\delta_i\}_{i=1}^n}{\text{maximize}} \ \varepsilon \tag{7}$$

$$\text{subject to } \Re[\lambda_1(\text{diag}(\beta_i) A_{\mathcal{G}} - \text{diag}(\delta_i))] \leqslant -\varepsilon, \tag{8}$$

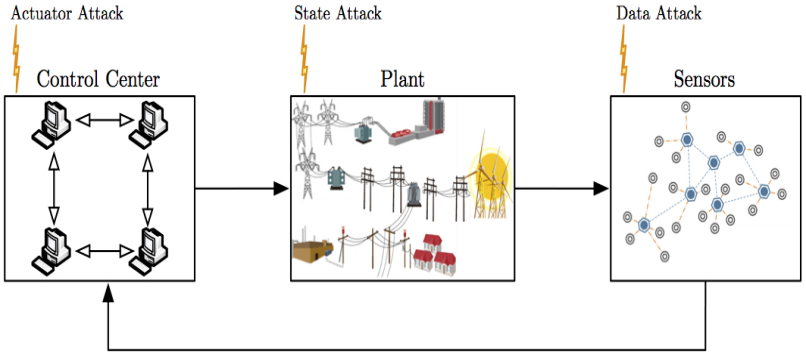
$$\sum_{i=1}^{n} f_i(\beta_i) + g_i(\delta_i) \leqslant C, \tag{9}$$

$$\underline{\beta}_i \leqslant \beta_i \leqslant \overline{\beta}_i, \tag{10}$$

$$\underline{\delta}_i \leqslant \delta_i \leqslant \overline{\delta}_i, \ i = 1, \ldots, n, \tag{11}$$

Cascades in Cyber-Physical Network & Challenges:

Networks of networks



Other issues

- Reliability of measurements
- Intermittent information transfer
- Need for distributed computations
- Design CPN networks robust to attacks?

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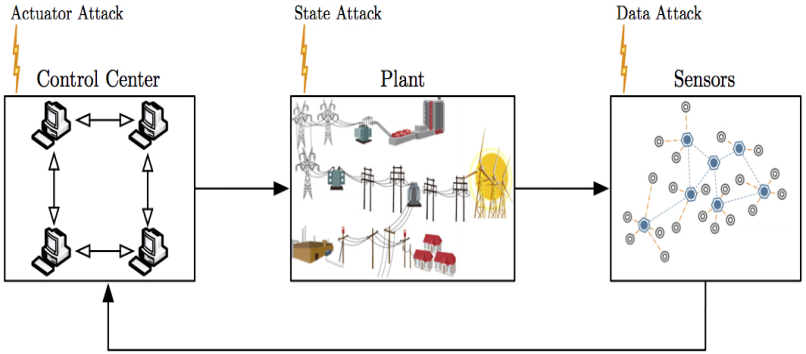

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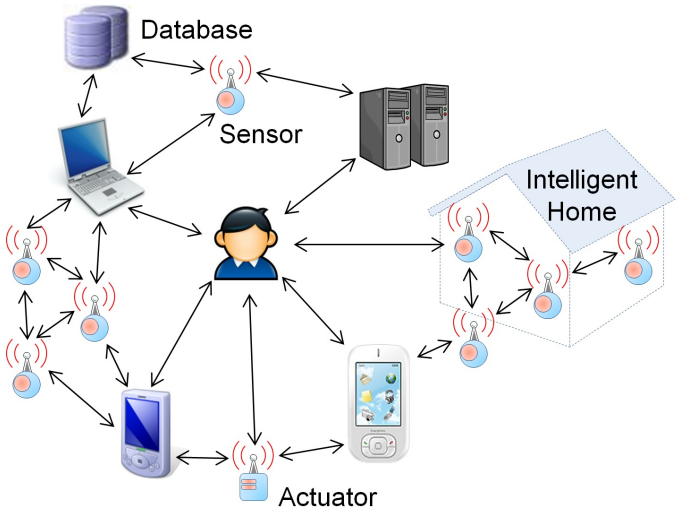
Not necessarily static



Dynamics not linear

$$\frac{\mathrm{d}\mathbf{p}\left(\mathrm{t}\right)}{\mathrm{dt}} = \left(\mathrm{B}\mathrm{A}_{\mathcal{G}} - \mathrm{D}\right)\mathbf{p}\left(\mathrm{t}\right)$$

- Existing paradigms for switched systems don't apply
- Add your questions to the list!

Stop by my poster to see key contribution

- exact convex characterization of resource allocation problem via Geometric Programming
- and chat :)

References

- Chinwendu Enyioha, Victor Preciado, and George Pappas. "Bio-inspired strategy for control of viral spreading in networks." Proceedings of the 2nd ACM international conference on High confidence networked systems. ACM, 2013.
- Victor Preciado, Michael Zargham, Chinwendu Enyioha, Ali Jadbabaie, and George J. Pappas. "Optimal Resource Allocation for Network Protection: A Geometric Programming Approach." IEEE Transactions on Control of Network Systems. 2014 To appear.
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