Coordination control in a cyberphysical environment[1]

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Workshop on the Control of Cyber-Physical Systems
University of Notre Dame London Centre
October 20-21, 2012

[1] Joint work with **Paolo Frasca** – Politecnico di Torino

Cyberphysical systems

Cyber-physical systems (CPS)

Engineered systems whose operations are monitored, **coordinated**, controlled and integrated by a $\underbrace{\text{computing and communication core}}_{\textbf{cyber-infrastructure}}$ (P. Antsaklis)

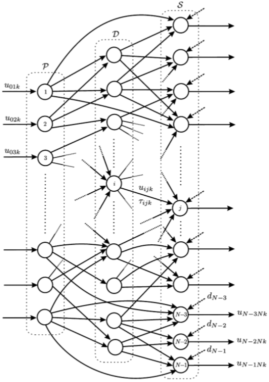
Engineered system	=	Distribution network
Coordination	=	Load balancing
Cyber infrastructure	=	measurement scheduling
		control computation
		actuation scheduling
		robustness to delays, quantization
		poor clock synchronization

Distribution network

$$\begin{aligned} \dot{x} &= Bu \\ z &= B^T x \end{aligned}$$

where

- $x_i \in \mathbb{R}$, stored quantity at the node $i \in I := \{1, 2, \ldots, n\}$
- $u_k \in \mathbb{R}$ flow through edge $k \in E := \{1, 2, \ldots, m\}$
- B incidence matrix of *undirected* graph G



Load balancing

Design edge controllers u_k, $k \in E$, such that

- u_k depends on $z_k := x_i - x_j$
- $z_k \to 0$ for all $k \in E$

Why (still) studying load balancing?

- It is a prototypical problem:
  solutions can be extended to more complex scenarios
- It is useful in many application fields:
  - robotic networks
  - sensors networks
  - data networks
  - opinion dynamics

- It is well studied:

Theorem (Standard consensus)

If the graph G is connected, the control law $u_k = -z_k$ guarantees that $\lim_{t\to\infty} z_k(t) = 0$ for all k. Moreover

$$x_i(t) \to \sum_{j=1}^{n} \frac{x_j(0)}{n}$$

Coordination in a cyber-physical environment

The algorithm requires continuous acquisition of information from neighbors

This is too demanding in a cyber-physical environment!

We instead want a scenario in which

- sensors collect information only upon need
- the continuous-time systems "naturally" interacts with the discrete-time information acquisition
- the whole system is robust against network uncertainties (delays, poor synchronization of local clocks, limited data rate communication, noise)

A hybrid coordination algorithm I

State variables ($i \in I$, $k \in E$)

- node quantities: $x_i \in \mathbb{R}$
- flows: $u_k \in \{-1, 0, +1\}$ (ternary controls)
- local clock variables: $\theta_k \in \mathbb{R}$

Continuous evolution when no information exchange occurs

$$\begin{cases} \dot{x}_i = \sum_{k \in E} b_{ik} u_k \\ \dot{u}_k = 0 \\ \dot{\theta}_k = -1 \end{cases}$$

Jumps occur at every t such that the set

$$\mathcal{I}(\theta, t) = \{k \in E \ : \ \theta_k = 0\} \neq \emptyset$$

A hybrid coordination algorithm II

Discrete evolution: how the exchange of information affects the systems

$$\begin{cases} x_i(t^+) = x_i(t) \quad \forall i \in I \\ u_k(t^+) = \begin{cases} -\operatorname{sign}_\varepsilon(z_k(t)) & \text{if } k \in \mathcal{I}(\theta, t) \\ u_k(t) & \text{otherwise} \end{cases} \\ \theta_k(t^+) = \begin{cases} f_k^\alpha(z_k(t)) & \text{if } k \in \mathcal{I}(\theta, t) \\ \theta_k(t) & \text{otherwise} \end{cases} \end{cases}$$

- $\operatorname{sign}_\varepsilon(z) = \begin{cases} \operatorname{sign}(z) & \text{if } |z| \geq \varepsilon \\ 0 & \text{otherwise} \end{cases}$
- $\varepsilon > 0$ is a *sensitivity* parameter
- $\alpha \in (0, 1)$ is a *robustness* parameter

Note: the law $u_k = -\operatorname{sign}(z_k)$ is known to imply finite-time convergence

A hybrid coordination algorithm III

Next sampling time

$$\theta_k(t^+) = \begin{cases} f_k^\alpha(z_k(t)) & \text{if } k \in \mathcal{I}(\theta, t) \\ \theta_k(t) & \text{otherwise} \end{cases}$$

where

$$f_k^\alpha(z_k) = \begin{cases} \dfrac{\alpha}{2(\deg_i + \deg_j)} |z_k| & \text{if } |z_k| \geq \varepsilon \\ \dfrac{\alpha}{2(\deg_i + \deg_j)} \varepsilon & \text{otherwise} \end{cases}$$

so that

- **dwell time** property holds: $t_{\ell+1}^k - t_\ell^k \geq \dfrac{\alpha\varepsilon}{2\deg_{\max}}$
- $\text{sign}(z_k(t))$ **constant** on $[t_\ell^k, t_{\ell+1}^k]$
- ε-**deadzone** to prevent Zeno

Hybrid coordination algorithm

Protocol

initialization: for all $k \in E$, set $\theta_k(0) = 0,\ u_k(0) \in \{-1, 0, +1\}$;
for all $i \in I$ **do**
    for all $k \in E_i$ **do**
        while $\theta_k(t) > 0$ **do**
            i applies the control $b_{ik} u_k(t)$;
        end while
        if $\theta_k(t) = 0$ **then**
            k polls nodes i, j and collects the information $z_k(t)$;
            k updates $\theta_k(t^+) = f_k^\alpha(z_k(t))$;
            k updates $u_k(t^+) = \mathrm{sign}_\varepsilon(z_k(t))$;
        end if
    end for
end for

Main result

Theorem (Practical balancing)

For every initial condition $\bar{x}$, let $x(t)$ be the solution to the Hybrid Coordination Algorithm such that $x(0) = \bar{x}$.
Then $x(t)$ converges in finite time to a point x^* belonging to the set

$$\mathcal{E} = \{x \in \mathbb{R}^n \ : \ ||\underbrace{B^T x}_{z}||_\infty < \varepsilon\}$$

Time cost (time to converge)

$$T := \inf\{t \geq 0 \ : \ x(t) \in \mathcal{E}\} \leq \frac{(\deg_{\max} + 1)}{\varepsilon} ||\bar{x}||^2$$

Communication cost (# updates to converge)

$$C := \max_{i \in I} \max\{k \ : \ t_k^i \leq T\} \leq \frac{4 \deg_{\max} (\deg_{\max} + 1)}{\varepsilon^2} ||\bar{x}||^2$$

Main result

Theorem (Practical balancing)

For every initial condition $\bar{x}$, let $x(t)$ be the solution to the Hybrid Coordination Algorithm such that $x(0) = \bar{x}$. Then $x(t)$ converges in finite time to a point x^* belonging to the set

$$\mathcal{E} = \{x \in \mathbb{R}^n \ : \ ||\underbrace{B^T x}_{z}||_\infty < \varepsilon\}$$

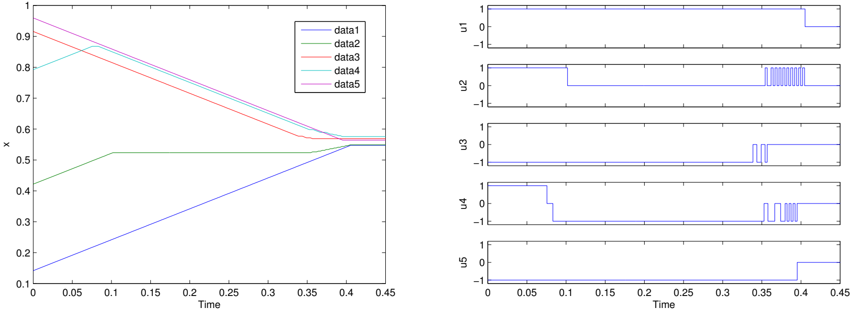
Proof Based on a Lyapunov-like argument for hybrid systems with

$$V(x) = x^T x$$

It satisfies

$$\dot{V}(t) \leq - \sum_{k \in E : |z_k(t_\ell^k)| \geq \varepsilon} \frac{|z_k(t_\ell^k)|}{2}$$

Simulations



Sample evolutions of states x and corresponding controls u on a ring with $n = 5$ nodes, $\varepsilon = 0.02$

Capacity constraints

- Ternary controllers satisfy edge capacity constraints
- Node capacity constraints can also be satisfied

Proposition

Let

$$0 \leq c_{\text{min}} \leq x_i(0) \leq c_{\text{max}}, \quad \text{for all } i \in I.$$

where $0 \leq c_{\text{min}} < c_{\text{max}}$ are bounds on the capacities of the nodes.
Then the solution $x(t)$ to the Hybrid Coordination Algorithm starting from $x(0)$ satisfies

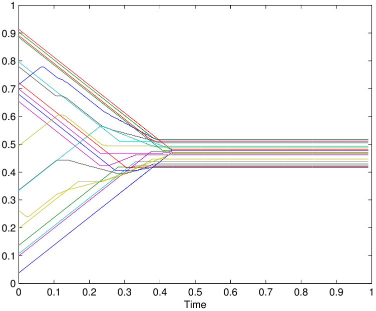
$$0 \leq c_{\text{min}} \leq x_i(t) \leq c_{\text{max}}, \quad \text{for all } i \in I,$$

for all $t \geq 0$.

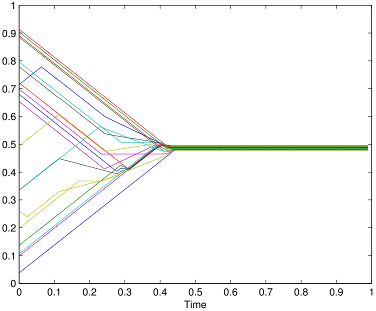
Asymptotical coordination

Asymptotical coordination: idea

Accuracy of practical balancing depends on ε



$n = 20, \varepsilon = 0.01$



$n = 20, \varepsilon = 0.001$

Practical convergence may not be satisfactory: can we do better?

Asymptotical coordination: idea

Underlying idea

- ε is a measure of the size of the region of convergence
- To achieve asymptotical coordination we let $\varepsilon(t) \to 0$
- To prevent Zeno, we must slow down both the information request process and the velocity of the systems

Information request	System velocity
$\frac{1}{\gamma(t)} f_k^{\alpha}(z_k)$	$\gamma(t) \sum_{k \in E} b_{ik} u_k$

in a consistent way, namely $\frac{\varepsilon(t)}{\gamma(t)} \geq c \quad \forall t \geq 0$

Asymptotical coordination: system

Continuous-time dynamics

$$\begin{cases} \dot{x}_i = \gamma(t) \sum_{k\in E} b_{ik} u_k \\ \dot{u}_k = 0 \\ \dot{\theta}_k = -1 \end{cases}$$

Discrete-time dynamics

$$\begin{cases} x_i(t^+) = x_i(t) \quad \forall i \in I \\ u_k(t^+) = \begin{cases} \mathrm{sign}_{\varepsilon(t)}(z_k(t)) & \text{if } k \in \mathcal{I}(\theta, t) \\ u_k(t) & \text{otherwise} \end{cases} \\ \theta_k(t^+) = \begin{cases} \frac{1}{\gamma(t)} f_k^{\alpha}(z_k(t)) & \text{if } k \in \mathcal{I}(\theta, t) \\ \theta_k(t) & \text{otherwise} \end{cases} \end{cases}$$

Asymptotical coordination: results

Theorem (Asymptotical consensus)

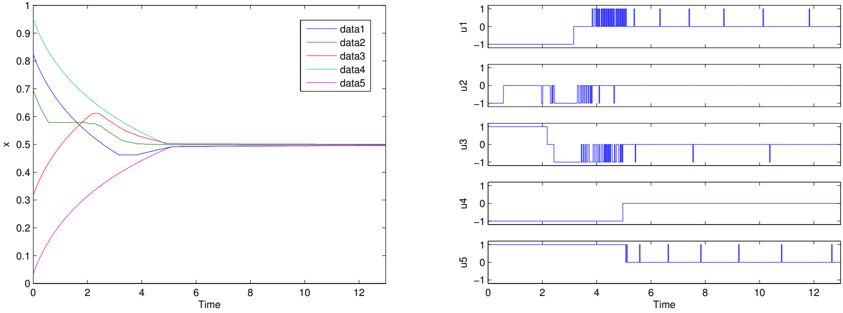
Let $x(\cdot)$ be the solution to the Hybrid Asymptotical Coordination Algorithm. Then, for every initial condition $\bar{x} \in \mathbb{R}^n$ there exists $\beta \in \mathbb{R}$ such that $\lim_{t\to\infty} x_i(t) = \beta$ for all $i \in I$, if and only if $\int_0^{+\infty} \gamma(s)ds$ is divergent

- Condition $\int_0^{+\infty} \gamma(s)ds = +\infty$ is necessary because a "persistent excitement" is needed to ensure convergence
- Dwell time property is satisfied

$$t_{\ell+1}^k - t_\ell^k \geq \frac{1}{\gamma(t_\ell^k)} f_k(x(t_\ell^k)) \geq \frac{\alpha}{4d_{\max}} \frac{\varepsilon(t_\ell^k)}{\gamma(t_\ell^k)} \geq c'$$

- Robustness: no need to have the same γ, ε: we can use different γ_k, ε_k

Simulations



Sample evolutions of x and u on a ring with $n = 5$, $\varepsilon(t) = \frac{0.05}{1+t}$, $\gamma(t) = \frac{0.25}{1+t}$: dwell time is 0.025

Conclusion

Conclusions

- Load balancing in a distribution network
- Coarse controllers and occasional information collection
- Protocols for practical & asymptotical convergence
- Robustness (delays, quantization, clock skews), guaranteed dwell-time
- Network of hybrid systems that synchronize asynchronously

C. De Persis and P. Frasca. Robust self-triggered coordination with ternary controllers. *IEEE Transactions on Automatic Control*, provisionally accepted. Available at `http://arxiv.org/abs/1205.6917`

Work in progress

$$\begin{cases} \dot{x} = Bu \\ z = B^T x \end{cases} \qquad \begin{cases} \dot{w} = \sigma(w) \\ \dot{x} = f(x) + Bu + Pw \\ z = B^T x \end{cases}$$

$$u = -z \qquad \begin{cases} \dot{\eta} = \Phi(\eta, z) \\ u = \Psi(\eta, z) \end{cases}$$

QUantized Information and Control for formation Keeping (QUICK)

NWO Netherlands Organisation for Scientific Research