Frontier: Collaborative Research: Correct-by-Design Control Software Synthesis for Highly Dynamic Systems

Jessy Grizzle, Aaron Ames, Hartmut Geyer, Huei Peng, and Paulo Tabuada

A Classical to Modern Transition

- Classical control geared toward
  - Asymptotically stabilize a point or set
  - Track a set of trajectories
  - Create and stabilize periodic behavior
- Modern engineered systems
  - require the composition of these classical components to meet higher level objectives
- Design requirements typically written in plain English, specifying desired sequence of intermediate objectives
  - start-up and shut-down
  - different modes of operation (e.g., adaptive cruise controllers)
  - priority requirements among conflicting objectives (e.g., safety critical components always take precedence over comfort)

NSF **CPS**

A Classical to Modern Transition

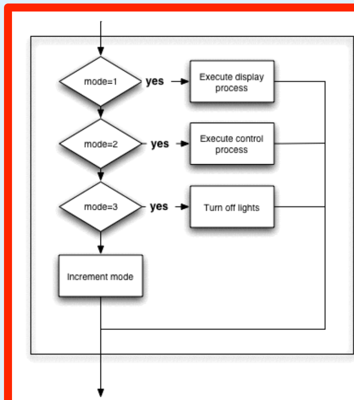
- Outstanding engineers write software attempting to <u>choreograph</u> a suite of classical controllers…

- …to produce complex controllers fulfilling all the design requirements.

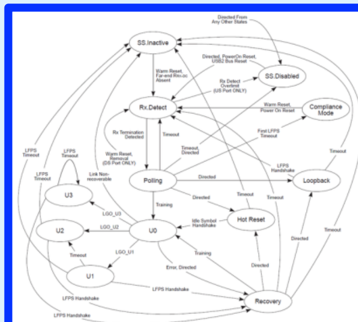
- At least we hope!

- Help! Verification required!

IF-THEN-ELSE



State Machines



Deadlock!

Big Picture: Our Project

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- "Choreography" in English replaced by a **specification in a formal logic (LTL)**

- From specification, **synthesize control software**

  - **that is correct by construction.**
  - for systems of ODEs
  - with time-critical safety requirements…

- Evaluate on hardware---robotics and automotive

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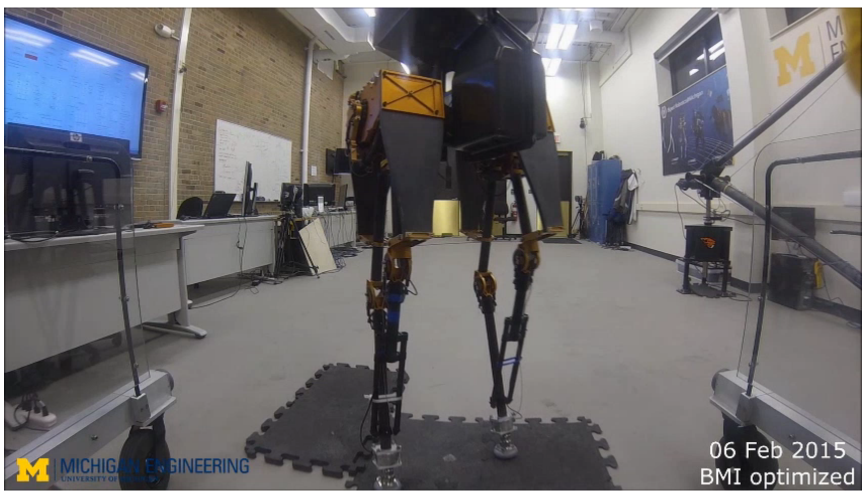
Big Picture: Bipedal Locomotion

- Robotics: bipedal locomotion

Understanding how to formally realize guarantees on dynamic locomotion can allow for richer interactions between humans and robots



→

Safety → Dependability → Trust

Big Picture: Automotive

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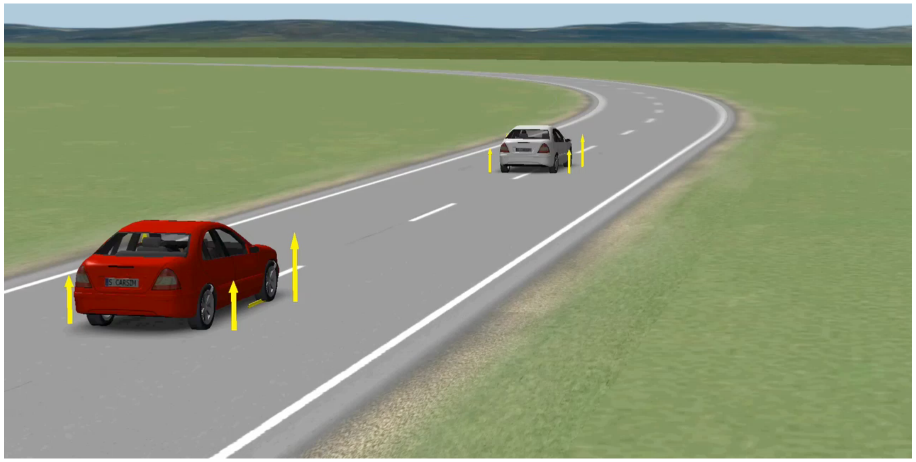
- Automotive: driver convenience and safety

Understanding how to formally realize guarantees on automotive systems can lead to dependability and trust in smart cities



Safety → Dependability → Trust

Industrial Partners

Ford

TOYOTA

Beginning discussions with Eaton Corp.

Input from Ford and Toyota

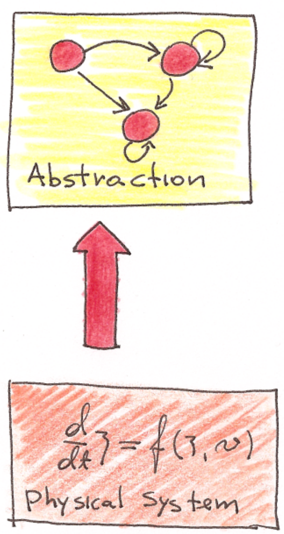
Suggested automotive problems

- Adaptive Cruise Control (ACC)
- Lane Keeping
- Obstacle avoidance
- Composition

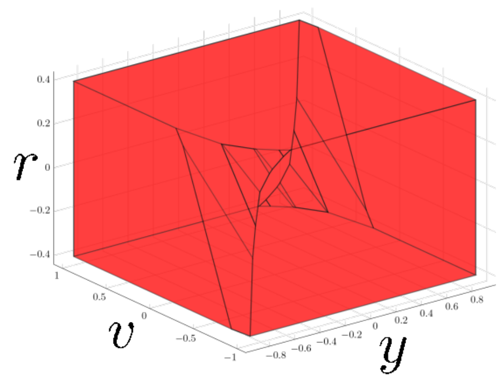
Approaches we are following

- PESSOA ←Abstraction via spatial and temporal discretization
- PCIS ← Linear models and controlled-invariant polyhedra
- Barrier Functions ← Adding formal safety guarantees to classical control

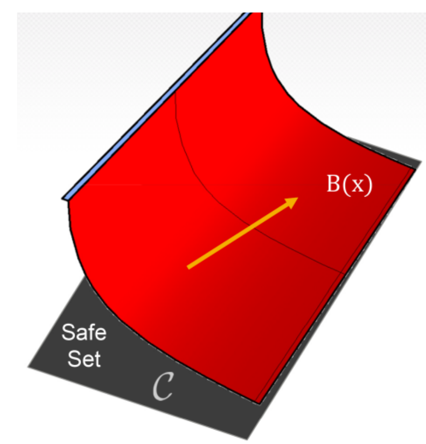
PESSOA



PCIS



Barrier Function



Features One May Care About

	PCIS	Pessoa	Barrier fun.
Complex specifications	TBD	Yes	TBD
Turn-key automation	For LTI	Almost	TBD
Approximation bounds	For LTI	For δ-ISS	Maybe
Parameter tuning	Yes	Some	Yes
Nonlinear	Not now	Yes	Yes
High-dimensional	Not now	Not now	Yes
Termination guarantees	Approximate	Yes	N/A

Supervisor of legacy software	**Yes**	**Yes**	**Yes**

Advances in Control Barrier Functions

Aaron Ames, Xiangru Xu
Jessy Grizzle, Paulo Tabuada

Motivation: Performance & Safety

Goal:

Unify **control objectives** with **safety specifications** in a formal and provably correct manner

Main Idea:

Define **control barrier functions** that allow for the unification in an optimization-based framework

Control Barrier Functions: Last Year

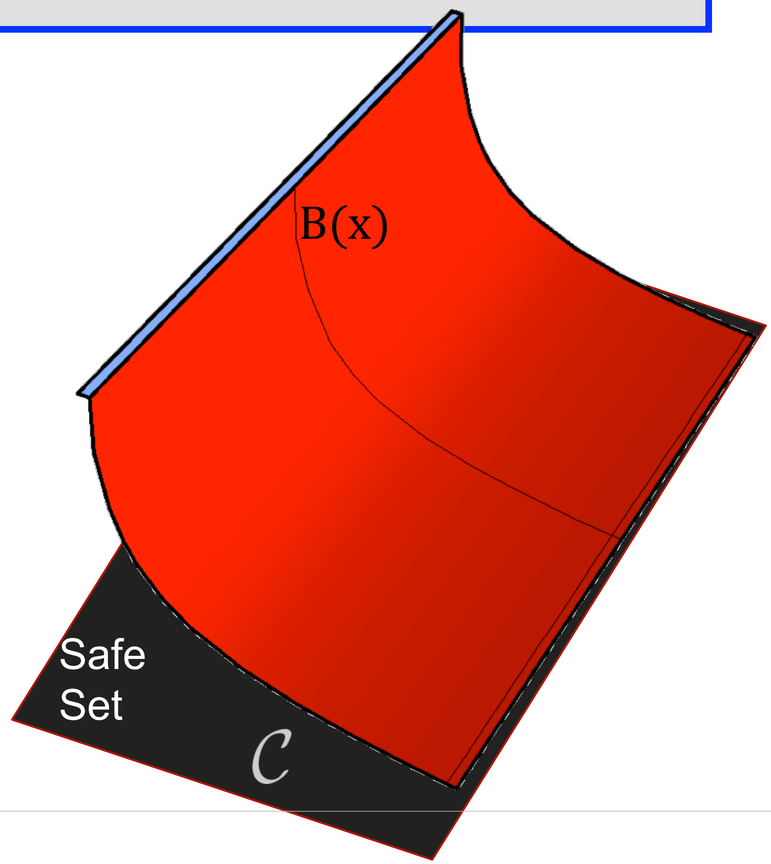
Control Barrier Functions: provably ensure satisfaction of safety specifications

- Define the safe set, $\mathcal{C}$
- Consider a **barrier** function:

$$\inf_{x \in \mathrm{Int}(\mathcal{C})} B(x) \geq 0, \qquad \lim_{x \to \partial \mathcal{C}} B(x) = \infty.$$

- Ensure invariance of the safe set through the requirement:

$$\inf_{u \in \mathcal{U}} \dot{B}(x,u) \leq \frac{\gamma}{B(x)}$$



Control Barrier Functions: Last Year

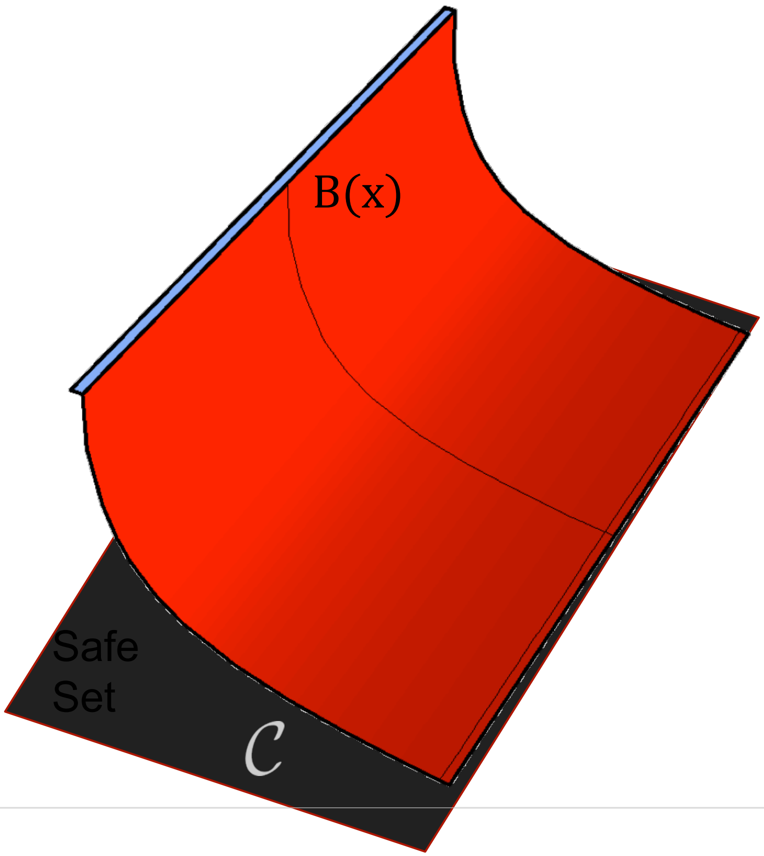
Reciprocal Barrier and Control Barrier Function

$$\inf_{x \in \mathrm{Int}(\mathcal{C})} B(x) \geq 0, \qquad \lim_{x \to \partial \mathcal{C}} B(x) = \infty.$$

and

$$\inf_{u \in \mathcal{U}} \dot{B}(x,u) \leq \frac{\gamma}{B(x)}$$

then the set $\mathcal{C}$ is forward invariant, i.e., the set is provably safe.



Starting Point for a New Barrier Function

- **Generality:** $$\inf_{u \in U} \dot{B}(x,u) \leq \frac{\gamma}{B(x)} \Rightarrow \mathcal{C} \ \text{ safe}$$

The converse? $$\mathcal{C} \ \text{safe} \ \Rightarrow \inf_{u \in U} \dot{B}(x,u) \leq \frac{\gamma}{B(x)} \ ?$$

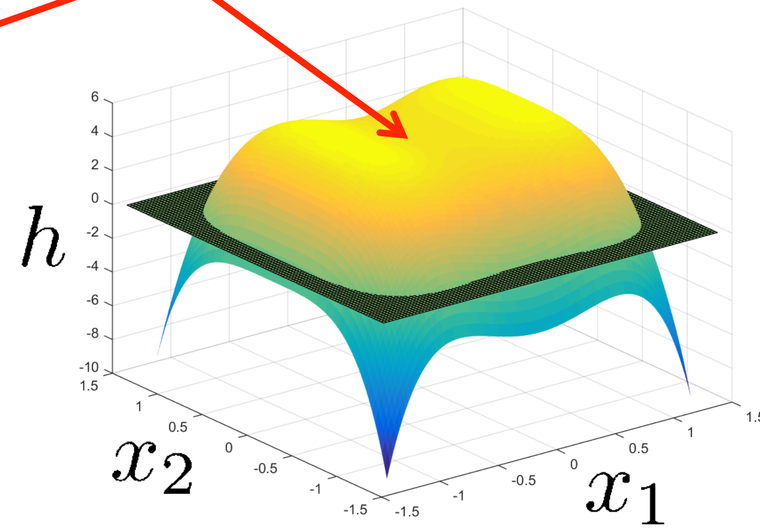
- **Why important?** Less restrictive safety condition implies more freedom for control performance.

- **Robustness:** Model uncertainty [rogue traffic] may force the system out of the safe set. What happens?

Zeroing Barriers

$$\mathcal{C} = \{x \in \mathbb{R}^n : h(x) \geq 0\}$$

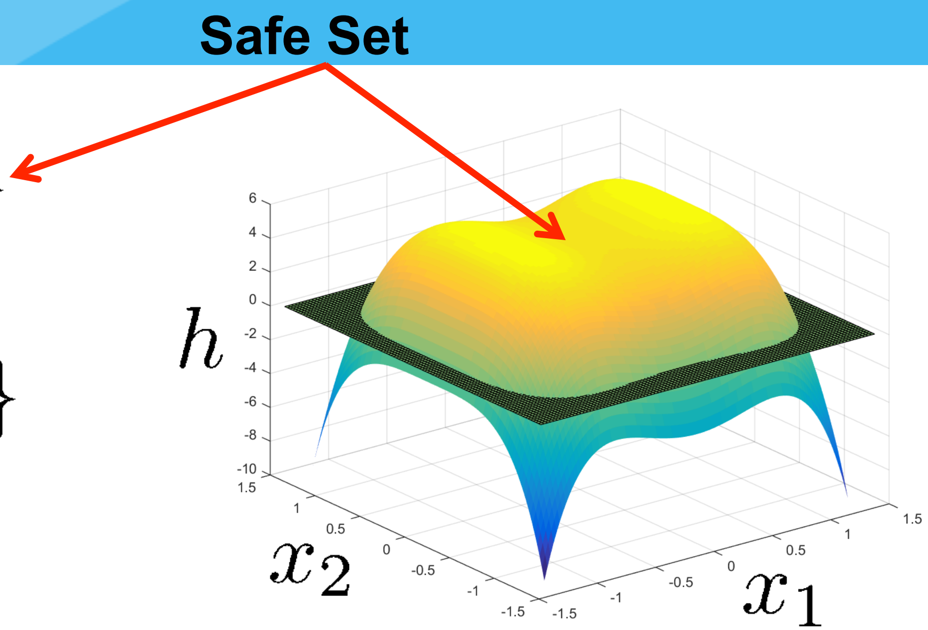
Safe Set



Zeroing Barriers

$$\mathcal{C} = \{x \in \mathbb{R}^n : h(x) \geq 0\}$$

$$\partial\mathcal{C} = \{x \in \mathbb{R}^n : h(x) = 0\}$$



Zeroing Barriers

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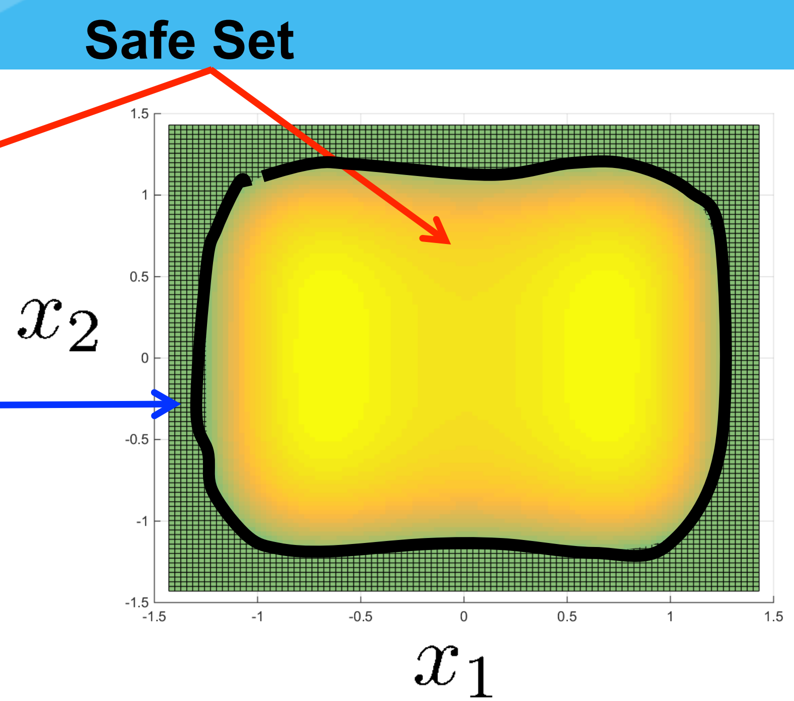
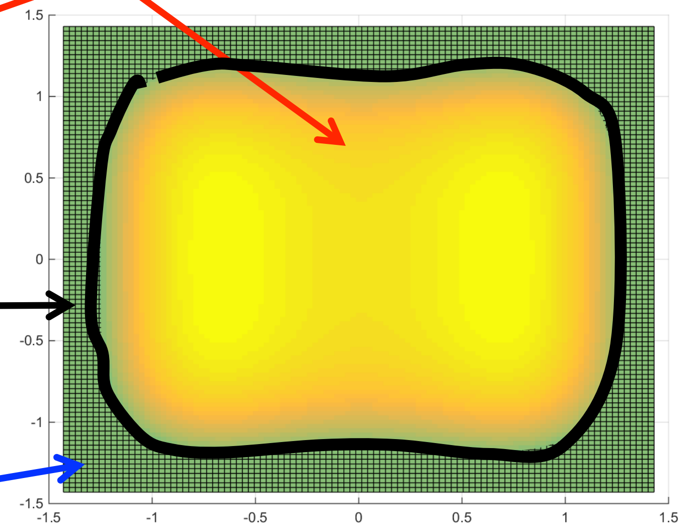


Zeroing Barriers

Safe Set

$$\mathcal{C} = \{x \in \mathbb{R}^n : h(x) \geq 0\}$$

$$\partial\mathcal{C} = \{x \in \mathbb{R}^n : \underline{h(x) = 0}\}$$



x_2

x_1

$$\mathcal{C} \subset \mathcal{D} \subset R^n$$

Def. $h : \mathcal{D} \to \mathbb{R}$ **Zeroing Barrier Function**

$$\dot{h}(x) \geq -\alpha(h(x)) = \begin{cases} > 0 & \text{outside } \mathcal{C} \\ 0 & x \in \partial\mathcal{C} \\ * & x \in \mathrm{Int}(\mathcal{C}) \end{cases}$$

Zeroing Barriers

Safe Set

$$\mathcal{C} = \{x \in \mathbb{R}^n : h(x) \geq 0\}$$

$$\partial\mathcal{C} = \{x \in \mathbb{R}^n : h(x) = 0\}$$

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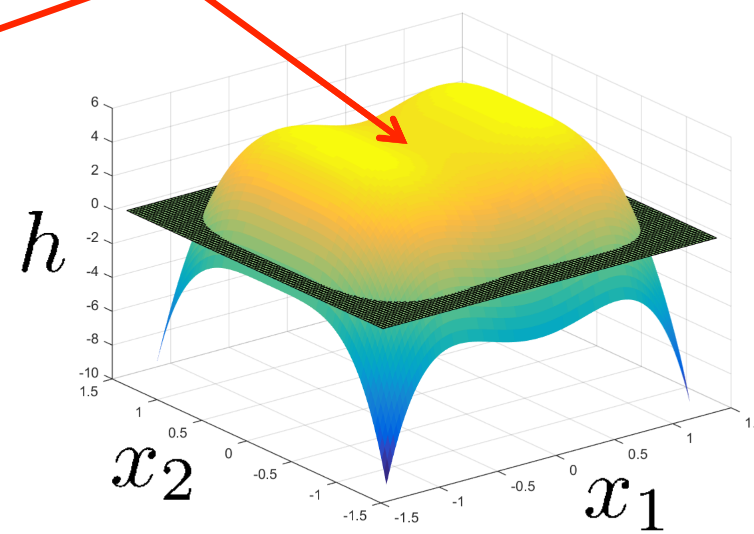


Def. $h : \mathcal{D} \to \mathbb{R}$ **Zeroing Control Barrier Function**

$$\inf_{u \in U} \dot{h}(x,u) \geq -\alpha(h(x)) = \begin{cases} > 0 & \text{outside } \mathcal{C} \\ 0 & x \in \partial\mathcal{C} \\ * & x \in \mathrm{Int}(\mathcal{C}) \end{cases}$$

Zeroing Barriers

Theorem: Selecting control inputs according to

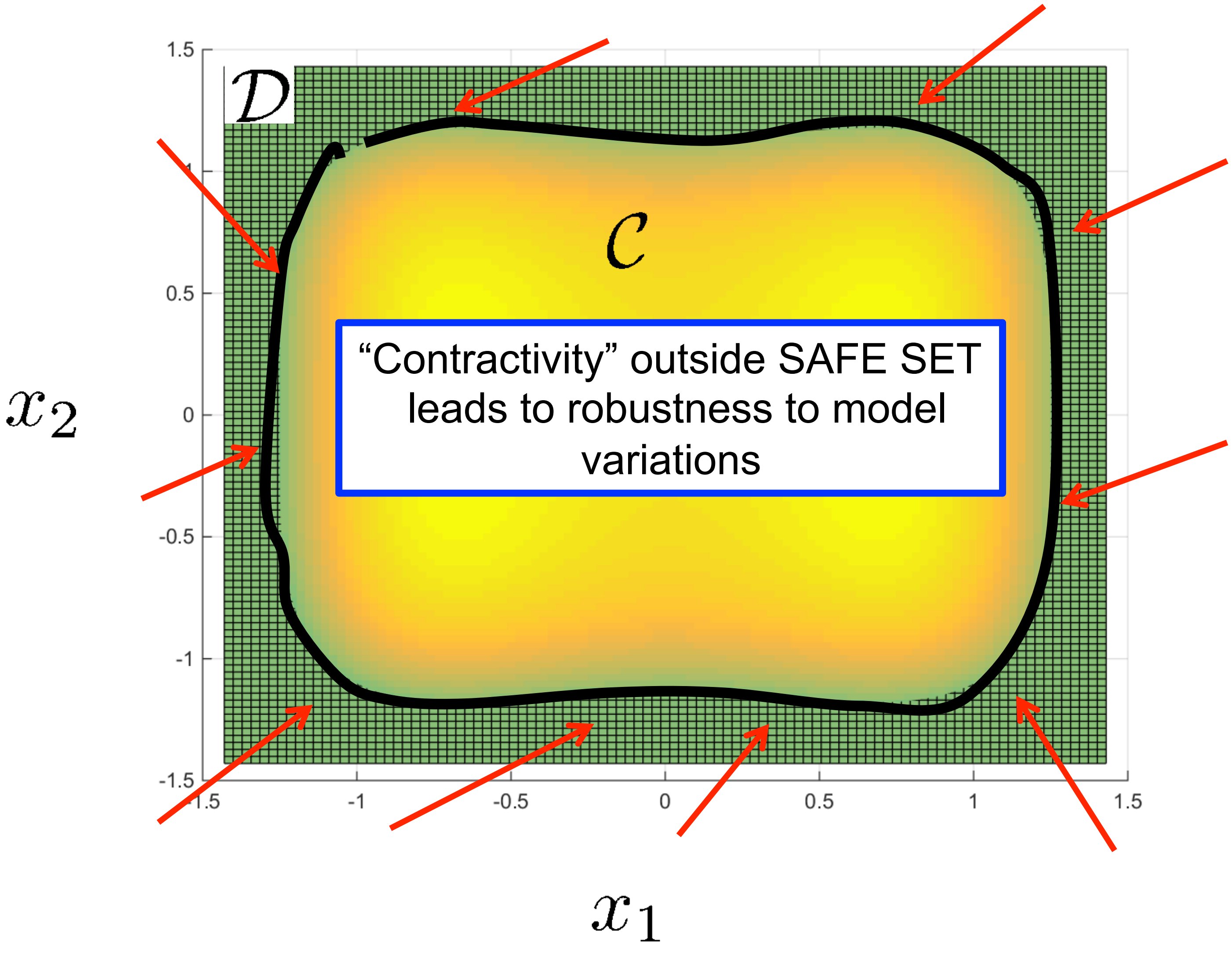
$$\inf_{u \in U} \dot{h}(x,u) \geq -\alpha(h(x))$$

guarantees forward invariance of $\mathcal{C}$ and hence, safety.

Def. $h : \mathcal{D} \to \mathbb{R}$ **Zeroing Control Barrier Function**

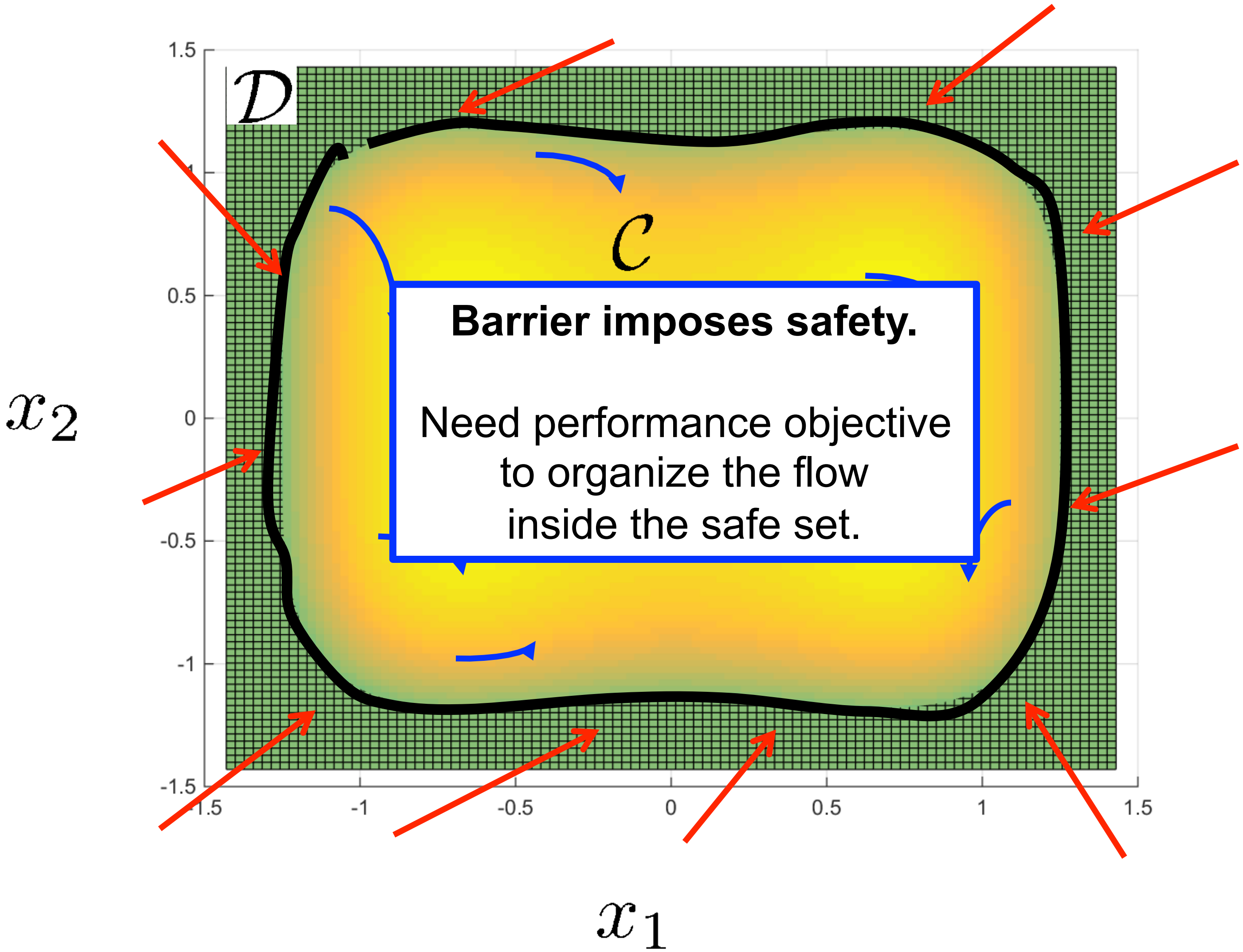
$$\inf_{u \in U} \dot{h}(x,u) \geq -\alpha(h(x)) = \begin{cases} > 0 & \text{outside } \mathcal{C} \\ 0 & x \in \partial \mathcal{C} \\ * & x \in \mathrm{Int}(\mathcal{C}) \end{cases}$$

Zeroing Barriers



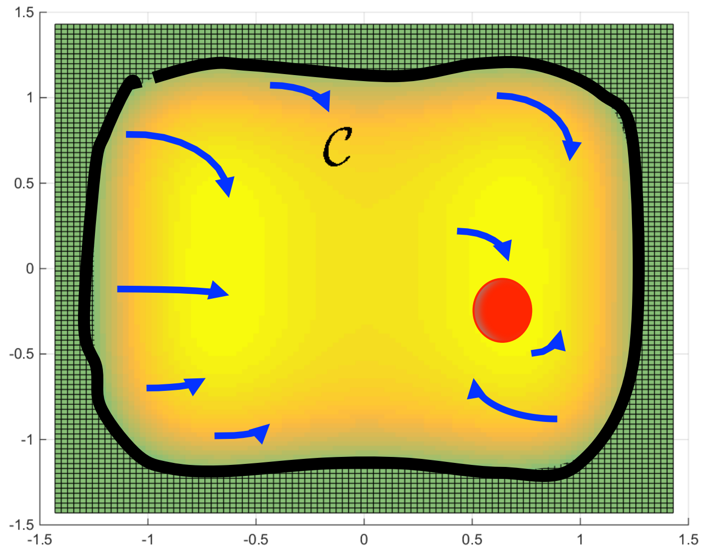
"Contractivity" outside SAFE SET leads to robustness to model variations

Zeroing Barriers

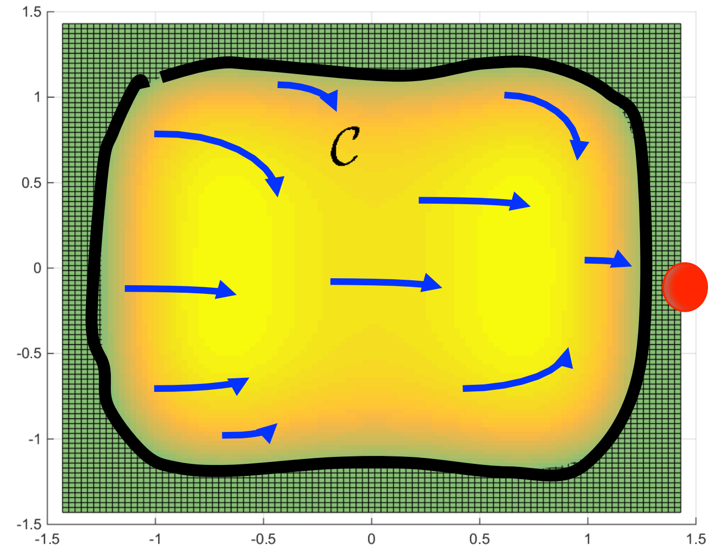


Safety + Performance: 2 Cases

Performance Objective In Safe Set



Performance Objective not in Safe Set



How to combine both cases in a unified framework?

Our answer: Quadratic Program

Multi-Objective QP

Safe Set & Barrier $$\mathcal{C} = \{x \in \mathbb{R}^n : h(x) \geq 0\}$$

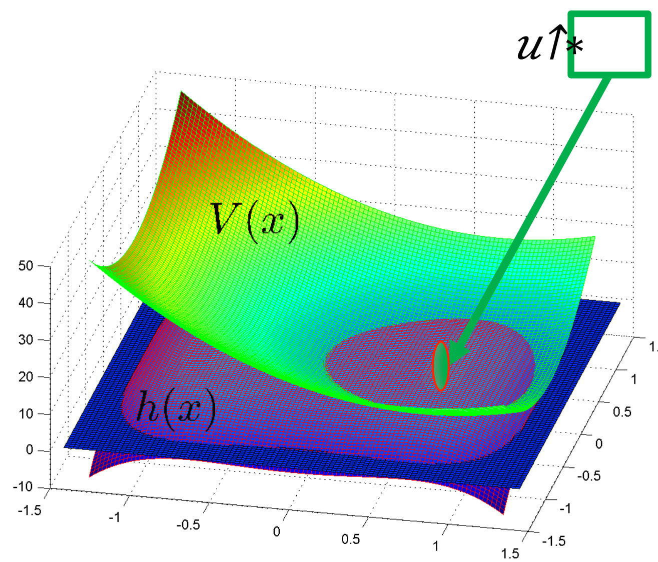
Control System $$\dot{x} = f(x) + g(x)u$$

u*(x) = smallest control input assuring safety

$$\dot{h}(x,u) \geq -\alpha(h(x))$$

…and performance as "close as possible"

$$\dot{V}(x,u) \approx -cV(x)$$



First: QP for Safety Only

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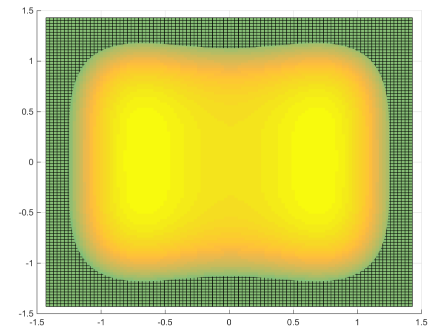
Safe Set & Barrier $\mathcal{C} = \{x \in \mathbb{R}^n : h(x) \geq 0\}$

Control System $\dot{x} = f(x) + g(x)u$



Smallest control input assuring safety

$$u^*(x) = \underset{u \in \mathbb{R}^m}{\operatorname{argmin}} \quad u^\top u$$

$$\text{s.t.} \quad \dot{h}(x,u) \geq -\alpha(h(x))$$

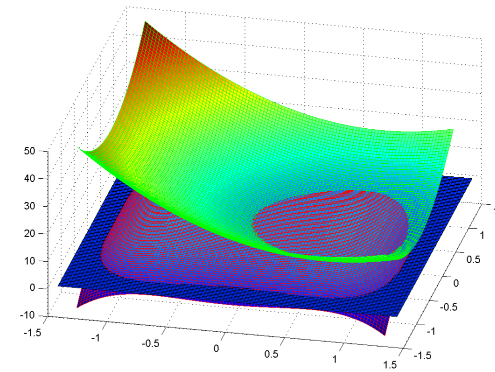
$$\dot{h}(x,u) = L_f h(x) + L_g h(x) u$$

CPS

Now: QP for Safety and Performance

Safe Set & Barrier $\mathcal{C} = \{x \in \mathbb{R}^n : h(x) \geq 0\}$

Control System $\dot{x} = f(x) + g(x)u$



Smallest control input assuring safety

$$u^*(x) = \underset{u \in \mathbb{R}^m}{\operatorname{argmin}} \quad u^\top u$$

$$\text{s.t.} \quad L_f h(x) + L_g h(x) u \geq -\alpha(h(x))$$

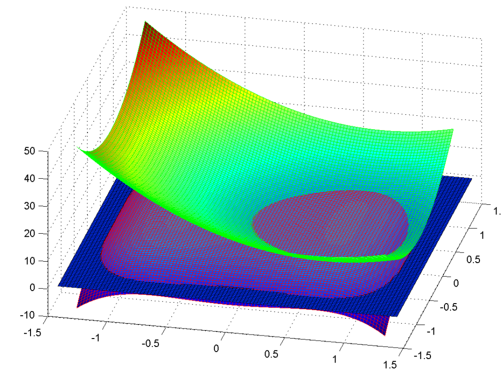
…AND performance as "close as possible"

$$\boxed{\dot{V}(x,u) \approx -cV(x)}$$

Multi-Objective QP

Safe Set & Barrier $\mathcal{C} = \{x \in \mathbb{R}^n : h(x) \geq 0\}$

Control System $\dot{x} = f(x) + g(x)u$



Smallest control input assuring safety

+ performance

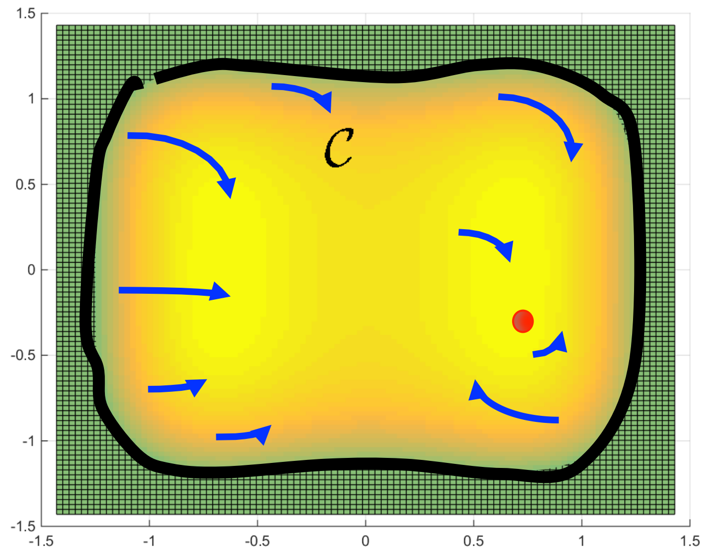
$$u^*(x) = \underset{u \in \mathbb{R}^m}{\operatorname{argmin}} \quad u^\top u + 10^2 \delta^2$$

$$\text{s.t.} \quad L_f h(x) + L_g h(x) u \geq -\alpha(h(x))$$

$$L_f V(x) + L_g V(x) u \leq -cV(x) + \delta$$

Multi-Objective QP

CPS



Feasible w/o relaxation

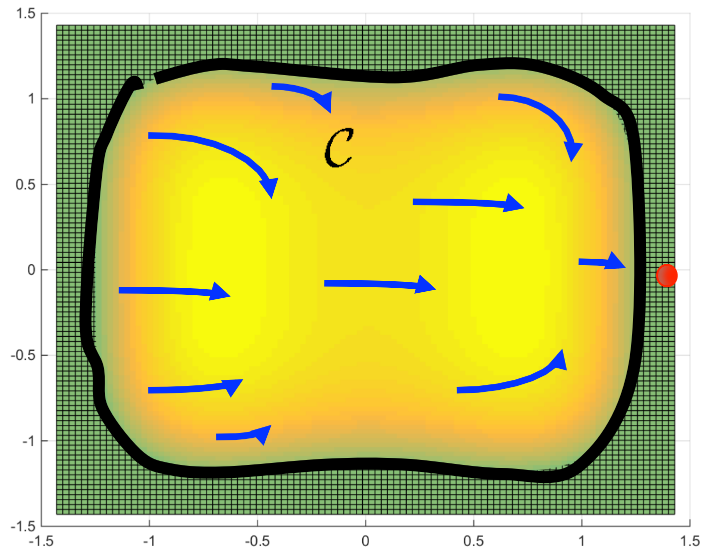
$$u^*(x) = \underset{u \in \mathbb{R}^m}{\operatorname{argmin}} \quad u^\top u + 10^2 \delta^2$$

$$\text{s.t.} \quad L_f h(x) + L_g h(x) u \geq -\alpha(h(x))$$

$$L_f V(x) + L_g V(x) u \leq -cV(x) + \delta$$

called relaxation parameter

Multi-Objective QP



Infeasible w/o relaxation

$$u^*(x) = \underset{u \in \mathbb{R}^m}{\text{argmin}} \quad u^\top u + 10^2 \delta^2$$

$$\text{s.t.} \qquad L_f h(x) + L_g h(x) u \geq -\alpha(h(x))$$

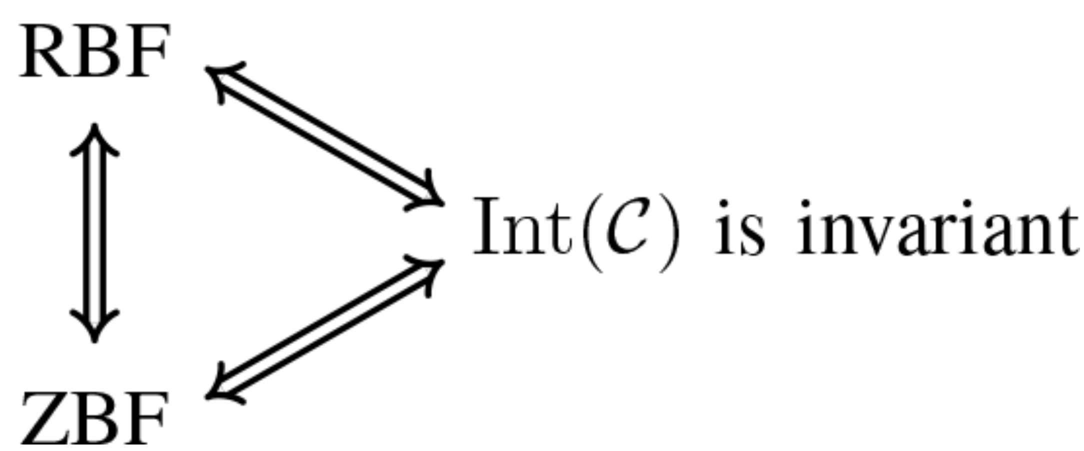
$$L_f V(x) + L_g V(x) u \leq -cV(x) + \delta$$

called relaxation parameter

Characterization

- When QPs produce Lipschitz controllers
- Robustness to disturbances
- Relationships

Theorem 2:
(Assume $\mathcal{C}$ is compact and contractive)



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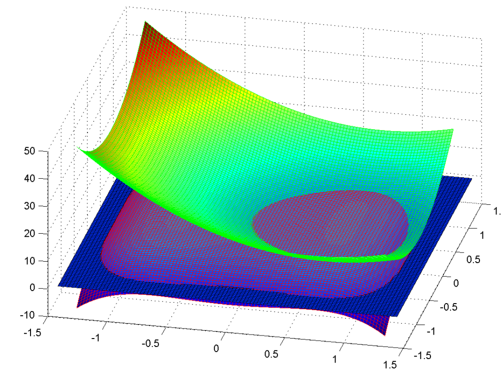
Lipschitz Continuity of QPs

Safe Set & Barrier $\mathcal{C} = \{x \in \mathbb{R}^n : h(x) \geq 0\}$

Control System $\dot{x} = f(x) + g(x)u$



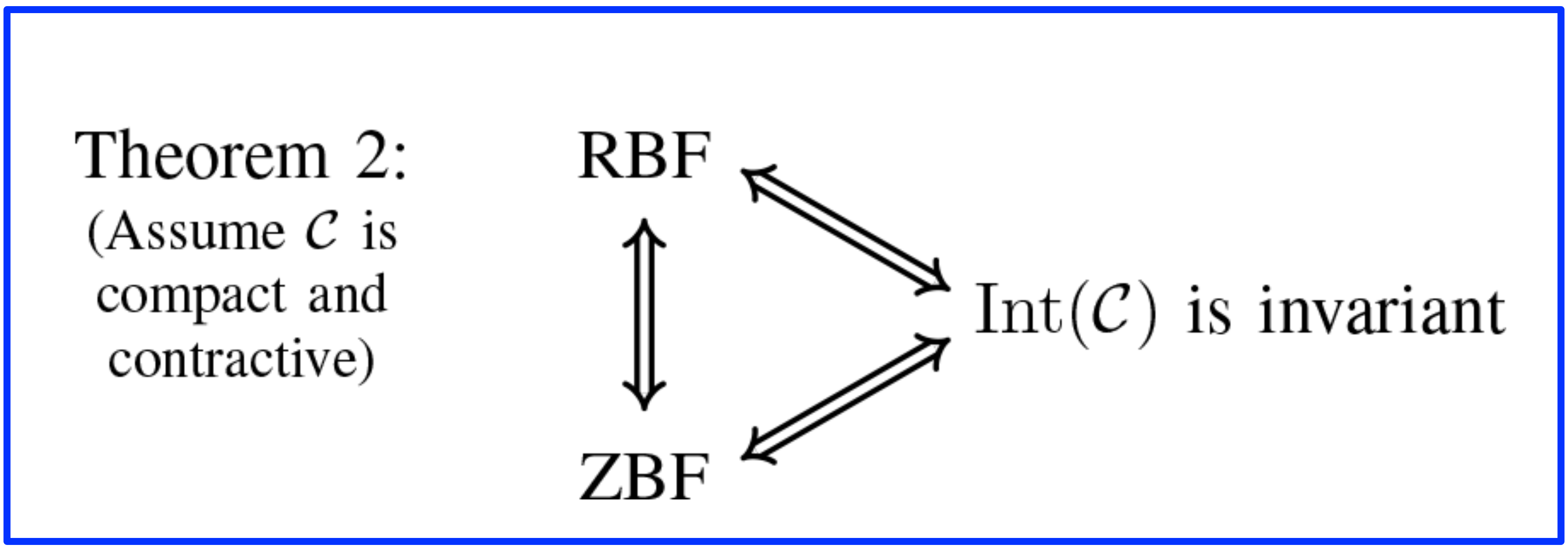
- Theorem: QPs produce Lipschitz continuous controls when

$$L_g h(x) = \frac{\partial h(x)}{\partial x} g(x) \neq 0, \quad x \in \mathcal{C}$$

$$\begin{aligned}
\mathbf{u}^*(x) = \quad & \underset{\mathbf{u}=(u,\delta)\in\mathbb{R}^m\times\mathbb{R}}{\operatorname{argmin}} \quad \frac{1}{2}u^\top u + 10^2\delta^2 \\
\text{s.t.} \quad & L_fV(x) + L_gV(x)u + c_3V(x) - \delta \leq 0, \\
& L_fh(x) + L_gh(x)u + \alpha(h(x)) \geq 0,
\end{aligned}$$

Relations

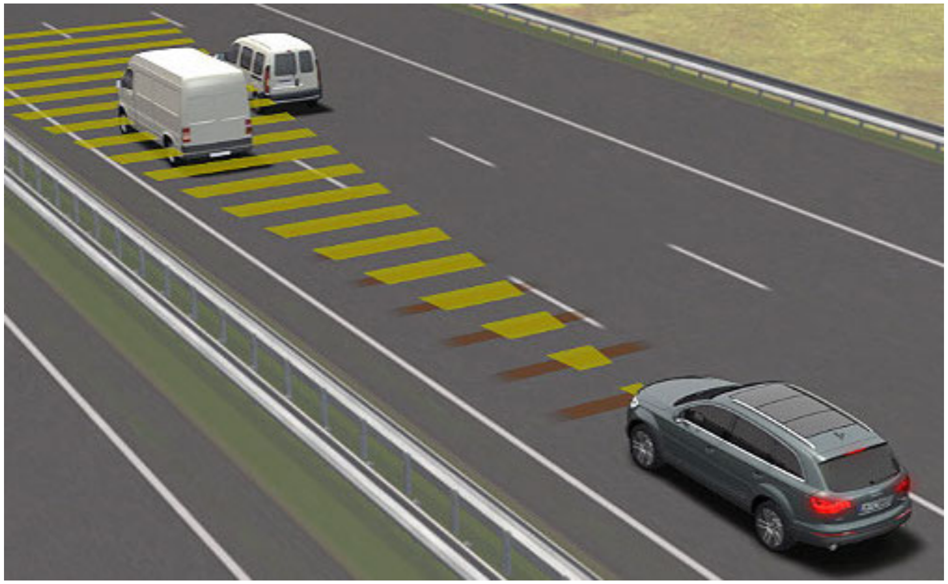
- **Relationship:** Under certain conditions, controlled invariance, RBF and ZBF are equivalent



Applications

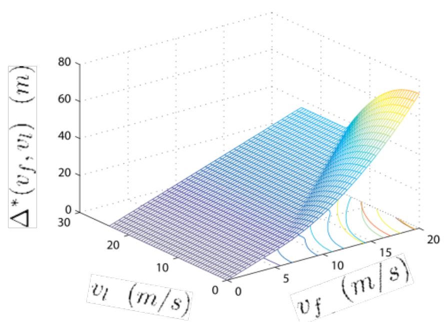
- ACC
- Lane Keeping
- *Working on* composition: LK + ACC
- Others are already using our work

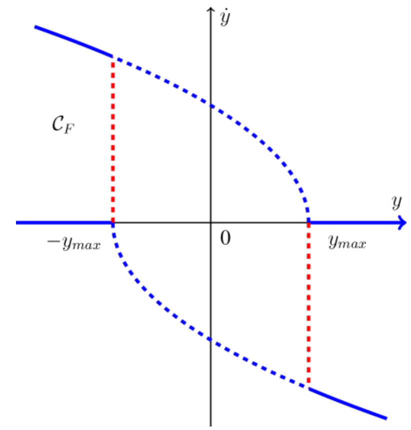
Applications



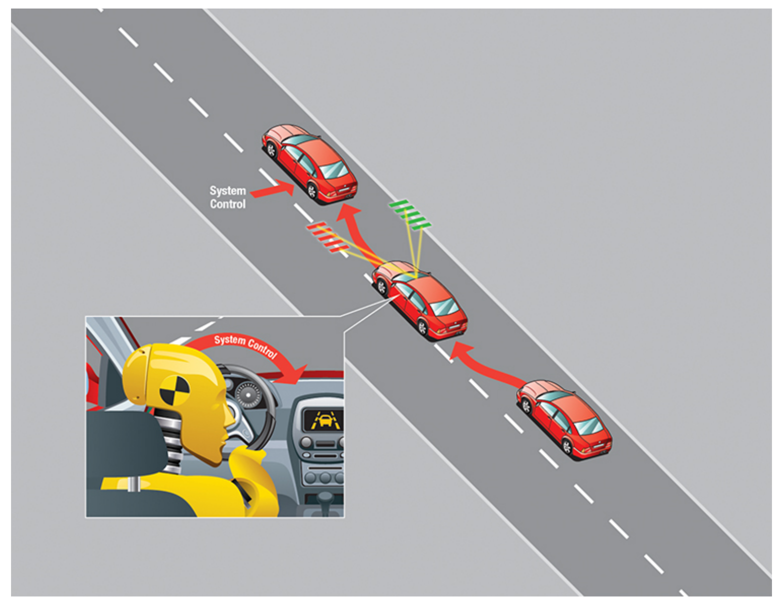
From http://www.proctorcars.com/how-does-adaptive-cruise-control-work/

Adaptive Cruise Control





Lane Keeping



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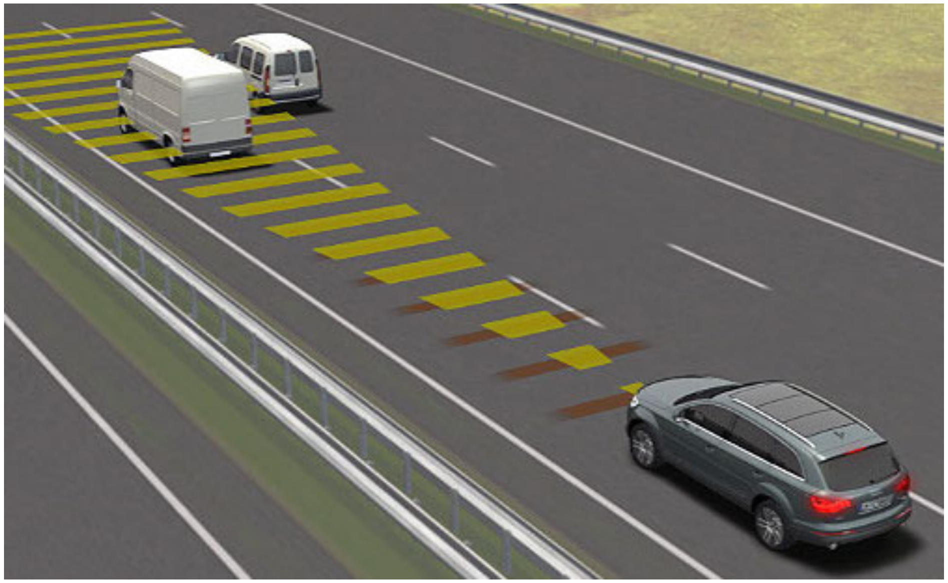
Applications



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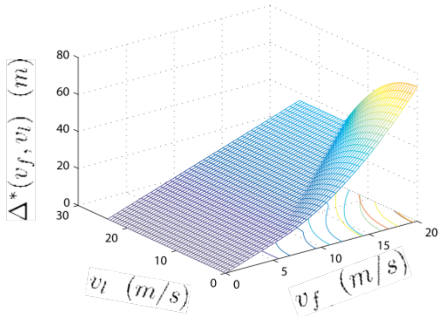
Adaptive Cruise Control



Robustness to Road Grade Uncertainty

Current Challenge Composition

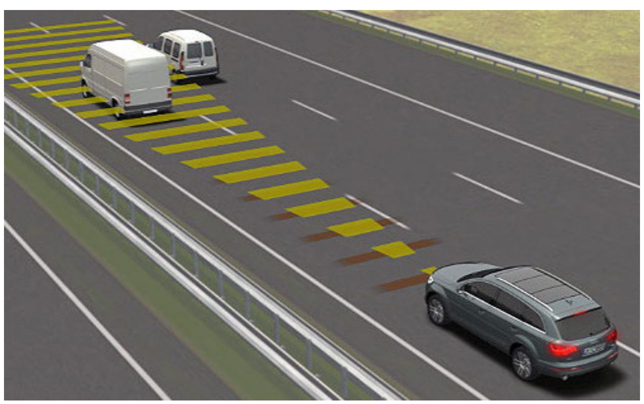
$$\Box S_U \wedge \Box \left(\bigwedge_{i=1}^{2} (\Box(M_i \implies \Diamond\Box T_i)) \right) \bigwedge ?$$

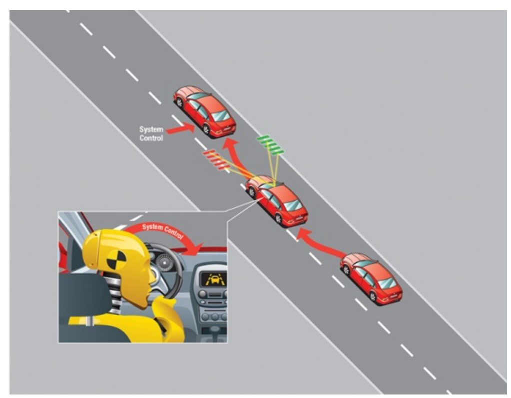
$$\Box(y \in [-0.9, 0.9] \wedge \ddot{y} \in [-0.3g, 0.3g])$$

Current Challenge Composition

$$\frac{d}{dt}\begin{bmatrix} y \\ \nu \\ \psi - \psi_d \\ r \end{bmatrix} = \begin{bmatrix} 0 & 1 & v_f & 0 \\ 0 & -\frac{C_{\alpha f}+C_{\alpha r}}{m\, v_f} & 0 & \frac{bC_{\alpha r}-aC_{\alpha f}}{m\, v_f} - v_f \\ 0 & 0 & 0 & 1 \\ 0 & \frac{bC_{\alpha r}-aC_{\alpha f}}{I_z\, v_f} & 0 & -\frac{a^2C_{\alpha f}+b^2C_{\alpha r}}{I_z\, v_f} \end{bmatrix} \begin{bmatrix} y \\ \nu \\ \psi - \psi_d \\ r \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{C_{\alpha f}}{m} \\ 0 \\ a\frac{C_{\alpha f}}{I_z} \end{bmatrix} \delta_f + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} r_d$$

$$\frac{d}{dt}\begin{bmatrix} v_f \\ v_l \\ D \end{bmatrix} = \begin{bmatrix} -F_r/m + u \\ a_L \\ v_l - v_f \end{bmatrix}$$





Current Challenge Composition

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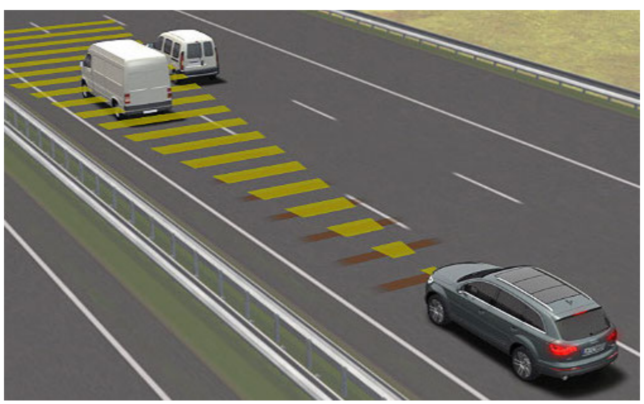
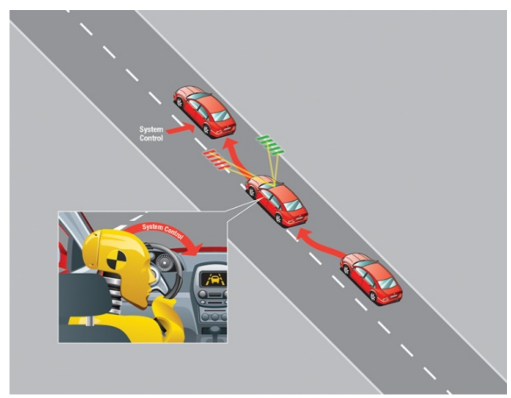
$$\frac{d}{dt}\begin{bmatrix} v_f \\ v_l \\ D \end{bmatrix} = \begin{bmatrix} -F_r/m + u/m - \nu r \\ a_L \\ v_l - v_f \end{bmatrix}$$





CarSim—16 DOF Model

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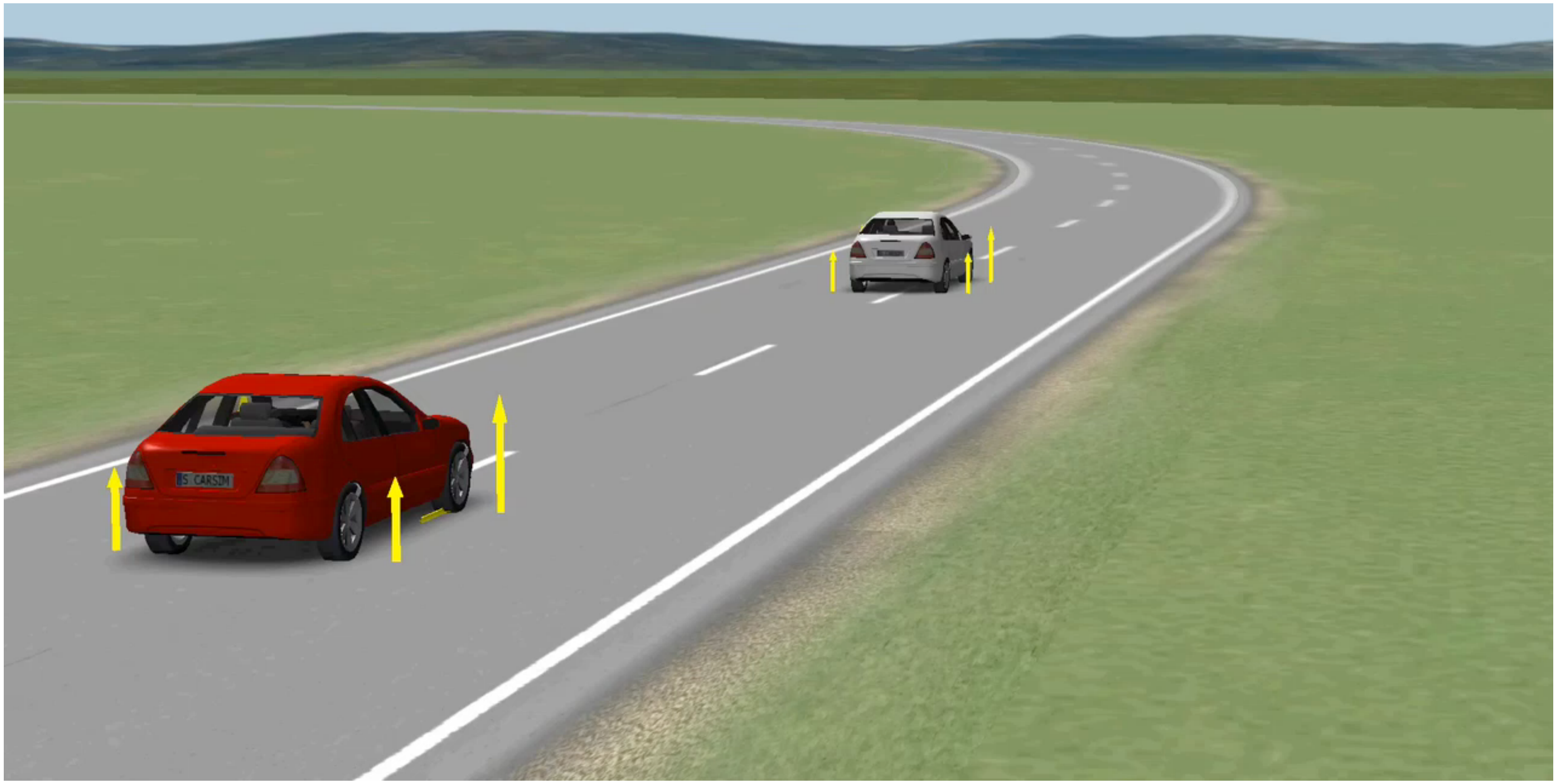
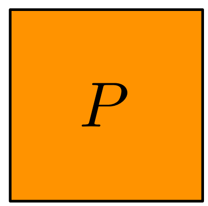
Have a start on it



Composition

Composing, decomposing, recomposing, …

Three approaches to the compositional synthesis problem:



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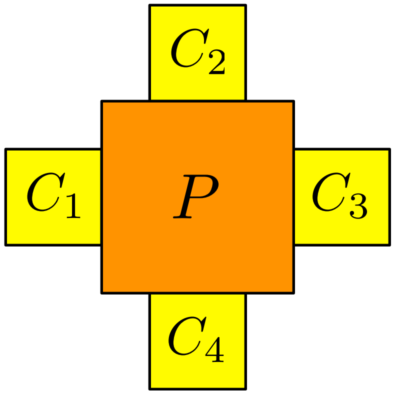

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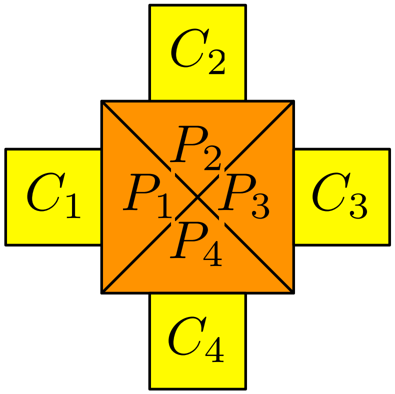

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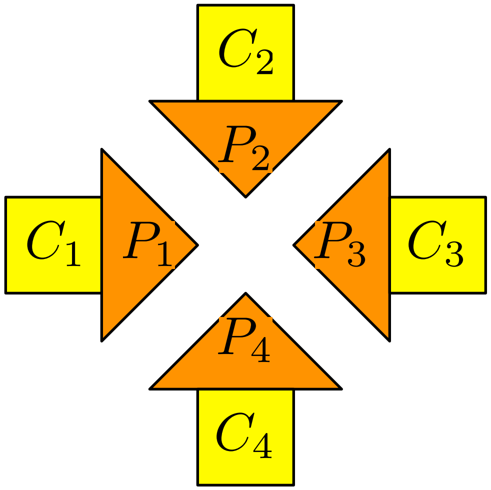

Composing, decomposing, recomposing, …

CPS

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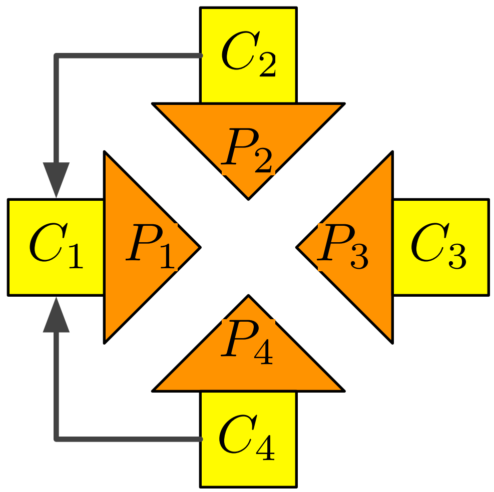

	[DT15]		
View	Internal		
Models	Discrete		
Key ingredient	Ranking functions		

[DT15] On Compositional Symbolic Controller Synthesis Inspired by Small-Gain Theorems. E. Dallal and P. Tabuada. To appear in CDC15, 2015.

Composing, decomposing, recomposing, …

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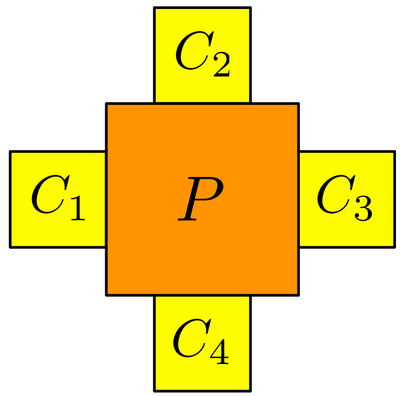

	[DT15]	[XOG15]	
View	Internal	Input-output	
Models	Discrete	Discrete/continuous	
Key ingredient	Ranking functions	Passivity indices	

[XOG15] Passivity Degradation in Discrete Control Implementations: An Approximate Bisimulation Approach, X. Xu, N. Ozay, V. Gupta, To appear in CDC15, 2015

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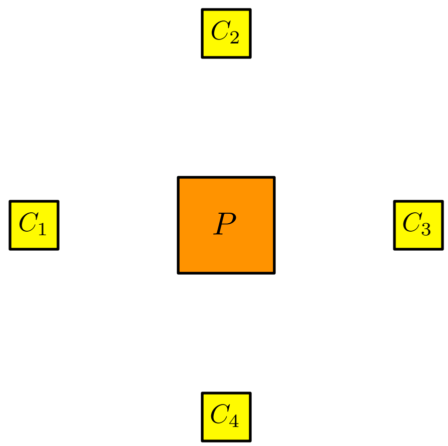

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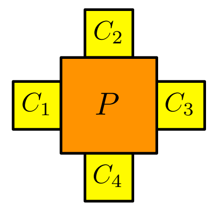

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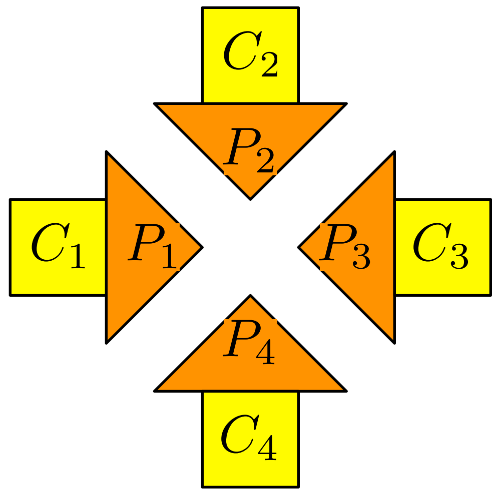

	[DT15]	[XO15]	[NO15]
View	Internal	Input-output	Internal
Models	Discrete	Discrete/continuous	Continuous
Key ingredient	Ranking functions	Passivity indices	Robust invariance

[NO15] Synthesis of separable controlled invariant sets for modular local control design. P. Nilsson and N. Ozay. Submitted to ACC15, 2015

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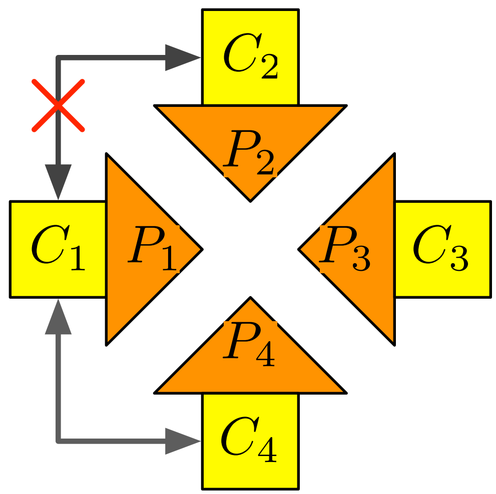

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Formal Controller Synthesis for Bipedal Robotic Walking

MICHIGAN

UCLA

GT

Motivation

Formal Methods in Robotics

Necessary to create and certify the next generation of robotic walking behaviors:

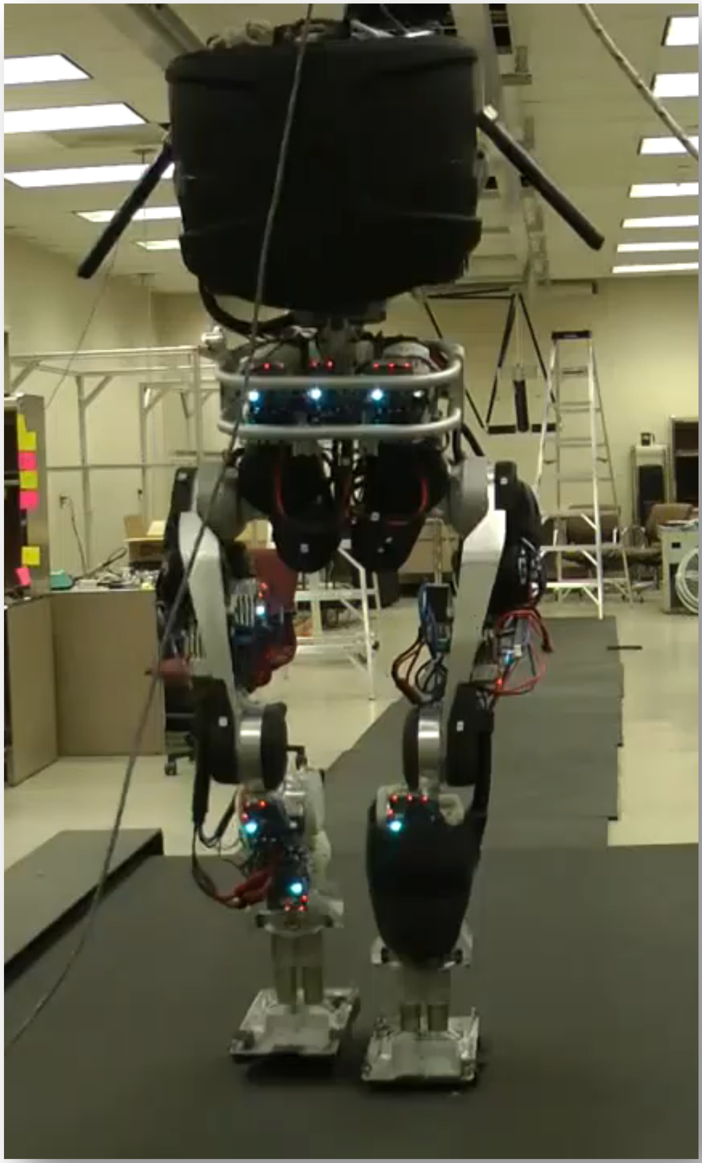
Correct by construction control…
if you design it, it will work!

Challenges:

- *High dimensional:* over 50 dimensions
- *Highly dynamic:* traditional notions of stability are not sufficient
- *Hybrid:* Dynamics involve both discrete and continuous behavior

Approach:

Unify formal methods and feedback control to overcome curse of dimensionality



What is Walking?

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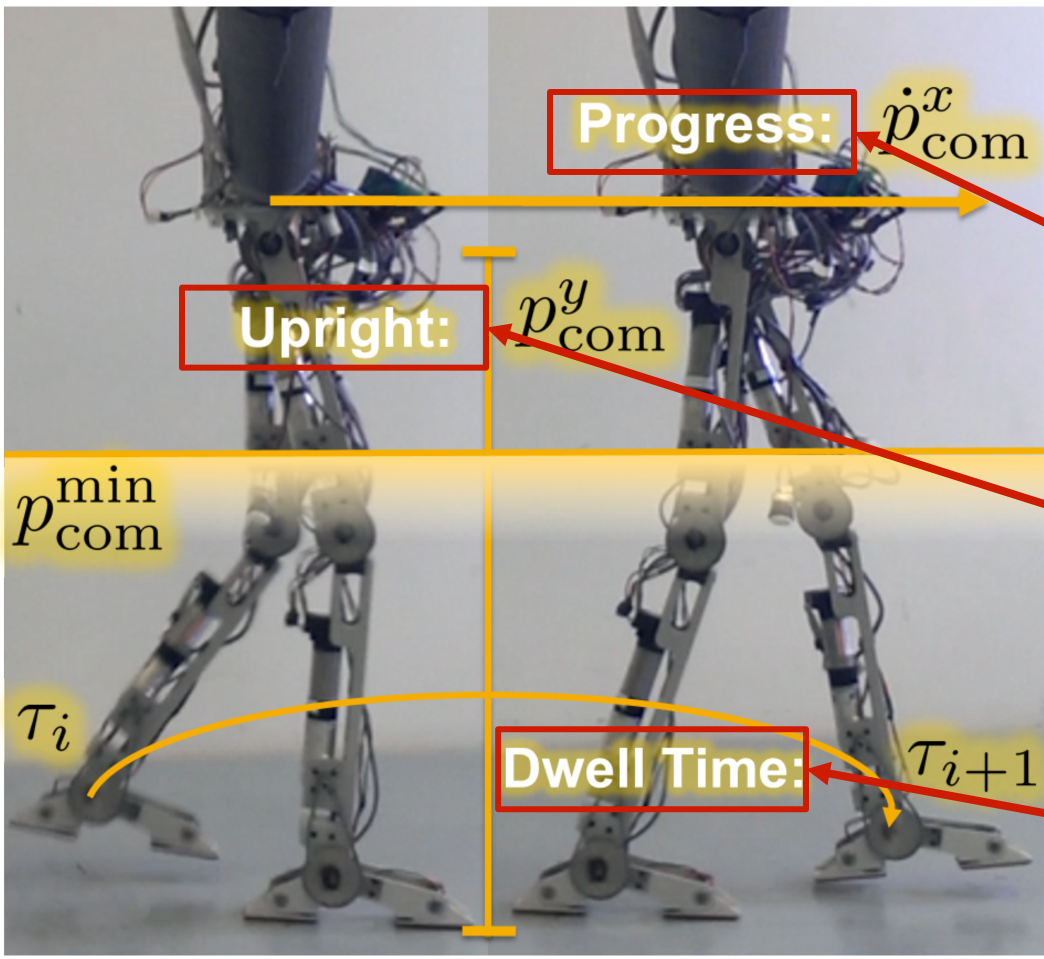
Synthesis Objective: $\Box(P_1 \cap P_2 \cap P_3)$, *i.e., render a subset* $P \subseteq P_1 \cap P_2 \cap P_3$ *invariant.*



- ☑ Forward progression of the center of mass
- ☑ Center of mass stays upright
- ☑ Each step takes nonzero time to complete

Specifications

CPS

Synthesis Objective: $\square(P_1 \cap P_2 \cap P_3)$, *i.e., render a subset* $P \subseteq P_1 \cap P_2 \cap P_3$ *invariant.*

Maximum torque is not exceeded

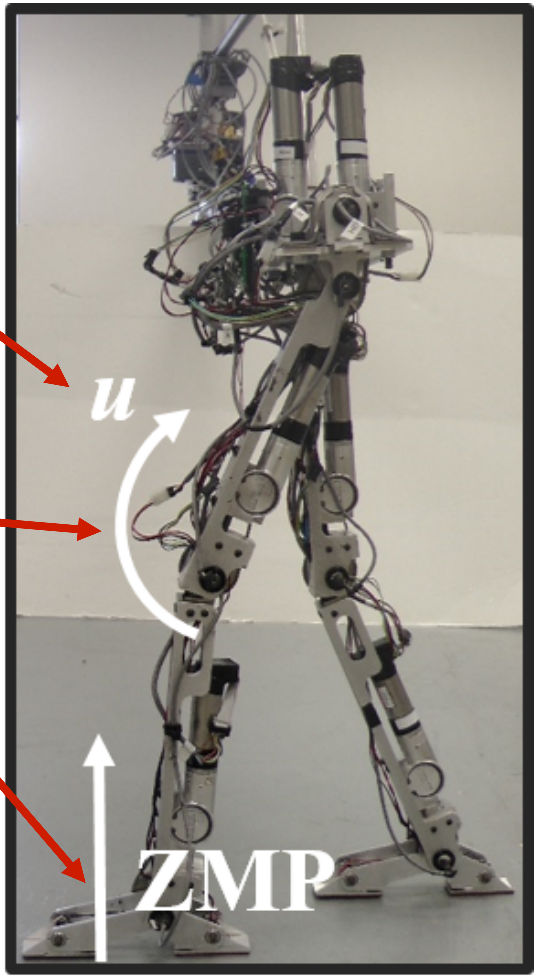
$$P_1 = \{z \in \mathcal{D}_{\mathbf{PZ}} : |u(\vartheta_r(z), \dot{\vartheta}_r(z))| < u_{\max}\}$$

Maximum speed is not exceeded

$$P_2 = \{z \in \mathcal{D}_{\mathbf{PZ}} : |\dot{\vartheta}_r(z)| < \dot{\theta}_{\max}\}$$

Foot stays flat on the ground (ZMP stays in the foot)

$$P_3 = \{z \in \mathcal{D}_{\mathbf{PZ}} : A_{\mathrm{ZMP}} F_{st}(\vartheta_r(z), \dot{\vartheta}_r(z), u(\vartheta_r(z), \dot{\vartheta}_r(z))) < 0\}$$



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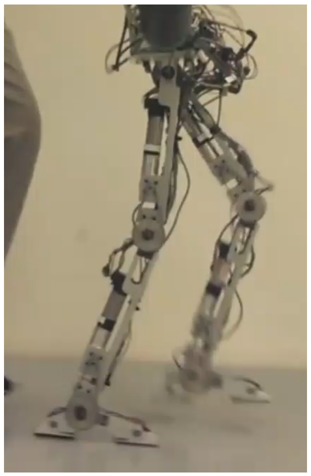
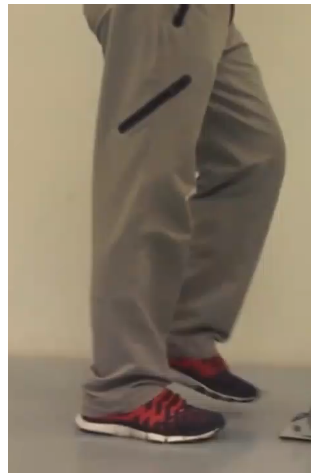
Controller Synthesis

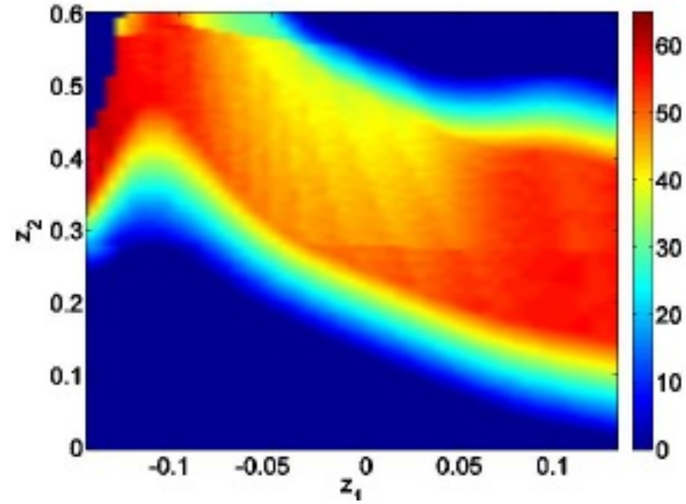


Actual Outputs: y_a

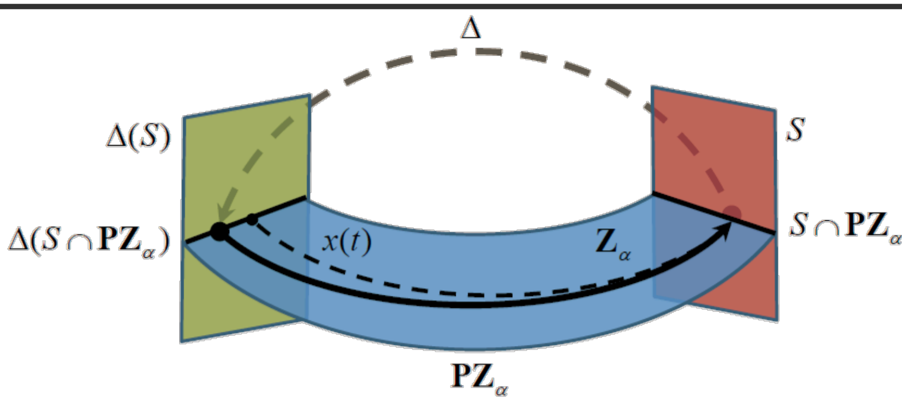
Drive: $y_a \to y_d$



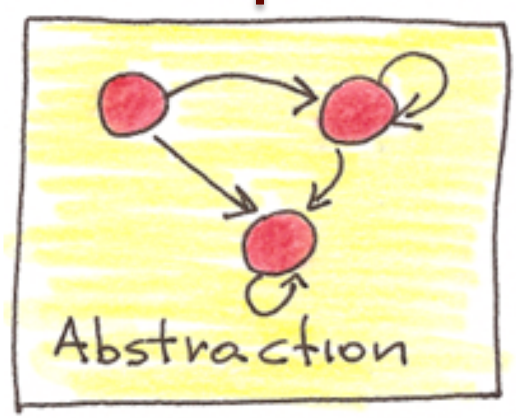
Desired Outputs: y_d



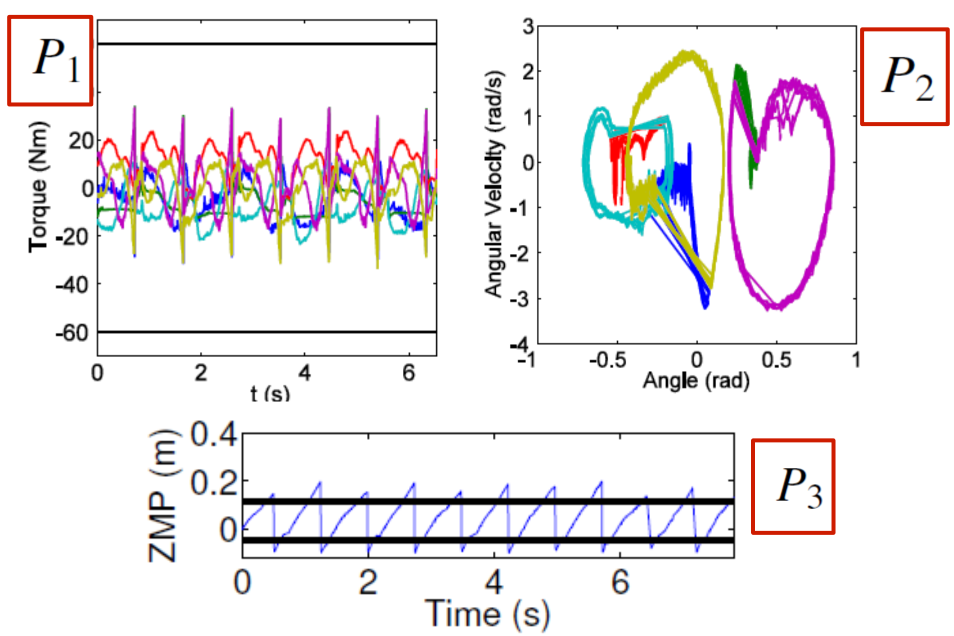
Safe set and number of inputs allowed by the controller in each state.



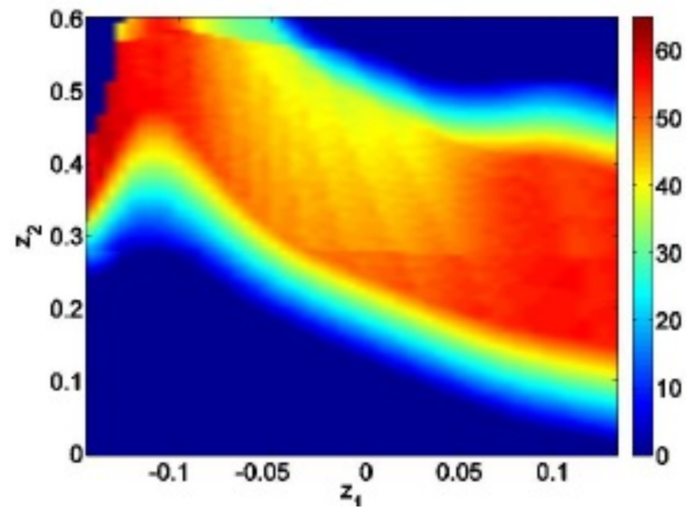
Reduced order representation given by hybrid zero dynamics



Physical Realization



All specifications are "satisfied": $\square(P_1 \wedge P_2 \wedge P_3)$



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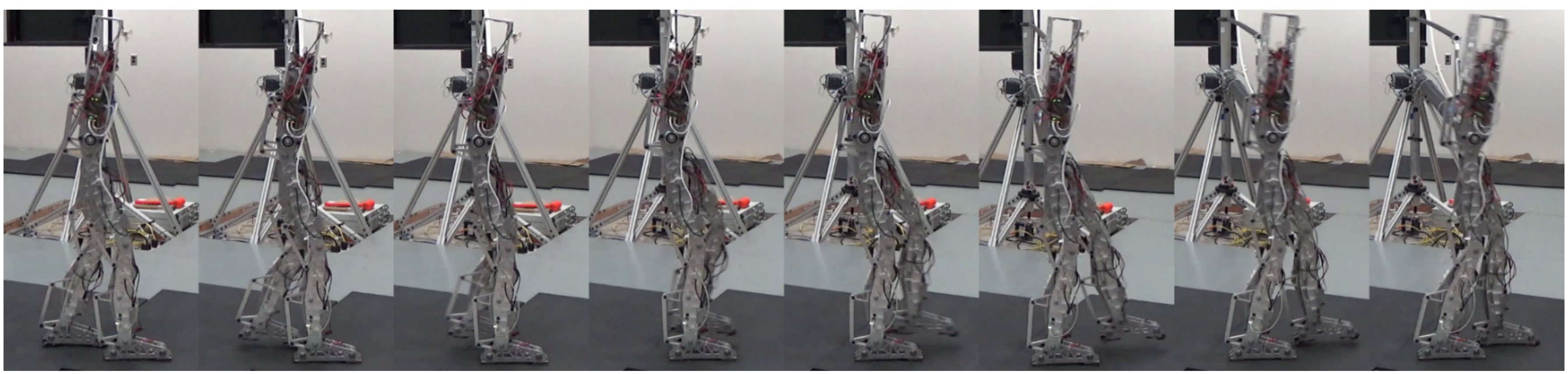
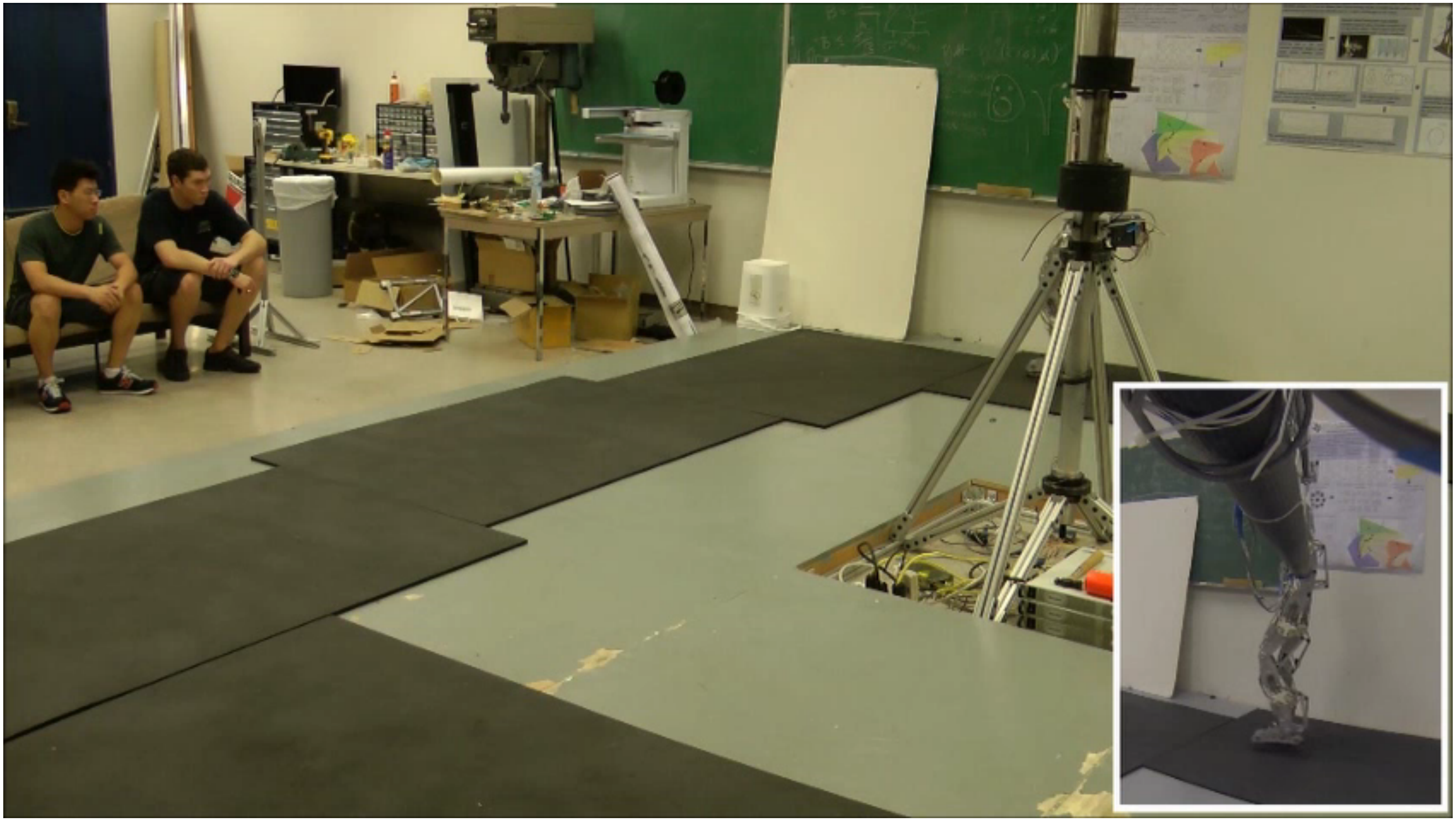


CPS

Formally Synthesized Walking

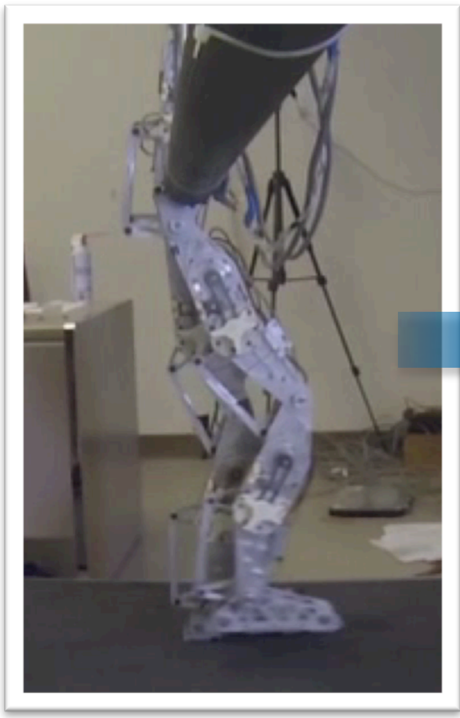


First experimental realization of formal methods on a bipedal walking robot

Future Work

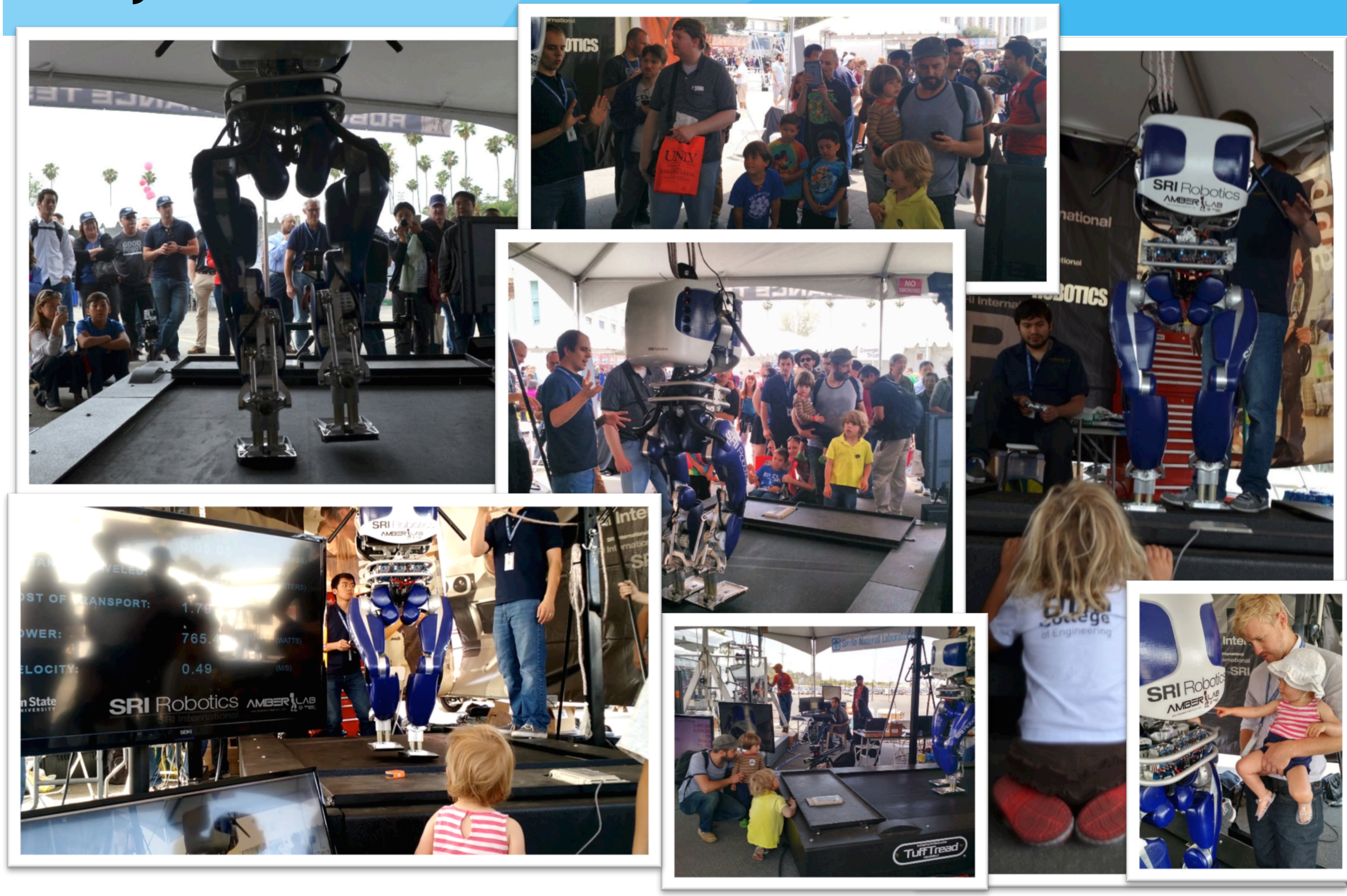
Next Steps

Implement correct-by-construction controllers experimentally on the humanoid robot DURUS



Why? Because robots are Cool…

CPS



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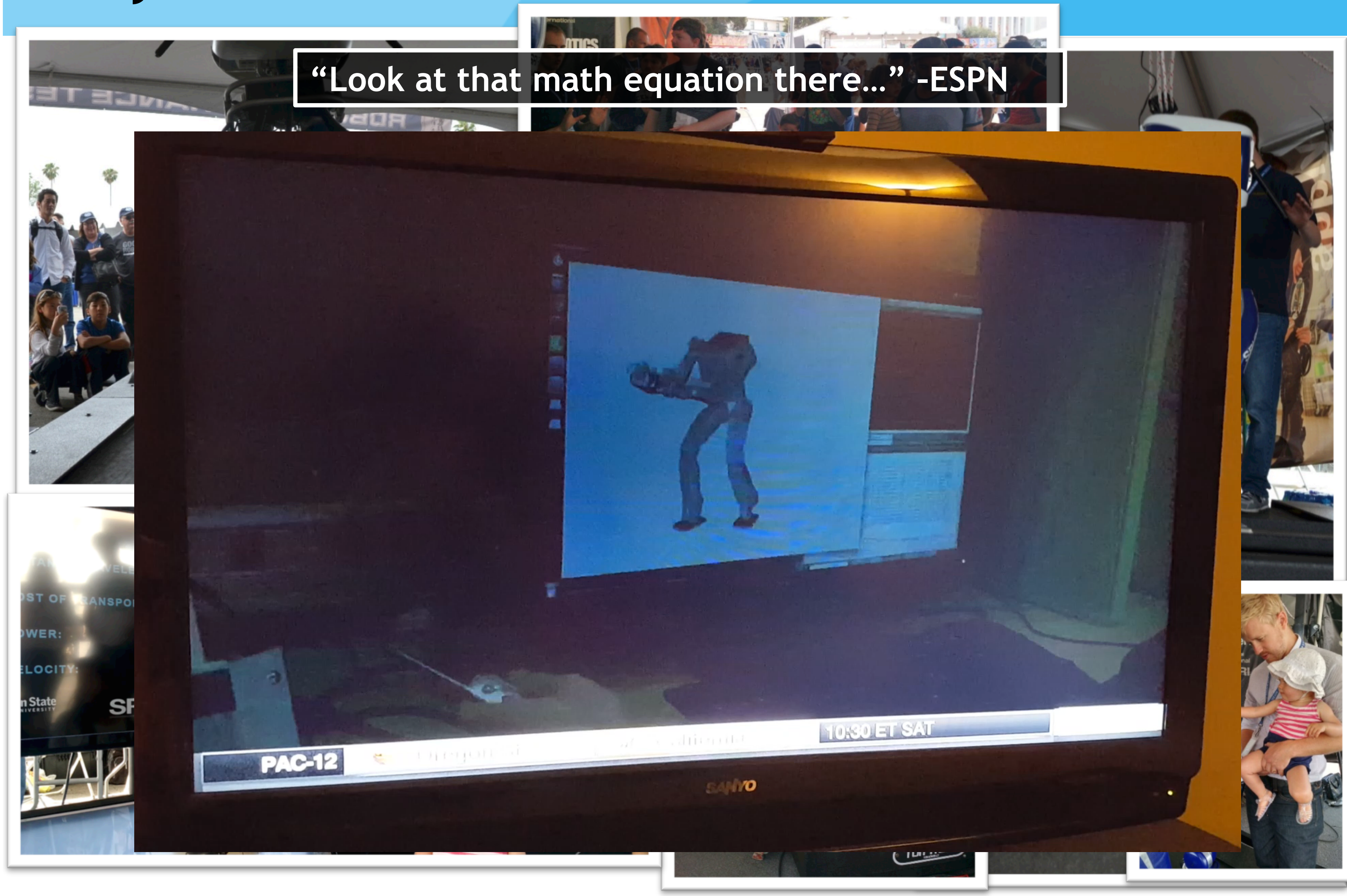
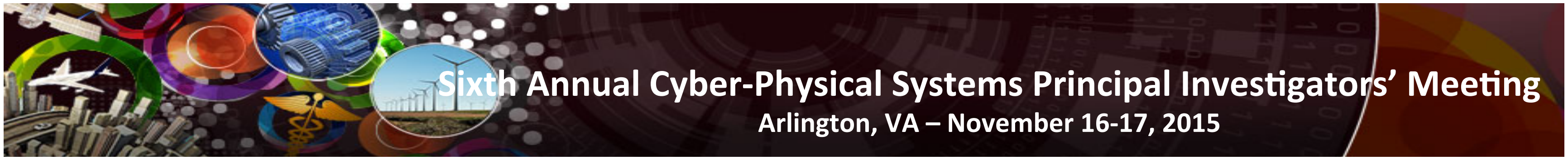




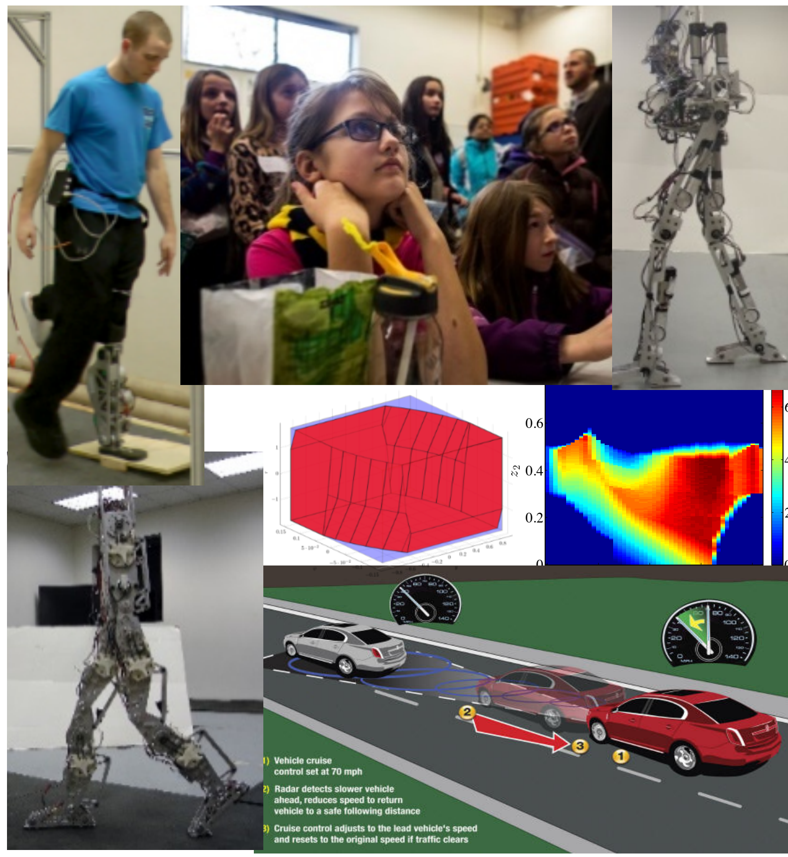
Correct-by-Design Control Software Synthesis for Highly Dynamic Systems

Challenge:

- From a formal specification, synthesize control software for highly dynamic CPS with nonlinear (hybrid) dynamics.
- Implement and evaluate on automotive and robotic hardware.

Solution:

- Two abstraction and fixed-point methods (PESSOA and PCIS).
- Control Barrier Functions.
- Safety and performance guaranteed integration via real-time quadratic programs.



CPS Awards 1239055, 1239037, and 1239085

J.W Grizzle and H. Peng (U. Michigan), A. Ames (GaTech), H. Geyer (CMU), and P. Tabuada (UCLA)

Scientific Impact:

- Use of models vetted by industry.
- Solutions to practical engineering problems with formal safety guarantees.
- Formal methods on a 14 dim. robot model with experiments.

Broader Impact:

- Two automotive safety systems:
  - Adaptive Cruise Control
  - Lane Keeping
- Presented to
  - Ford, Toyota and Eaton Corp.

Use of our Work

CPS

Obtaining precise footstep placements with periodic walking controllers (HZD) was always a problem, **not any more** - thanks to your fantastic creation of CBFs in your CDC 2014 paper. Here's a video

Prof. Koushil Sreenath, CMU

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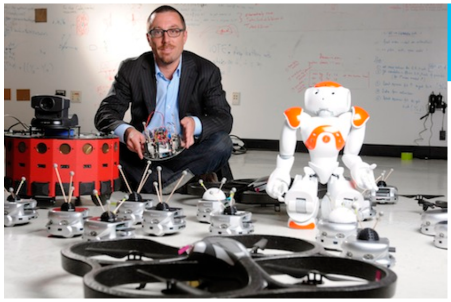
Dim. 10 model

Precise Footstep Placement for Dynamic Bipedal Walking with Control Barrier Functions

Dim. 10 model

CPS

Use of our Work



Controlling robot swarms with CBFs

Prof. Magnus Egerstedt, Georgia Tech