NRI: Decentralized Feedback Control Design for Cooperative Robotic Walking with Application to Powered Prosthetic Legs

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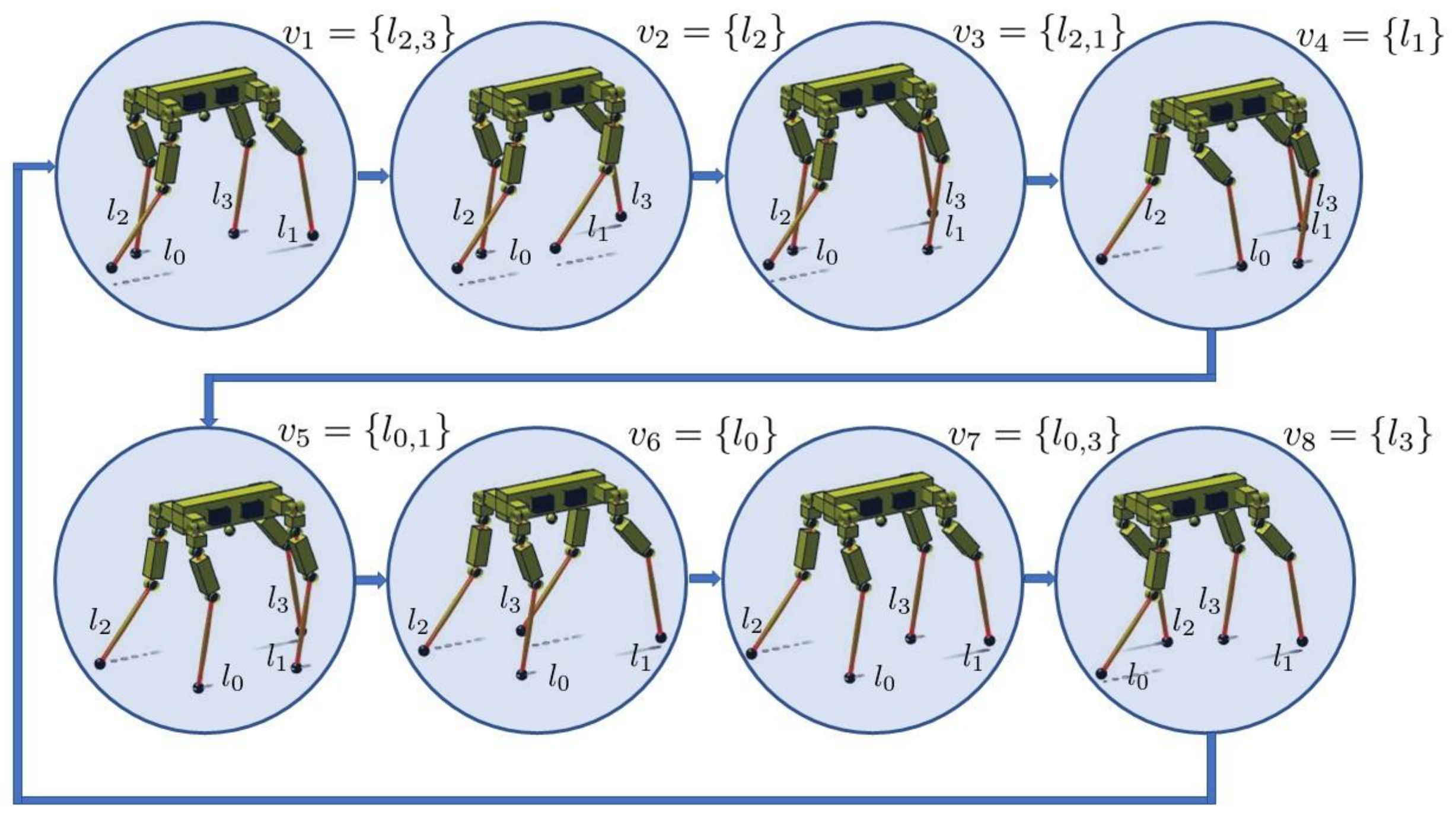
Goals and Objectives

- To investigate a ***potentially transformative*** decentralized feedback control framework to robustly stabilize walking trajectories of legged robots
- To derive optimization algorithms that provably generate robust decentralized controllers for hybrid systems
- To investigate decentralized feedback control architectures for robotic quadruped locomotion
- To investigate decentralized feedback control architectures for robotic prosthetic legs by human amputees subjects

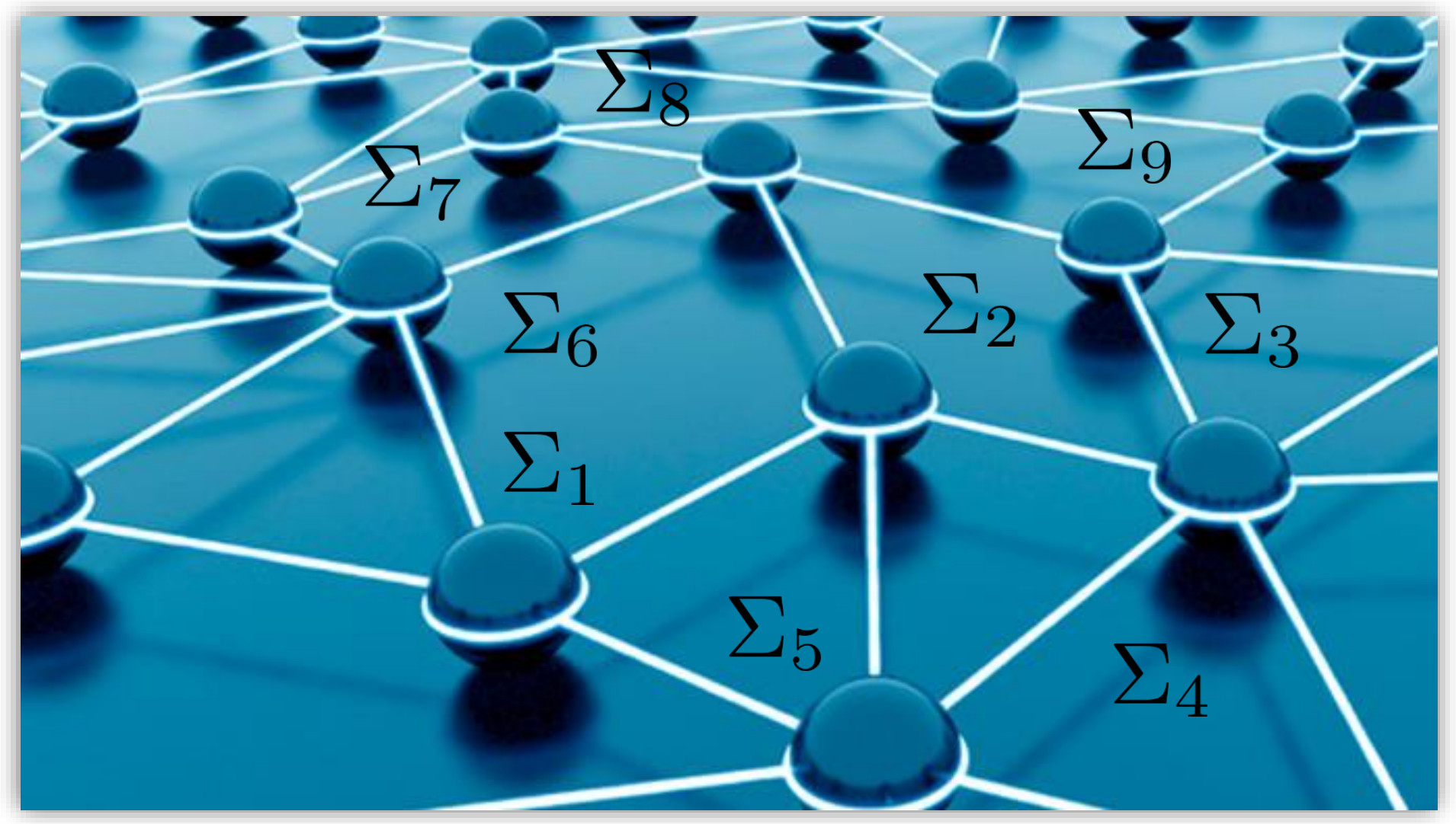
Multi-Domain Hybrid Models of Locomotion



Cooperative Hybrid Subsystems

$$\Sigma_i : \begin{cases} \dot{x}_i = f_i\left(x_1, x_2, \cdots, x_N\right) + \sum_{j=1}^{N} g_{ij}\left(x_1, x_2, \cdots, x_N\right) u_j, & x^- \notin \mathcal{S} \\ x_i^+ = \Delta_i\left(x_1^-, x_2^-, \cdots, x_N^-\right) + d_i, & x^- \in \mathcal{S} \end{cases}$$

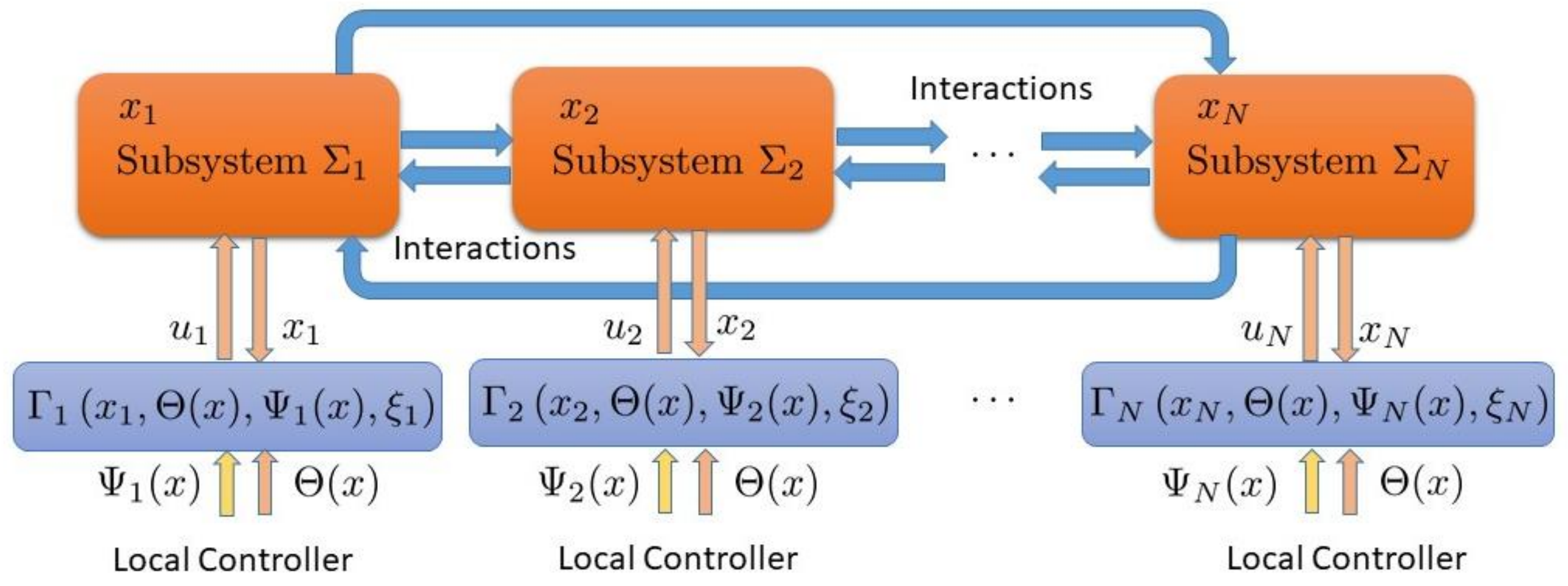
$x_i \in \mathcal{X}_i \subset R^{n_i}$: Local State Variables
$u_i \in \mathcal{U}_i \subset R^{m_i}$: Local Control Inputs
$d_i \in \mathcal{D}_i \subset R^{n_i}$: Local Uncertainty

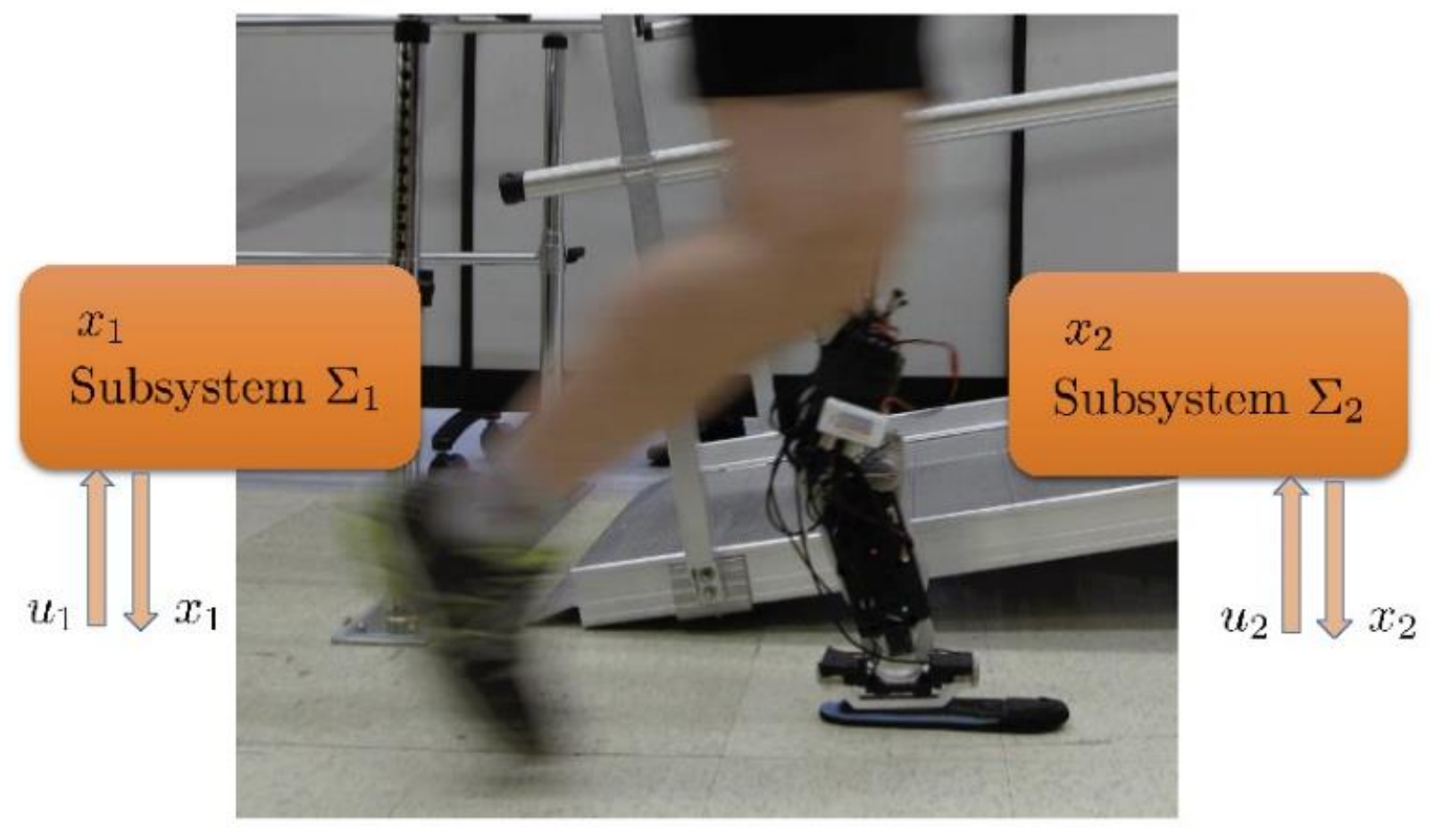


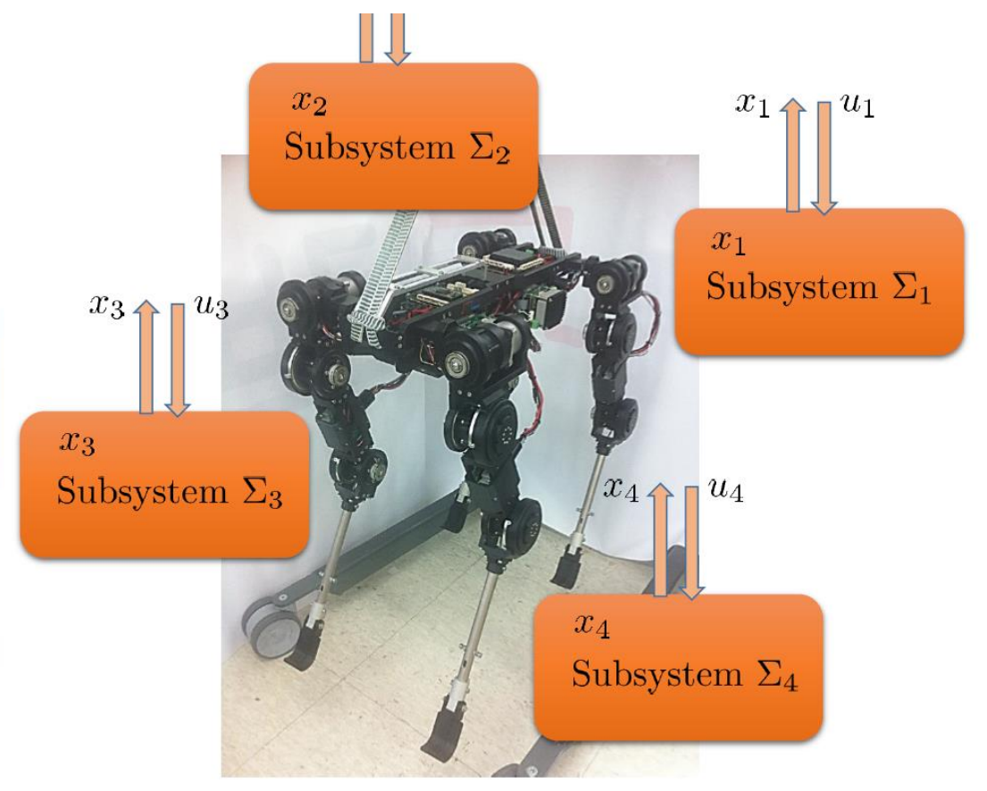
Proposed Decentralized Control

- The local feedback controllers are *parameterized* and *general nonlinear feedback laws* which have access to their own local measurements (i.e., local states) x_i as well as a subset of external measurable global variables $\Psi_i(x)$
- In order to *coordinate* the action of local controllers, we make use of a *common* phasing variable $\Theta(x)$ that is measurable for all subsystems Σ_i for $i \in \{1, 2, \cdots, N\}$

$$u_i = \Gamma_i\left(x_i, \Theta(x), \Psi_i(x), \xi_i\right), \quad i \in \{1, 2, \cdots, N\}$$


x_1 Subsystem Σ_1
x_2 Subsystem Σ_2
Interactions
$\cdots$
x_N Subsystem Σ_N
Interactions
u_1 x_1
u_2 x_2
u_N x_N
$\Gamma_1\left(x_1, \Theta(x), \Psi_1(x), \xi_1\right)$
$\Gamma_2\left(x_2, \Theta(x), \Psi_2(x), \xi_2\right)$
$\cdots$
$\Gamma_N\left(x_N, \Theta(x), \Psi_N(x), \xi_N\right)$
$\Psi_1(x)$ $\Theta(x)$
$\Psi_2(x)$ $\Theta(x)$
$\Psi_N(x)$ $\Theta(x)$
Local Controller
Local Controller
Local Controller


x_1 Subsystem Σ_1
x_2 Subsystem Σ_2
u_1 x_1
u_2 x_2


x_2 Subsystem Σ_2
x_1 u_1
x_1 Subsystem Σ_1
x_3 u_3
x_3 Subsystem Σ_3
x_4 u_4
x_4 Subsystem Σ_4

Robust Stabilization

- $\mathcal{H}_2$- and $\mathcal{H}_\infty$-optimal control problems on the Poincaré section

$$\mathcal{P} : \begin{cases} x[k+1] = P\left(x[k], \xi, d[k]\right) \\ c[k] = c\left(x[k]\right) \end{cases}$$

$d[k] \in \mathcal{D}$: discrete-time uncertainty (exogenous inputs)
$c[k] \in \mathcal{C}$: discrete-time outputs to be controlled

Iterative BMI Algorithm for Tuning Decentralized Controllers

We develop an *effective numerical algorithm* based on a sequence of iterative and offline optimization problems involving Bilinear Matrix Inequalities (BMIs) to overcome specific difficulties arising from the lack of closed-form expression for the Poincaré map, high dimensionality, and underactuation in tuning the parameters of decentralized feedback controllers.

Steps of the BMI Algorithm

- **Sensitivity Analysis**

$$\partial\mathcal{P} : \begin{cases} \delta x[k+1] = \hat{A}\left(\xi^j, \Delta\xi\right) \delta x[k] + \hat{B}\left(\xi^j, \Delta\xi\right) d[k] \\ \delta c[k] = \frac{\partial c}{\partial x}\left(x^\star\right) \delta x[k] \end{cases}$$

- **BMI Optimization**

$$\begin{aligned} \min_{W, Z, \Delta\xi, \mu, \eta} \quad & w\,\mu + \eta \\ \text{s.t.} \quad & \begin{bmatrix} W & \hat{A}\left(\xi^j, \Delta\xi\right) W & \hat{B}\left(\xi^j, \Delta\xi\right) \\ \star & W & 0 \\ \star & \star & I \end{bmatrix} > 0 \\ & \begin{bmatrix} W & C\,Z \\ \star & Z \end{bmatrix} > 0 \\ & \begin{bmatrix} I & \Delta\xi \\ \star & \eta \end{bmatrix} > 0 \\ & \operatorname{trace}(W) < \mu \end{aligned}$$

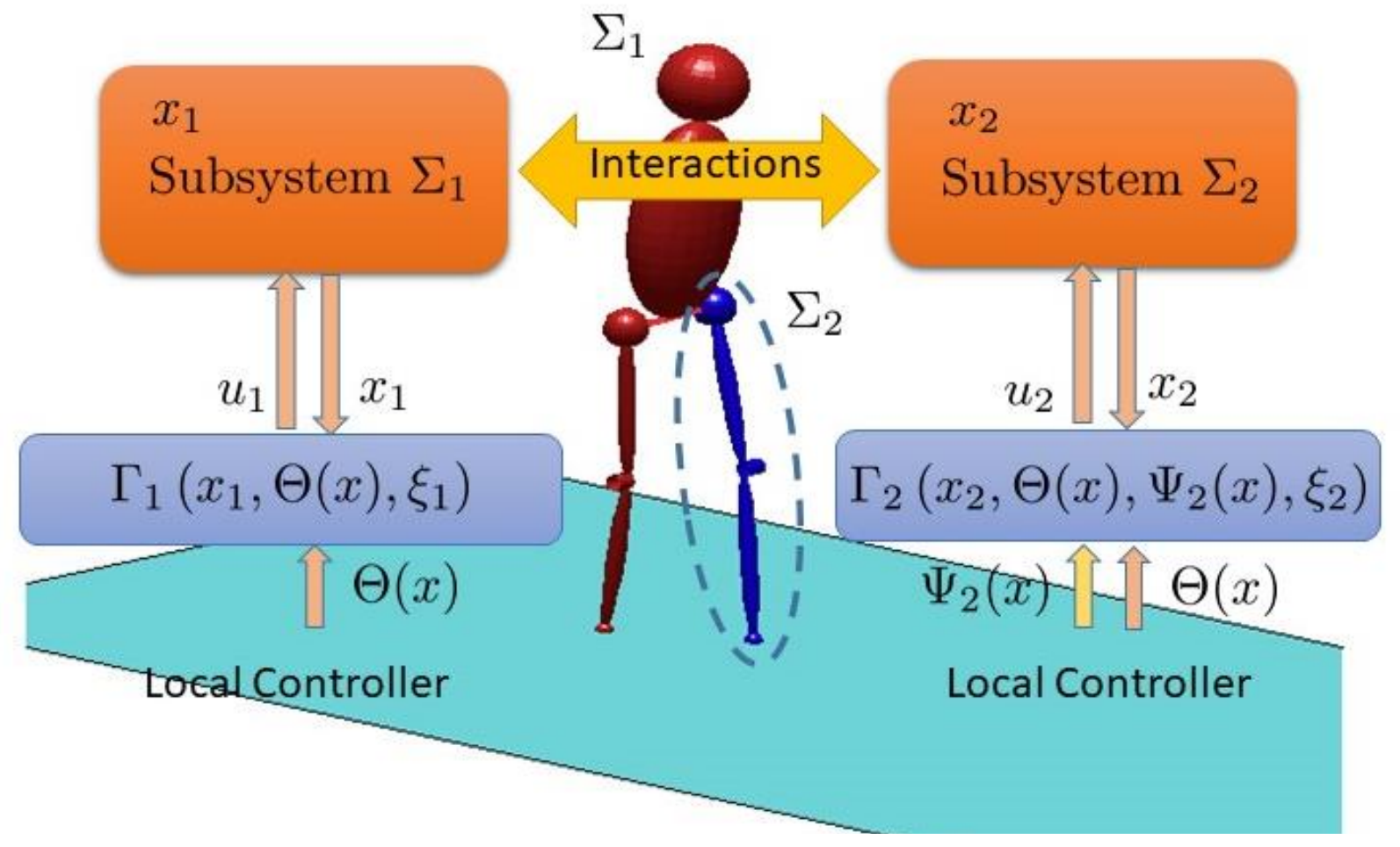
- **Iteration**

$$\xi^{j+1} = \xi^j + \Delta\xi$$

Results

- Designing decentralized virtual constraints and local I-O feedback linearizing controllers for stable walking of amputee locomotion with a transpelvic prosthetic leg
- No expensive force sensor and no high-gain controller for dealing with nonlinear interactions
- 45% improvement in the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms

$$y_i = h_i(q_i, \Theta, \Psi_i, \xi_i), \quad i = 1, 2$$


Σ_1
x_1 Subsystem Σ_1
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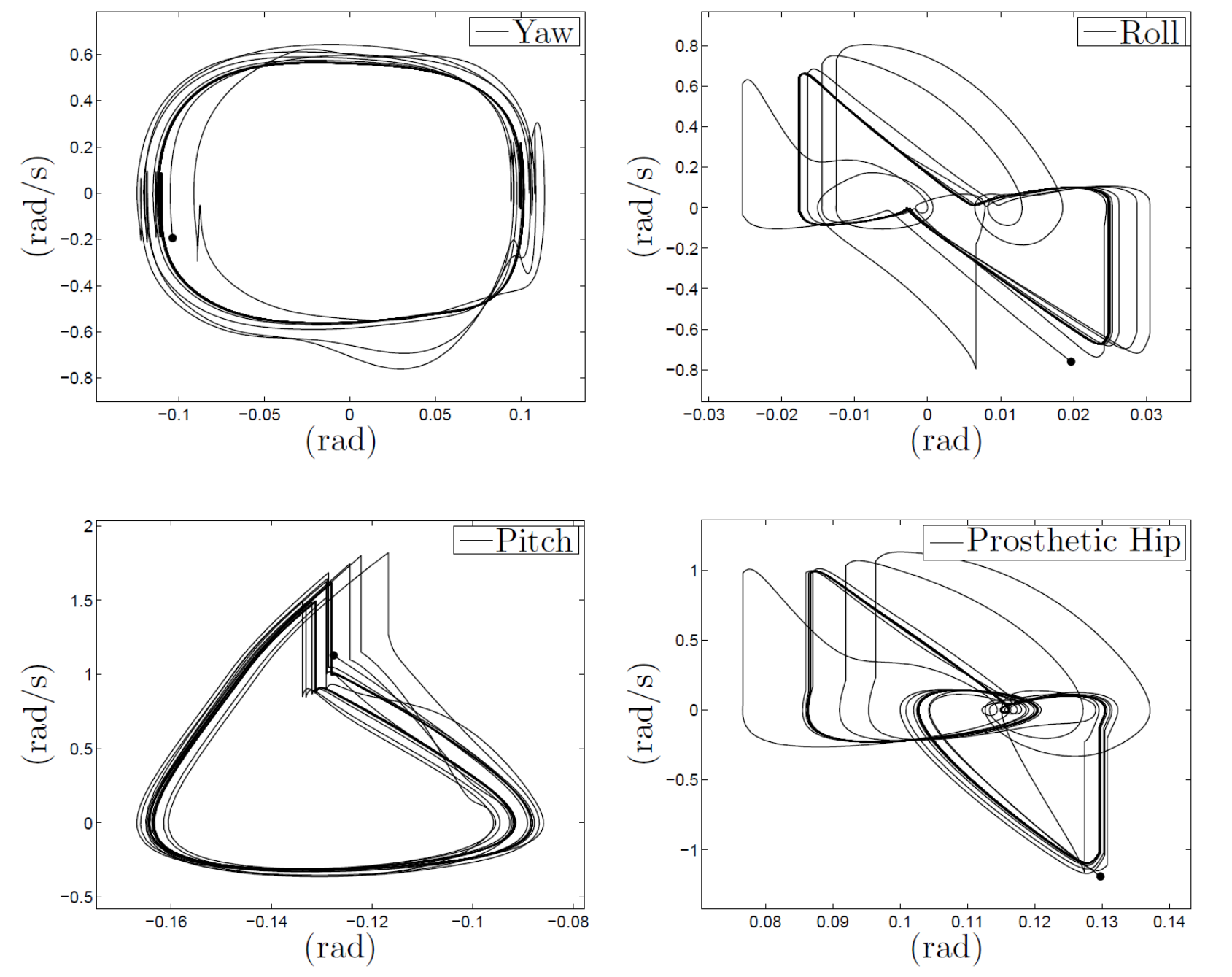
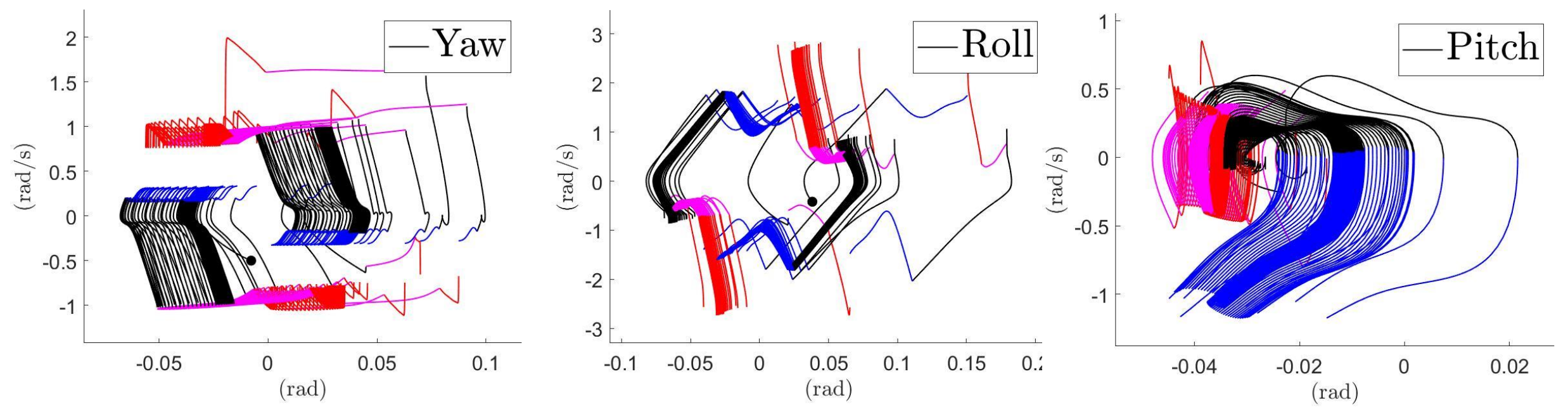
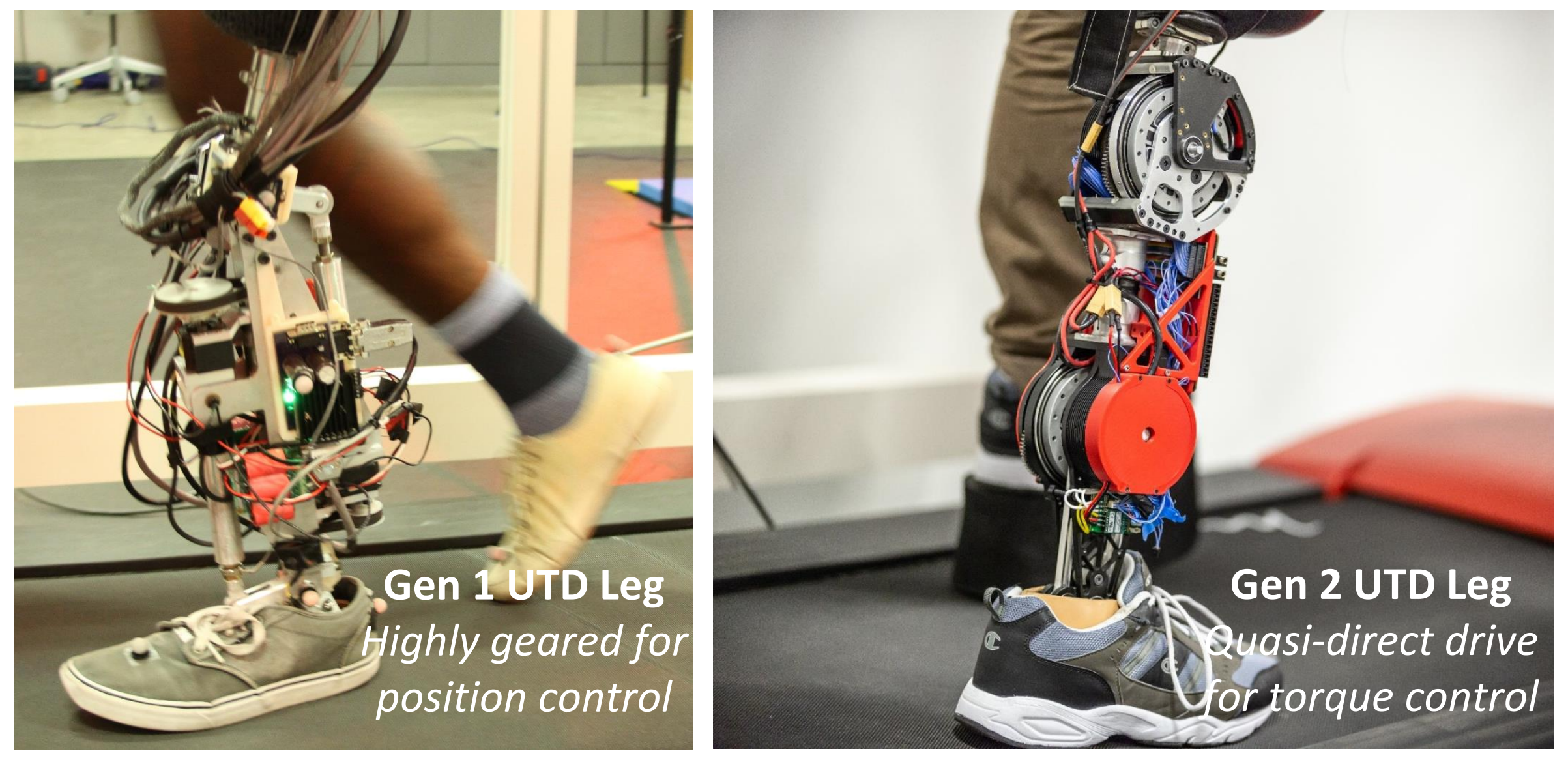
$\Gamma_2\left(x_2, \Theta(x), \Psi_2(x), \xi_2\right)$
$\Theta(x)$
$\Psi_2(x)$ $\Theta(x)$
Local Controller
Local Controller

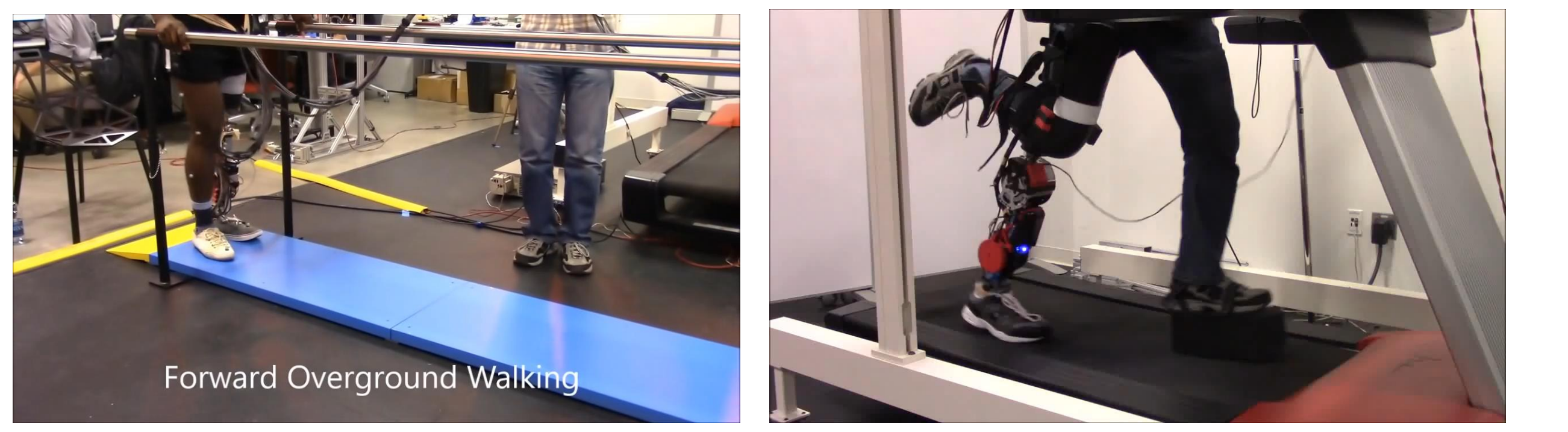


- Designing virtual constraint controllers for stable 3D quadruped locomotion (amble gait) of a simulation model of Vision 60



- Two prosthesis testbeds for decentralized control





Publications

- K. Akbari Hamed and R. D. Gregg, "Decentralized event-based controllers for robust stabilization of hybrid periodic orbits," *IEEE Transactions on Automatic Control*, In Press, August 2018
- K. Akbari Hamed and R. D. Gregg, "Decentralized feedback controllers for robust stabilization of periodic orbits of hybrid systems: Application to bipedal waking," *IEEE Transactions on Control Systems Technology*, 2017
- D. Quintero, D. J. Villarreal, D. J. Lambert, S. Kapp and R. D. Gregg, "Continuous-phase control of a powered knee-ankle prosthesis: Amputee experiments across speeds and inclines," *IEEE Transactions on Robotics,* 2018
- K. Akbari Hamed, R. D. Gregg, A. D. Ames, "Exponentially stabilizing controllers for multi-contact 3D bipedal locomotion," *American Control Conference*, 2018
- K. Akbari Hamed, A. D. Ames, and R. D. Gregg, "Observer-based feedback controllers for exponential stabilization of hybrid periodic orbits: Application to underactuated bipedal walking," *American Control Conference*, 2018
- K. Akbari Hamed, W. Ma, A. D. Ames, "Dynamically stable 3D quadruped walking with multi-domain hybrid system models and virtual constraint controllers," *American Control Conference*, Under Review, 2018
- K. Akbari Hamed and A. D. Ames, "Nonholonomic hybrid zero dynamics for the stabilization of periodic orbits: Application to underactuated robotics walking," *IEEE Transactions on Control Systems Technology*, Under Review, 2018

Acknowledgements

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