Design of in-line controllers for continuously operating networks with structural uncertainty (CAREER #1553504)

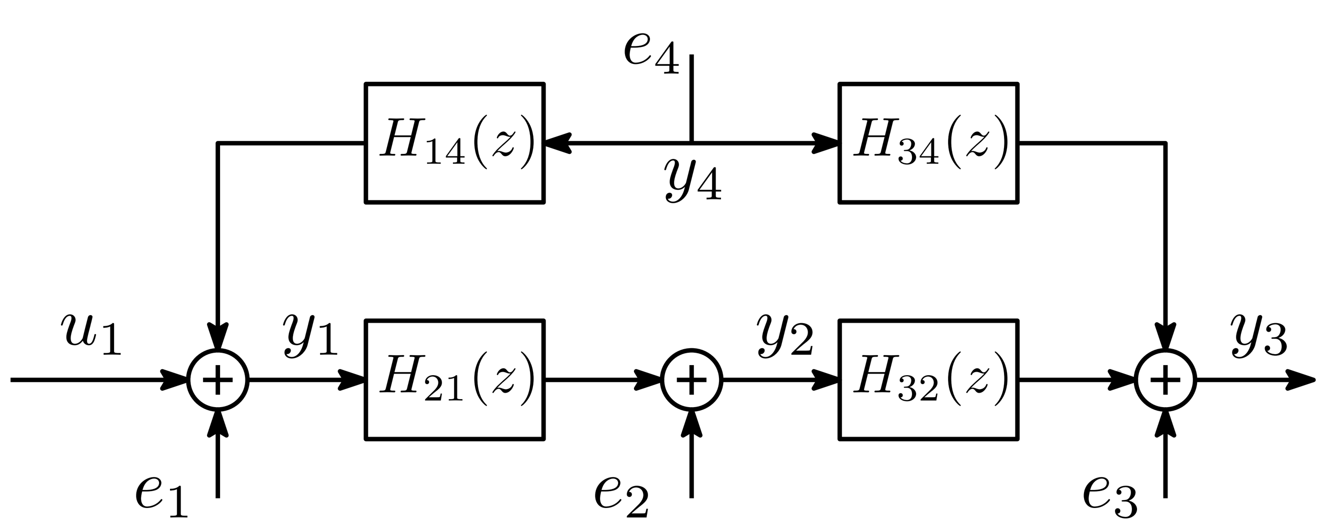
P.I.: Donatello Materassi, Electrical Engineering and Computer Science Department, University of Minnesota

Linear Dynamic Influence Networks: an I/O model for noninvasively sensed systems

$$\begin{pmatrix} y_1\\ \vdots \\ y_n\end{pmatrix} = \begin{pmatrix} u_1\\ \vdots\\ u_n\end{pmatrix} + \begin{pmatrix} H_{11}(z) & \dots & H_{1n}(z)\\ \vdots & \ddots & \vdots \\ H_{n1}(z) & \dots & H_{nn}(z)\end{pmatrix}\begin{pmatrix} y_1\\ \vdots\\ y_n\end{pmatrix}$$

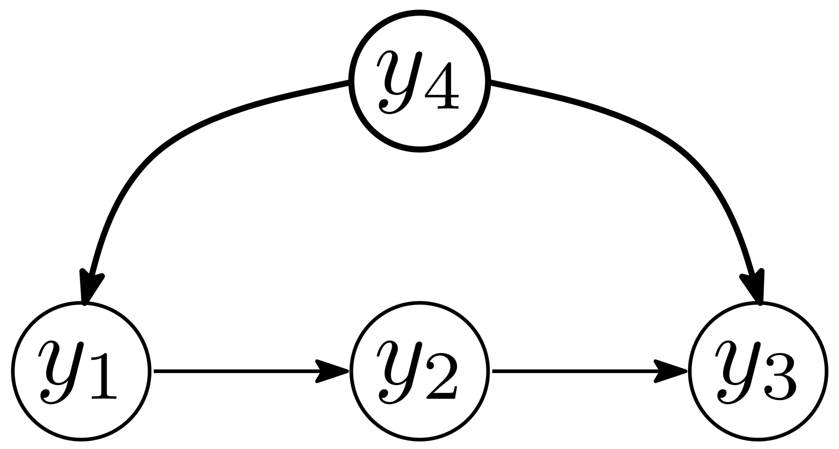
- The inputs y_i are observable
- The inputs u_i are unknown
- u_i and u_j are independent, for $i \neq j$

A graphical model interpretation



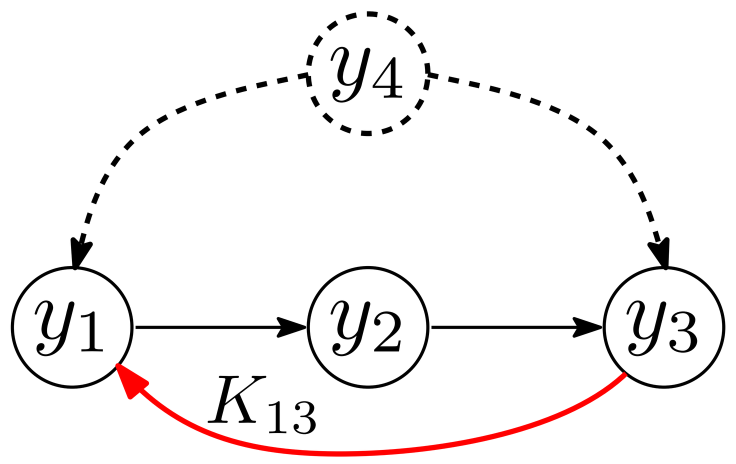
Given a block diagram

- the outputs are nodes
- if $H_{ji}(z) \neq 0$ draw, a link from y_i to y_j.



This representation extends graphical models of random variables to describe stochastic processes

Design Control Challenge I



- Transfer functions are unknown
- Graph is partially known (with unknown structure)
- y_4 is not measurable (and not known if it is present)
- Known Power Spectral Densities $\Phi_{y_iy_j},\ i,j=1,2,3$

Find the closed loop behavior

In the absence of y_4 the solution is easy

$$H_{32}(z)H_{21}(z) = \frac{\Phi_{y_3y_2}}{\Phi_{y_2}}\frac{\Phi_{y_2y_1}}{\Phi_{y_1}}$$

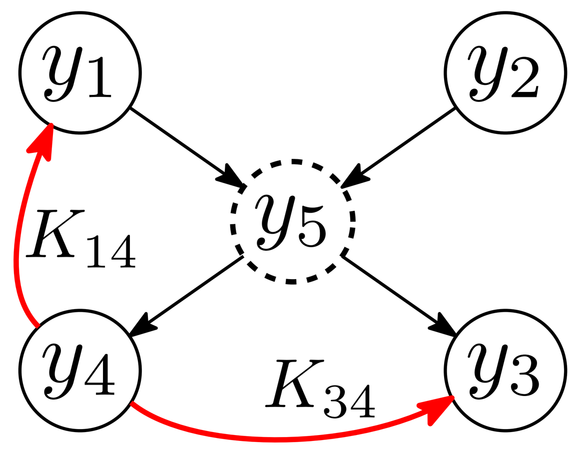
$$G_{y_1\to y_3} = \frac{H_{32}H_{21}}{1-H_{32}H_{21}} = \frac{\frac{\Phi_{y_3y_2}}{\Phi_{y_2}}\frac{\Phi_{y_2y_1}}{\Phi_{y_1}}}{1-K_{13}\frac{\Phi_{y_3y_2}}{\Phi_{y_2}}\frac{\Phi_{y_2y_1}}{\Phi_{y_1}}} = \frac{\frac{\Phi_{y_3y_1}}{\Phi_{y_1}}}{1-K_{13}\frac{\Phi_{y_3y_1}}{\Phi_{y_1}}}$$

In the presence (or absence) of y_4 (see [?])

$$G_{y_1\to y_3} = \frac{\frac{\Phi_{y_2y_1}}{\Phi_{y_1}}\begin{pmatrix}\Phi_{y_3y_1} & \Phi_{y_3y_2}\end{pmatrix}\begin{pmatrix}\Phi_{y_1} & \Phi_{y_1y_2}\\ \Phi_{y_2y_1} & \Phi_{y_2}\end{pmatrix}^{-1}\begin{pmatrix}0\\1\end{pmatrix}}{1-K_{13}\frac{\Phi_{y_2y_1}}{\Phi_{y_1}}\begin{pmatrix}\Phi_{y_3y_1} & \Phi_{y_3y_2}\end{pmatrix}\begin{pmatrix}\Phi_{y_1} & \Phi_{y_1y_2}\\ \Phi_{y_2y_1} & \Phi_{y_2}\end{pmatrix}^{-1}\begin{pmatrix}0\\1\end{pmatrix}}$$

This formula is robust with respect to uncertainties in the structure: a novel notion of robustness!

Design Control Challenge II



- Transfer functions are unknown
- Graph is a tree (with unknown structure)
- y_5 is not measurable and it is not known if it is present
- Known Power Spectral Densities $\Phi_{y_iy_j},\ i,j=1,2,3,4$

Find the closed loop behavior

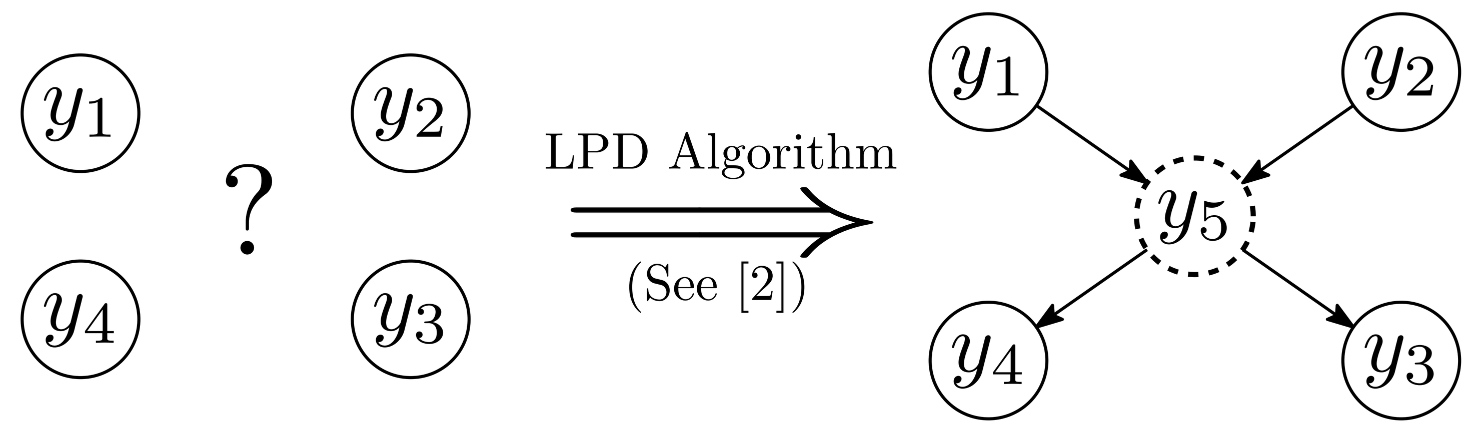
A distance that is additive along paths of a rooted tree, such as

$$d(y_i,y_j) = -\int_0^{2\pi}\log\left(\frac{|\Phi_{e_ie_j}|^2}{|\Phi_{e_i}||\Phi_{e_j}|}\right)d\omega,$$

enables the detection of hidden nodes (see [?])

$$y_5 \text{ is present} \Leftrightarrow d(y_1,y_4)-d(y_4,y_3) \neq d(y_1,y_3)$$

enables the inference of skeleton and link orientations (see [?])



$$G_{y_1\to y_4} = \frac{\Phi_{y_4y_1}}{\Phi_{y_1}-K_{14}\Phi_{y_4y_1}}$$

$$G_{y_3\to y_4} = 0$$

References

[1] F. Sepeher, D. Materassi, *Inferring the structure of polytree networks of dynamic systems with hidden nodes*, IEEE Transactions on Automatic Control, 2019

[2] F. Sepeher, D. Materassi, *An Algorithm to Learn Polytree Networks with Hidden Nodes*, NeurIPS, 2019

[3] J. Constanzo, D. Materassi, B. Sinopoli, *Using Viterbi and Kalman to detect topological changes in dynamic networks*, American Control Conference, 2017