



# Digital Control of Hybrid Systems via Simulation and Bisimulation

Ji-Woong Lee (PI)<sup>1</sup>Constantino Lagoa<sup>1</sup>Heath Hofmann<sup>2</sup><sup>1</sup>Pennsylvania State University, University Park<sup>2</sup>University of Michigan, Ann Arbor

### Project Goals

- Unification:** Symbolic models and Lyapunov analysis
- Scalability:** Increasing sequence of symbolic models
- Convexity:** Linear matrix inequality conditions
- Nonconservatism:** Symbolic models convergent to true model
- Robustness and Risk:** Modeling error vs. computational complexity
- Test Case:** Risk adjusted Optimal AC Power Flow

### Cyber-Physical System (CPS) Approach

#### Analysis

Symbolic Model  $\Rightarrow$  Lyapunov Analysis  $\Rightarrow$  Updated Symbolic Model  $\Rightarrow$  Improved Lyapunov Analysis  $\Rightarrow \dots$

#### Control Design

#### Control Objectives

- Stabilization
- Performance (i.e., relative stability) optimization
- Robustness against uncertainty  
(e.g., modeling error, external; bounded perturbations)

#### A Synthesis Procedure

- Partition state-input space adapting to control objective. (**Coarsest partitions do not work!**)
- Obtain simulation (i.e., symbolic model) of plant.
- Achieve closed-loop bisimulation (e.g., optimal stability region).

Simulation  $\rightarrow$  Updated Simulation  $\rightarrow \dots \rightarrow$  Closed-Loop Bisimulation

### Handling Risk

Moments Approach	Scenario Approach
$\max_x \text{Prob}_\delta \{g(x, \delta) \geq 0\}$ s.t. $f(x) \leq \gamma$	$\min_x f(x)$ s.t. $\text{Prob}_\delta \{g(x, \delta) \geq 0\} \geq 1 - \varepsilon$
$\max_{\mu, \mu_x} \int d\mu$ s.t. $\mu \preccurlyeq \mu_x \times \mu_q$ $\text{supp}(\mu_x) \subseteq \{x : f(x) \leq \gamma\}$ $\text{supp}(\mu) \subseteq \{(x, \delta), g(x, \delta) \geq 0\}$	$\min_x f(x)$ s.t. $g(x, \delta^{[i]}) \geq 0; i = 1, 2, \dots, N$
Moments representation of measures Linear Matrix Inequality approximation	
<b>+ Asymptotically exact approximations</b> <b>- Computational complexity increases "fast" with degree of approximation</b>	
<b>+ Easy setup</b> <b>- Approximations to risk are "loose"</b> <b>- May require non-convex solvers</b>	

### AC Power Flow Under Uncertainty

Electricity generated by a wind farm depends on the wind speed and location of installation

Loads can exhibit high variability

Renewable Gen.      Generator      Load      Variable load

Bus  $i$       Line  $(i, k)$       Bus  $k$       Bus  $j$

admittance  $y_{ij}$

$P_{R_i}^0 + Q_{R_i}^0 i + \delta_i$        $P_{G_k}^0 + \alpha_k \sum \delta_i$        $P_{L_j}^0 + Q_{L_j}^0 i + \delta_j$

**Simulation – England 39-bus**

- 21 loads with a total demand of 6254 MW
- 46 transmission lines
- 11 tap changing transformers
- 10 order IV synchronous generators with max capacitance of 8404 MW
- TG and AVR control devices
- Individual cost parameters for every generator

Nominal design

Risk-adjusted design