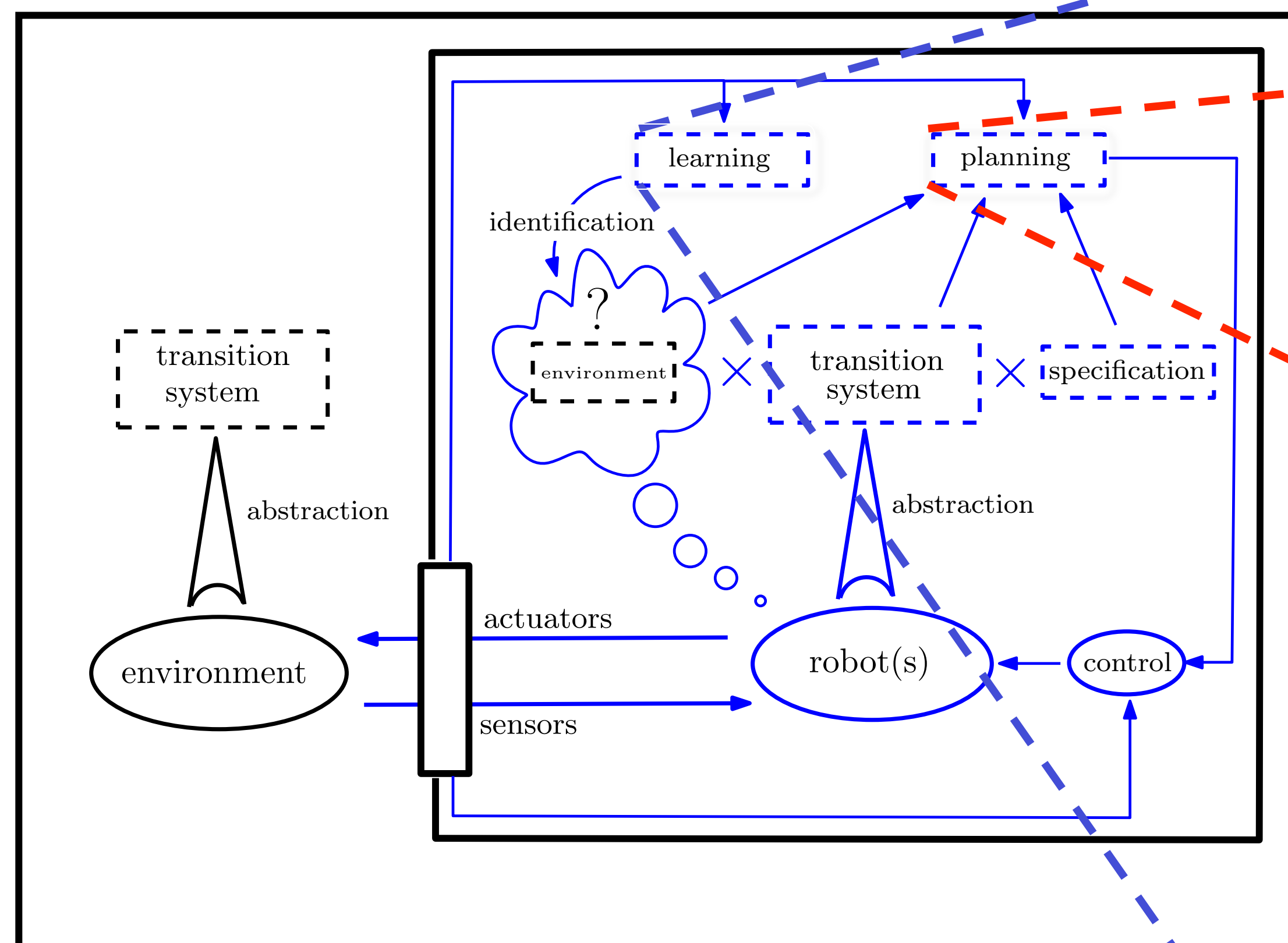


## Scientific Goal

Coordinate a group of heterogeneous autonomous cyber-physical systems to satisfy temporal logic control specifications in a partially unknown and dynamically changing environment.

### A Bird's-Eye View



### Learning and Adaptation

**Goal**  
Learn from observations the behavior of the environment a multi-agent system interacts with while attempting to satisfy its specification.

**Assumptions**

- Knowledge of the class of formal languages the environment behavior falls into
- Fully observable environment evolution

**Challenges**

- Requirements for guaranteed asymptotic convergence of learning algorithm
- Full generalization without overfitting

**Approach**

- Formulate the problem as learning a repeated two-player turn-based game
- Adapt grammatical inference algorithms for learning games

**Example**  
Suppose the dynamics of the unknown language can be modeled with a Strictly  $k$ -Piecewise ( $SP_k$ ) language [7]. This class of languages is learnable with a string extension learner [8].

**Strictly Piecewise Dynamics**

- String  $v = a_1 a_2 \dots a_n$  is a subsequence of  $w$  iff  $w \in \Sigma^* a_1 \Sigma^* a_2 \Sigma^* \dots \Sigma^* a_n \Sigma^*$ .
- Let  $f_k(w) = \{v \mid v \text{ is a subsequence of } w \text{ and } |v| \leq k\}$ .
- Example:  $f_2(abcad) = \{\lambda, a, b, c, d, ab, aa, ac, ad, ba, bc, bd, cd\}$ .
- $L \in SP_k$  iff there exists a finite set  $S \subseteq \Sigma^{\leq k}$  such that  $f_k(L) = S$ .
- This finite set can be viewed as the *grammar* generating  $L$ .

**String Extension Learning**

- A text  $T$  for  $L$  is an infinite sequence of elements of  $L$  such that each element of  $L$  occurs at least once in  $T$ .
- $T(i)$  is the  $i$ th element of  $T$ , and  $T[i]$  is the finite sequence  $T(1), T(2), \dots, T(i)$ .
- Given any text  $T$  for any  $SP_k$  language  $L$ , the learning function  $\phi_k$  converges to a grammar for  $L$ .

$$\phi_k(T[i]) = \begin{cases} \emptyset & i = 0 \\ \phi_k(T[i-1]) \cup f_k(T(i)) & \text{otherwise} \end{cases}$$

### Temporal Logic Synthesis

**Goal:**  
Synthesize a control policy to satisfy a mission specified using temporal logic

**Example Mission:**  
"Visit regions  $\pi_1$  and  $\pi_2$  infinitely often and always avoid the adversary" (see Example of Scenario below)

**Challenges:**

- Ensure progress towards goal despite unknown actions by the adversary
- Computational issues arising from planning for multiple agents and adversary

**Technical Approach:**

- Translate mission specification from Linear Temporal Logic to a Büchi Automaton
- Construct Product Automaton from Game Transition System (see Learning and Adaptation at left) and Büchi Automaton to capture agent/environment interactions and satisfaction of specification
- Use "energy" function to determine distance to accepting states in the Product Automaton
  - Function is computed using backward induction
  - Adversary is assumed to choose most antagonistic actions
- Incrementally update the Product when new elements in the grammar are learned
  - Add new transitions in Game Transition System
  - For each new transition, add appropriate transitions in Product Automaton
  - Re-compute energy function
- At each step, control policy is the action that leads to a state with lower value for energy function, if one exists, otherwise report failure

**Result:**

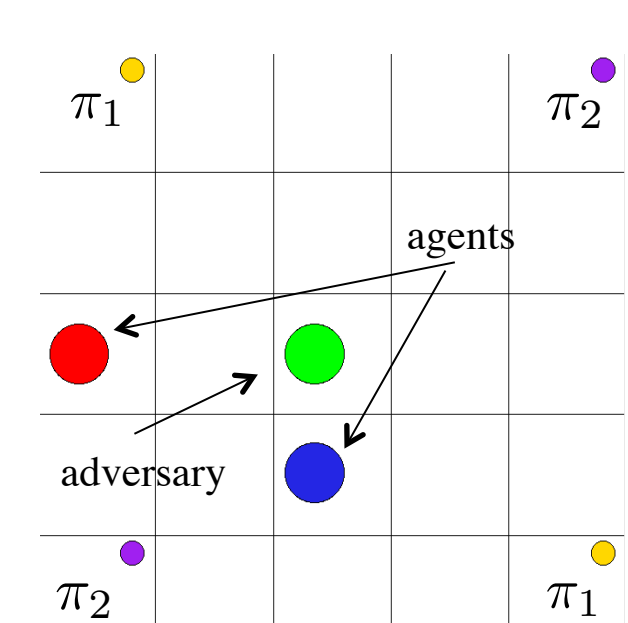
- Given that the learned model for adversary behavior is correct, our algorithm guarantees a control policy to satisfy the specification, if one exists

## Integration of Learning and Control in Cyber Physical Systems Operating Under Uncertainty

### Example of scenario

"Agents should visit the corners of the grid infinitely often while avoiding the adversary"

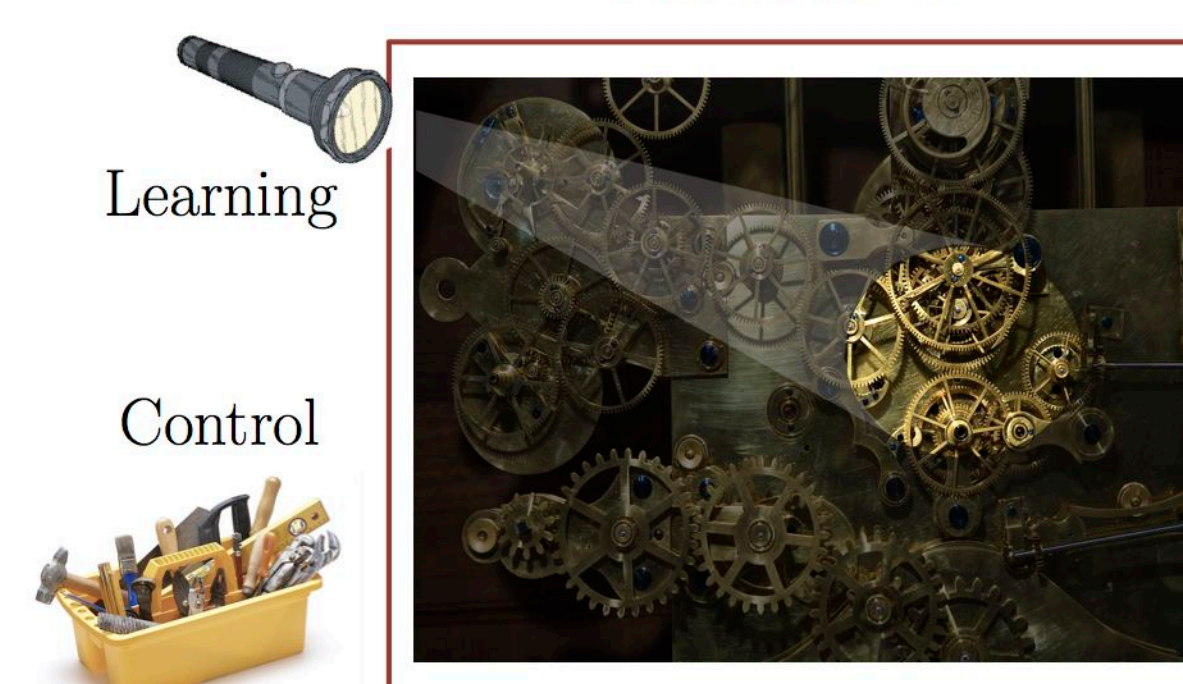
LTL:  $\varphi = \square \diamond \pi_1 \wedge \square \diamond \pi_2 \wedge \square \neg \pi_{\text{capture}}$



- Agents constraints: move in compass directions
- Adversary constraints: unknown, but its behavior is an SP2 language

### Approach

Environment

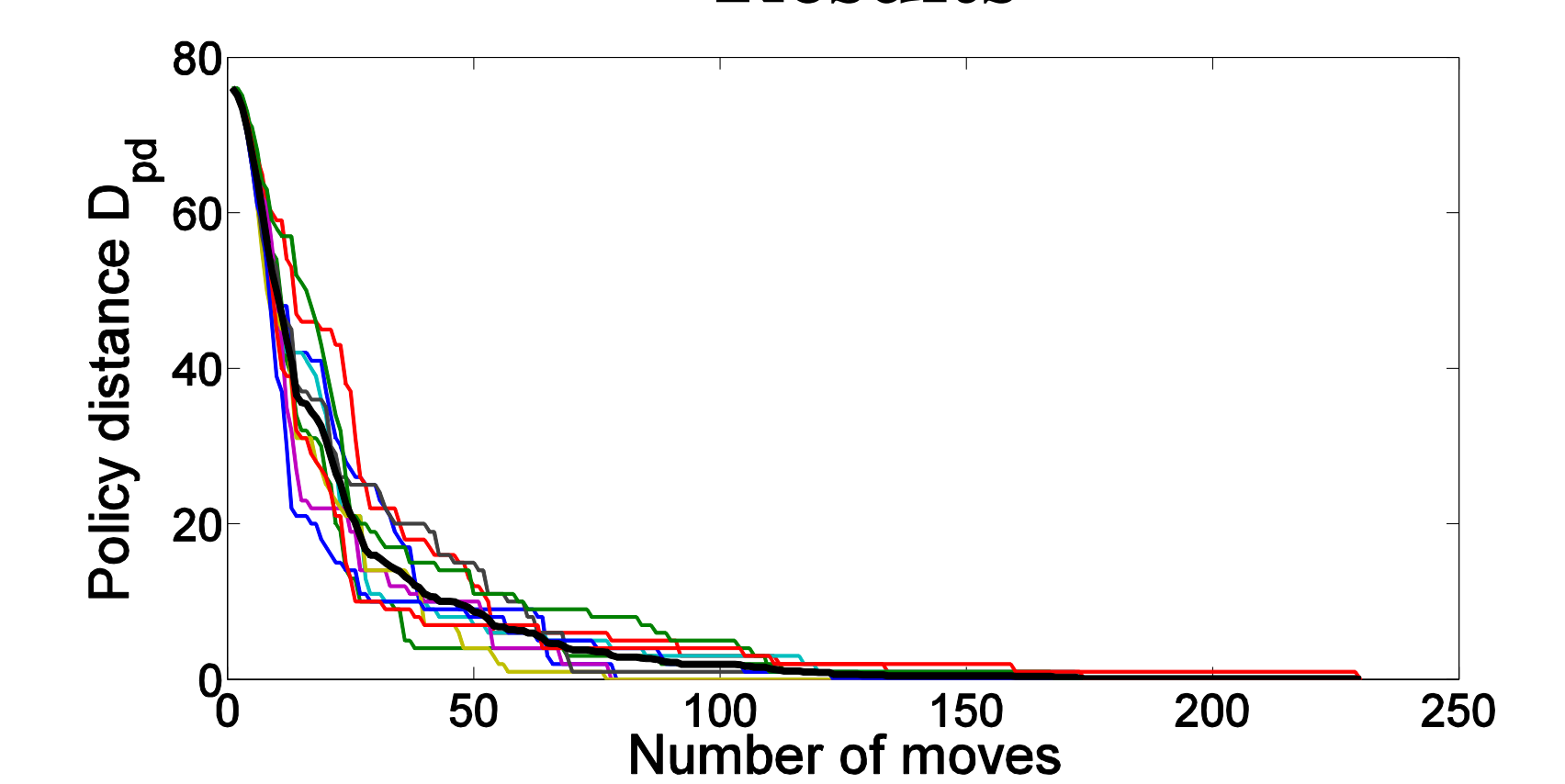


- Observe the environment and build a model for it
- Refine the model in real-time
- Use the refined model to update control strategies

### Methodology

- Interaction between agents and environment takes the form of a deterministic zero-sum game
- On the game graph, progress toward satisfaction of the LTL spec is quantified
- Agents strategize assuming their hypothesis about the adversary plays its best move
- Control strategy is synthesized along standard model-checking approaches
- Adversary can move diagonally but not along compass more than once (agents do not know any of that at first)
- Agents' prior knowledge is that the environment behavior is in a specific class of formal languages
- They observe adversary actions and incrementally built a model for it
- The model is **guaranteed** to asymptotically converge to the true environment model
- After finite turns agents can recover the performance of full knowledge of their adversary dynamics

### Results



In every game, as the environment model is refined and converges, the computed policy converges to the policy that would have been computed if environment dynamics were completely known.

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Supported by NSF CNS Under Awards  
#103588  
#103577

