CPS: Synergy: Collaborative Research:

Efficient Traffic Management: A Formal Methods Approach

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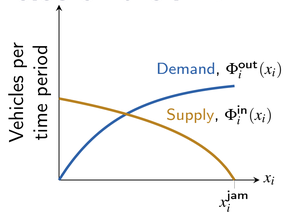
Overview

This project is bringing tools from formal methods to traffic management to meet control objectives expressed in temporal logic. This approach is being applied to signal timing and ramp metering strategies for signalized intersections and freeway traffic control. It will next be advanced to integrated control of freeways and arterials with a hierarchical control architecture that is compatible with the infrastructure.

Traffic Network Model

- For each link $\ell \in \mathcal{L}$, the state $x_\ell[t] \in [0, x_\ell^{\text{cap}}]$ represents the number of vehicles on the link
- Each link has:
  - Demand $\Phi_\ell^{\text{out}}(x_\ell)$ to move downstream
  - Supply $\Phi_\ell^{\text{in}}(x_\ell)$ to accept upstream flow

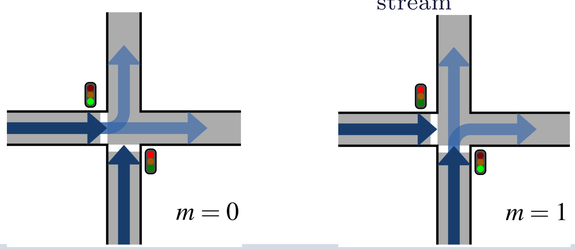


Dynamics:

$$x_\ell^+ = x_\ell + f_\ell^{\text{in}}(x) - f_\ell^{\text{out}}(x) =: F_\ell(x)$$

- Turn ratios $\beta_{\ell k}$ divide demand among downstream links and supply ratios $\alpha_{\ell k}$ divide supply among upstream links
- Signal variable $s_\ell \in \{0, 1\}$ indicates if link ℓ is active

$$f_\ell^{\text{out}}(x) = s_\ell \cdot \min\left\{\Phi_\ell^{\text{out}}(x_\ell),\ \min_{k \in \substack{\text{Down-}\\ \text{stream}}} \left\{\frac{\alpha_{\ell k}}{\beta_{\ell k}} \Phi_k^{\text{in}}(x_k)\right\}\right\}$$



Mixed Monotonicity

- Traffic networks are *mixed monotone* systems:

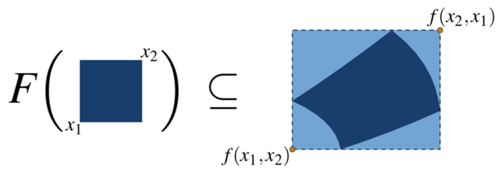
$$\exists \delta_{\ell k} \in \{-1, 1\} \quad \text{s.t.} \quad \delta_{\ell k} \frac{\partial F_\ell(x, d)}{\partial x_k} \geq 0 \quad \forall \ell, k$$

- Increasing and decreasing components
- Decomposition function $f(x, y, d)$
- Congestion causes nonmonotone behavior

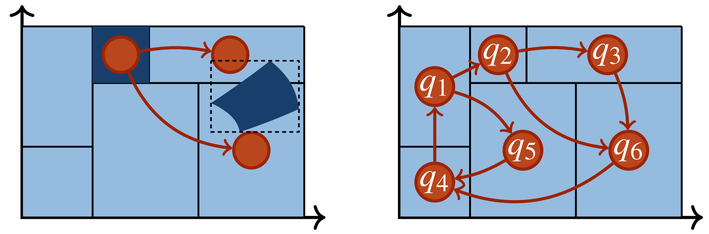
Abstraction

- Mixed Monotonicity leads to efficient abstraction:

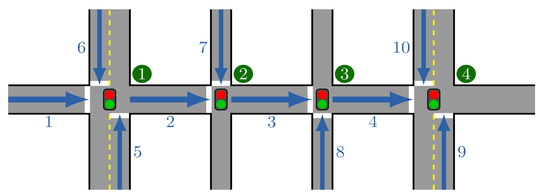
$$\left(\underline{x} \leq x \leq \overline{x}\right) \implies \left(f(\underline{x}, \overline{x}, d) \leq F(x, d) \leq f(\overline{x}, \underline{x}, d)\right)$$



- The one-step reachable set from a box of initial conditions is tightly over-approximated by computing the decomposition function at only two points
- This allows a scalable abstraction algorithm

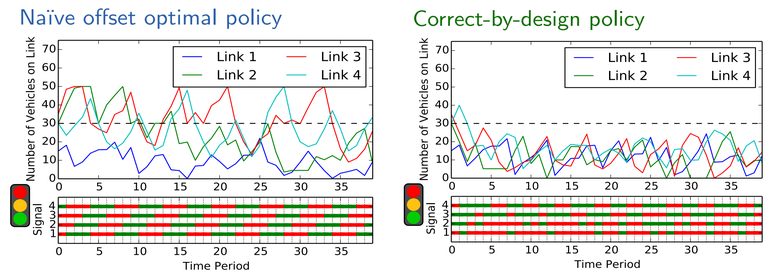


Example



LTL Specification:

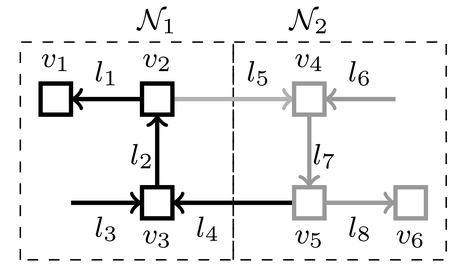
- Each signal actuates cross street traffic infinitely often
- Eventually, links 1, 2, 3, and 4 have fewer than 30 vehicles on each link and this remains true for all time
- The signal at junction 4 must actuate cross street traffic for at least two sequential time-steps



Compositional Synthesis

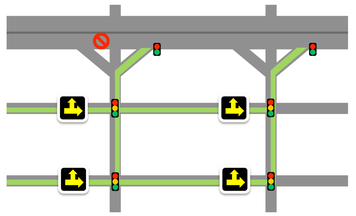
- Contracts between neighboring subnetworks to limit demand and guarantee adequate supply
- Neighbors' guarantees enable decoupled subnetwork models and decentralized controllers
- Original specifications for each subnetwork must be augmented with promises made to neighbors

$$\phi_i^{\text{new}} = \phi_i^{\text{original}} \wedge \phi_i^{\text{supply}} \wedge \phi_i^{\text{demand}} \qquad i = 1, 2, \ldots$$



Current Research

- More flexible contracts
- Probabilistic transition models from statistical data
- Adding optimality criteria to specifications
- Freeway onramp and arterial signaling coordination
- Validation with hybrid freeway / arterial simulation



Publications

- Coogan, Gol, Arcak, Belta, "Traffic network control from temporal logic specifications" IEEE Trans. Control of Network Systems, accepted 2015.
- Kim, Arcak, Seshia "Compositional controller synthesis for vehicular traffic networks" CDC 2015.
- Coogan and Arcak, "Efficient finite abstraction of mixed monotone systems" HSCC 2015.
- Coogan, Gol, Arcak, Belta, "Controlling a network of signalized intersections from temporal logic specifications" ACC2015.