

## CAREER: Embracing Complexity: A Fractal Calculus Approach to the Modeling and Optimization of Medical Cyber-Physical Systems

## Quantifying Fractal Behavior in Biological Systems: Novel Mathematics and Algorithms for Understanding the Brain

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# Description

### Challenges:

- Physiology exhibits *non-Gaussian, multi-fractal* and *non-stationary* characteristics
- Modeling *inter-patient asymmetric variability* & *interdependence* among physiological processes
- Accurate yet compact models of physiology

## Approach and Scientific Impact:

- Compact accurate mathematical models
- Fast and accurate model identification algorithms
- Fractal control algorithms

## Spatio-Temporal Fractal (STM) modeling of physiological processes:

- Proposed a *maximum causal non-extensive entropy principle* by exploiting micro-dynamics (increments) of the processes to construct STF
- Robust data-driven algorithm to infer *fractal* directed interdependence of physiological processes described by STF
- Guaranteed optimality to achieve full observability of the physiological system described by STF with minimal sensing efforts



# **Findings**

Finding 1: Mathematical & Algorithmic Approach for Reconstructing Complex Interdependency in **CPS under Malicious Interventions** 0.3

0.25 Causal inference tool capable to 0.2 decipher complex interdependency  $\mathbb{R}_{0.15}$ of a composable CPS with unknown 0.1 component dynamics and 0.05 unknown attacks

#### Degree Finding 2: Curvature-based Approach to Decomposing and **Mining Cyber-Physical Systems Interdependency**

Ollivier-Ricci curvature can reveal the hierarchical & community structure of a CPS interdependency without a priori knowledge 🍒 on the number and size of communities (in polynomial time)

#### Finding 3: The Mathematics of Approximate Submodular Functions (ASMF) for $(g_2, \alpha_2, \delta_2 = 1)$ Nonsubmodular Optimization in CPS

10

20

30

Inference by network model

Baseline

Proposed

 $\alpha = 10$ 

50

60

0.6 0.6

0.60.6

0.6

0.6

True

combined  $G_0$ 

0.3 0.6

 $(g_1, lpha_1, \delta_1)$ 

 $(g_i, \alpha_i, \delta_i)$ 

Inference by attack model

 $G_{0'1}$ 

50%

 $G_{0'2}$ 

- Nonsubmodular function f is  $\boldsymbol{\delta}$  away from a submodular surrogate g
- Lemma:  $\boldsymbol{\delta}$  cannot be arbitrarily low for any function f being limited by  $\delta \geq \frac{1-\gamma_f}{1+\gamma_f}$ , where  $\gamma_f$  is submodularity ratio
- *Theorem*:  $\delta$ -ASMF has constant performance guarantee penalized by  $\delta$ 
  - Applications: Bayesian D-optimality, minimum eigenvalue, Bayesian E-optimality