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Event-Triggered Output Feedback Control for Networked Control Systems using Passivity: Time-varying Network Induced Delays

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***Abstract*— When network induced delays are considered in the event-triggered control literature, they are typically delays from the plant to the controller and a tight bound on the admissible delays is usually imposed based on the analysis of inter-event time. In [19], a dynamic output feedback based event-triggered control scheme is introduced for stabilization of input feed-forward output feedback passive (IF-OFP) networked control systems (NCSs), which is a more general case compared with the passive and the output feedback passive (OFP) systems studied in [18]. Based on the results shown in [19], we propose a dynamic output feedback based event-triggered control set-up for NCSs which allows us to consider network induced delays both from the plant to the controller and from the controller to the plant. We show that based on the proposed set-up, finite-gain $\mathcal{L}_2$ stability can be achieved in the presence of arbitrary constant network induced delays or time-varying delays with bounded jitters.**

I. Introduction

Event-triggered control has been introduced for the possibility of reducing resources usage (i.e., sampling rate, CPU time, network access frequency) while preserving system's stability in networked control systems (NCSs) and embedded control systems[11]. Although there are heterogeneous terminologies in the literature which refer to the triggering mechanism as event-based-sampling[11], to event-driven sampling[12], Lebesgue sampling[4], deadband control[13], level-crossing sampling[14], state-triggered sampling[5] and self-triggered sampling[8] with slightly different meanings, in all cases, the triggering mechanisms are referring to the situation in which the control signals are kept constant until the violation of a condition on certain signals of the plant triggers the re-computation of the control signals.

Although the advantages of event-triggered control are well-motivated and even practical applications show its potential, there are still some problems that need to be addressed before event-triggered control can be fruitfully applied in NCSs. Most of the work on event-triggered control considers static state feedback controllers, which assumes that full plant's state can be measured. As in many control applications the full state information may not be available for measurement, it is important to study stability and performance of event-triggered control systems with static and dynamical output feedback based controllers. However, there are not many theoretical results on this problem in the literature.

An early work on event-triggered control using dynamic output feedback based controllers is presented in [14]. But a thorough analysis of the minimum time between two subsequent events, the so-called inter-event time, is not available in there. A recent work on output feedback based event-triggered control scheme with guaranteed $\mathcal{L}_\infty$-gain for linear time-invariant control system is introduced in [17], where the event-triggered control system is modeled as an impulsive system and linear matrix inequalities are used to study the stability and performance of event-triggered control systems. The framework shown in [17] cannot be easily extended to nonlinear control systems, and the triggering mechanism requires a synchronization between the plant and the network controller which is not very practical to be implemented in NCSs. In our previous work [18], a static output feedback based event-triggered control scheme is introduced for stabilization of passive and output feedback passive(OFP) NCSs, where a static output feedback gain and a triggering condition are derived based on the output feedback passivity indices of the plant. In [19], we propose an event-triggered control scheme for stabilization of Input Feed-forward Output Feedback Passive(IF-OFP) systems, which is a more general framework compared with our previous results in [18]. The triggering condition is derived based on the passivity theorem which characterizes a large class of output feedback stabilizing controllers. Moreover, $\mathcal{L}_2$ stability is guaranteed in the presence of bounded external disturbances as long as network induced delays from the plant to the network controller is upper bounded by the inter-event time.

However, in real time NCSs, the network induced delay is usually unknown, it is very likely having delay larger than the inter-event time. Moreover, in the presence of external disturbances, the admissible network induced delays derived based on the triggering condition could be every small as discussed in [19]. Furthermore, non-trivial delays from the controller to the actuator could also jeopardize the stability of the control system. Thus, when we apply event-triggered control to NCSs, it is important to take those problems just mentioned into consideration. Based on the passivity framework of dynamic output feedback based event-triggered control strategy investigated in [19], we propose an event-triggered control set-up for NCSs which allows us to consider network induced delays both from the plant to the controller and from controller to the plant. We show that based on the proposed set-up, finite-gain $\mathcal{L}_2$ stability can be achieved in the presence of arbitrary constant network induced delays or delays with bounded jitters. The rest of this paper is organized as follows: we introduce some background on passive and dissipative systems in section II; the problem is

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stated in section III; our main results are provided in section IV and followed by the examples provided in section V; concluding remarks are made in section VI.

II. Background Material

Consider the following control system, which could be linear or nonlinear:

$$H_p : \begin{cases} \dot{x}_p = f_p(x_p, u_p) \\ y_p = h_p(x_p, u_p) \end{cases} \tag{1}$$

where $x_p \in X_p \subset \mathbb{R}^n$, $u_p \in U_p \subset \mathbb{R}^m$ and $y_p \in Y_p \subset \mathbb{R}^m$ are the state, input and output variables, respectively, and X_p, U_p and Y_p are the state, input and output spaces, respectively. The representation $\phi_p(t, t_0, x_{p0}, u_p)$ is used to denote the state at time t reached from the initial state x_{p0} at time t_0.

Definition 1(Supply Rate)[1]: The supply rate $\omega_p(t) = \omega_p(u_p(t), y_p(t))$ is a real valued function defined on $U_p \times Y_p$, such that for any $u_p(t) \in U_p$ and $x_{p0} \in X_p$ and $y_p(t) = h_p(\phi_p(t, t_0, x_{p0}, u_p), u_p)$, $\omega_p(t)$ satisfies

$$\int_{t_0}^{t_1} |\omega_p(\tau)| d\tau < \infty. \tag{2}$$

Definition 2(Dissipative System)[1]: System H_p with supply rate $\omega_p(t)$ is said to be **dissipative** if there exists a nonnegative real function $V_p(x) : X_p \to \mathbb{R}^+$, called the storage function, such that, for all $t_1 \geq t_0 \geq 0$, $x_{p0} \in X_p$ and $u_p \in U_p$,

$$V_p(x_{p1}) - V_p(x_{p0}) \leq \int_{t_0}^{t_1} \omega_p(\tau) d\tau, \tag{3}$$

where $x_{p1} = \phi_p(t_1, t_0, x_{p0}, u_p)$ and $\mathbb{R}^+$ is a set of nonnegative real numbers.

Definition 3(Passive System)[1]: System H_p is said to be **passive** if there exists a storage function $V_p(x_p)$ such that

$$V_p(x_{p1}) - V(x_{p0}) \leq \int_{t_0}^{t_1} u_p^T(\tau) y_p(\tau) d\tau, \tag{4}$$

if $V_p(x_p)$ is $\mathcal{C}^1$, then we have

$$\dot{V}_p(x_p) \leq u_p^T(t) y_p(t), \ \forall t \geq 0. \tag{5}$$

One can see that passive system is a special class of dissipative system with supply rate $\omega_p(t) = u_p^T(t) y_p(t)$.

Definition 4(IF-OFP systems)[2]: System H_p is said to be **Input Feed-forward Output Feedback Passive**(IF-OFP) if it is dissipative with respect to the supply rate

$$\omega_p(u_p, y_p) = u_p^T y_p - \rho_p y_p^T y_p - \nu_p u_p^T u_p, \ \forall t \geq 0, \tag{6}$$

for some $\rho_p, \nu_p \in \mathbb{R}$.

For the rest of this paper, we will denote an $m-$inputs $m-$outputs dissipative system with supply rate (6) by *IF-OFP*$(\nu_p, \rho_p)^m$ and we will call (ν_p, ρ_p) *passivity indices* of the system.

Theorem 1 (Passivity Theorem)[15]: Consider a well-posed feedback interconnection as shown in Fig.1, and suppose each feedback component satisfies the inequality

$$\dot{V}_i \leq u_i^T y_i - \rho_i y_i^T y_i - \nu_i u_i^T u_i, \ \text{for } i = 1, 2, \tag{7}$$

for some storage function $V_i(x_i)$. Then, the closed-loop map from $\omega = [\omega_1^T, \omega_2^T]^T$ to $y = [y_1^T, y_2^T]^T$ is finite gain $\mathcal{L}_2$ stable if

$$0 < \rho_1 + \nu_2 < \infty, \quad 0 < \rho_2 + \nu_1 < \infty. \tag{8}$$

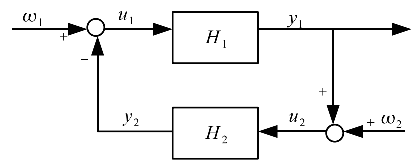

Fig. 1: Feedback Interconnection of Two IF-OFP Systems

Lemma 1[16]: Without loss of generality the domain of ρ_p, ν_p in IF-OFP system (6) is considered by $\Omega = \Omega_1 \cup \Omega_2$ with $\Omega_1 = \{\rho_p, \nu_p \in \mathbb{R} | \rho_p \nu_p < \frac{1}{4}\}$, $\Omega_2 = \{\rho_p, \nu_p \in \mathbb{R} | \rho_p \nu_p = \frac{1}{4}; \rho_p > 0\}$.

III. Problem Statement

We consider the control system given in (1). We assume H_p is IF-OFP$(\nu_p, \ \rho_p)^m$ with storage function V_p. Based on Theorem 1, we know that if we design an IF-OFP$(\nu_c, \ \rho_c)^m$ controller with storage function V_c such that $0 < \rho_c + \nu_p < \infty$, $0 < \rho_p + \nu_c < \infty$, then the closed-loop system is finite gain $\mathcal{L}_2$ stable.

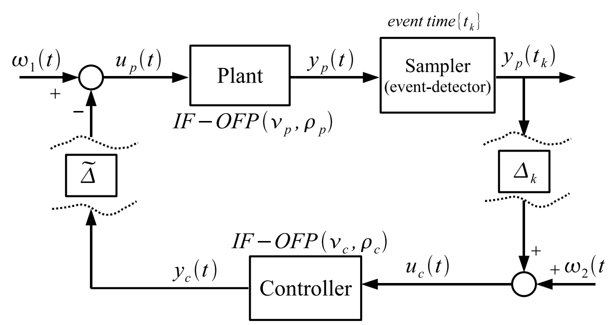

Fig. 2: Event-Triggered NCSs

In real time NCSs, the implementation of the feedback control law is typically done by sampling the plant output $y_p(t)$ at time instants $t_0, \ t_1, \ \ldots$, computing the control action $y_c(t)$ and updating the input to the plant at time instants $t_0 + \Delta_0, \ t_1 + \Delta_1, \ \ldots$, where $\Delta_k \geq 0$, for $k = 0, 1, 2, \ldots$ represents the network induced delay from the plant to the network controller(here, we assume the delay from the controller to the actuator is negligible). This means a sequence of measurements $y_p(t_k)_s$ corresponds to a sequence of controller's input updates $u_c(t_k + \Delta_k)_s$. In event-triggered NCSs, an event-detector (an embedded hardware in the sampler) is used to monitor the output of the plant with sufficiently fast sampling rate, an updated output measurement is sent to the network controller only when the size of the output novelty error $\tilde{e}_p(t) = y_p(t) - y_p(t_k)$ (for $t \in [t_k, t_{k+1})$) exceeds a certain threshold(triggering condition), where $y_p(t_k)$ denotes the last transmitted output information of the plant. However, in most set-up of event-triggered NCSs studied in the literature (see [5],[6],[8], [18]), only network induced delay (Δ_k as shown in Fig.2) from the plant to the controller has been considered and a bound on the admissible network induced delay is obtained based on the inter-event time to schedule data transmission, while the network induced delay from the controller to the actuator ($\widetilde{\Delta}$ as shown in Fig.2) is neglected.

As discussed in Section I, it is important to consider network induced delays both from the plant to the controller and from the controller to the actuator in event-triggered NCSs, and we also need to take care of the case when network induced delay is larger than the inter-event time. In the following section, we propose a set-up to solve those problems based on our previous results in [19].

IV. Main Results

We first state a main result in our companion paper [19], which is important to arrive at the triggering condition in our proposed set-up.

Theorem 2 ([19]). Consider the control system as shown in Fig.2, where the plant is IF-OFP$(\nu_p, \rho_p)^m$ with storage function V_p, the controller is IF-OFP$(\nu_c, \rho_c)^m$ with storage function V_c, and $0 < \nu_c + \rho_p < \infty$, $0 < \nu_p + \rho_c < \infty$. Assume that the network induced delay $\Delta_k \equiv 0$ and $\widetilde{\Delta} \equiv 0$. If the event time t_k is explicitly determined by the following triggering condition

$$\|\tilde{e}_p(t)\|_2 = \frac{\delta}{\zeta}\left[\sqrt{\beta(\rho_p+\nu_c)+\frac{\nu_c^2}{\zeta^2}}-\frac{|\nu_c|}{\zeta}\right]\|y_p(t)\|_2,\ \forall t \geq 0, \tag{9}$$

$$\text{where}\quad \zeta = \left[\frac{1}{4\alpha(\nu_p+\rho_c)}+|\nu_c|-\nu_c\right]^{\frac{1}{2}}, \tag{10}$$

with $\delta \in (0,1]$ and $0 < \alpha,\ \beta < 1$, then the control system is $\mathcal{L}_2$ stable from $\omega(t) = [\omega_1^T(t),\ \omega_2^T(t)]^T$ to $y(t) = [y_p^T(t),\ y_c^T(t)]^T$.

The triggering condition (9) in Theorem 2 determines when a new sampled output information of the plant should be sent to the network controller for control action update to ensure $\mathcal{L}_2$ stability of the control system in the absence of network induced delays. We could further get the admissible network induced delay from the plant to the controller by following similar analysis shown in [5], [6] and [7], [8], and as long as the delay is upper bounded by the inter-event time implicitly determined by the triggering condition, $\mathcal{L}_2$ stability of the system can be assured. However, the admissible delay obtained by following those methods might be very conservative and in the presence of external disturbances, the admissible delay could be extremely small [19].

For simplification of presentation, let

$$\sigma_o = \frac{1}{\zeta}\left[\sqrt{\beta(\rho_p+\nu_c)+\frac{\nu_c^2}{\zeta^2}}-\frac{|\nu_c|}{\zeta}\right]. \tag{11}$$

In the following proposition, we consider network induced delays both from the plant to the controller and from the controller to the plant and we introduce a set-up which guarantees $\mathcal{L}_2$ stability of the control system in the presence of arbitrary constant network induced delays or delays with bounded jitter.

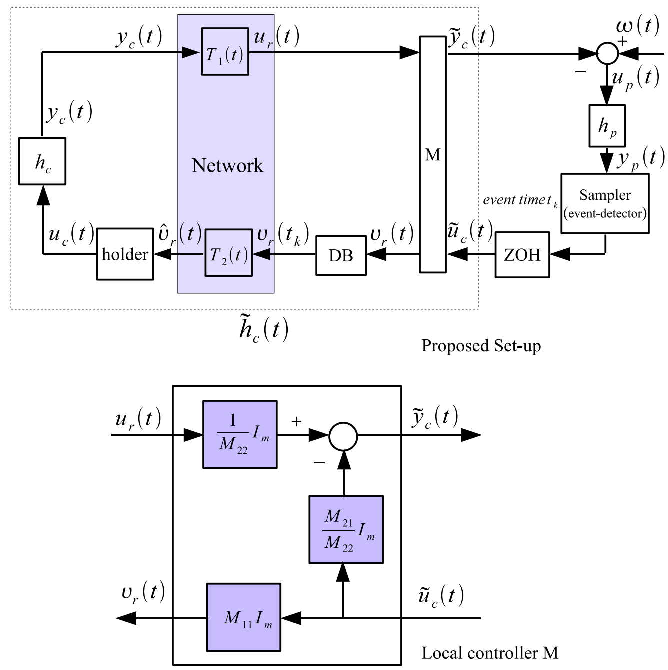

Fig. 3: Proposed Set-up for Event-Triggered NCSs

Proposition 1. Consider the set-up for event-triggered networked control system as shown in Fig.3. The plant h_p is an IF-OFP$(\nu_p, \rho_p)^m$ system with storage function V_p and the network controller h_c is an IF-OFP$(\nu_c, \rho_c)^m$ system with storage function V_c; $T_1(t)$ represents the network induced delay from the network controller to the plant, and $T_2(t)$ represents the network induced delay from the plant to the network controller; the "ZOH" block denotes the zero-order holder; the "DB" block represents the dead-band control so that the signal $\upsilon_r(t)$ can only be transmitted when

$$\|\upsilon_r(t)-\upsilon_r(t_k)\|_2 = \delta\sigma_o\|\upsilon_r(t)\|_2 \text{ and } \|\upsilon_r(t)\|_2 \geq \Delta_{min}, \tag{12}$$

where Δ_{min} is some lower bound on the dead-band designed for practical application; $\upsilon_r(t_k)$ denotes the last transmitted signal at the time instant t_k, $\upsilon_r(t_0) = M_{11}y_p(t_0)$, and $\delta \in (0,1]$; M is a local controller implemented at the plant side such that

$$\begin{bmatrix}\upsilon_r(t)\\ u_r(t)\end{bmatrix} = M\begin{bmatrix}\tilde{u}_c(t)\\ \tilde{y}_c(t)\end{bmatrix} = \begin{bmatrix}M_{11}I_m & 0\\ M_{21}I_m & M_{22}I_m\end{bmatrix}\begin{bmatrix}\tilde{u}_c(t)\\ \tilde{y}_c(t)\end{bmatrix} \tag{13}$$

where $I_m \in \mathbb{R}^{m\times m}$ is the identity matrix and M_{11}, M_{21}, M_{22} are chosen such that

$$\begin{aligned} M_{11}^2 &= \frac{\frac{1}{4\rho_c}-\nu_c}{\frac{1}{2\rho_c}+|\nu_c|},\quad M_{21}^2 = \frac{1}{2(1-d_1)\rho_c^2}\\ M_{22}^2 &= \frac{2}{1-d_1},\quad M_{21}M_{22} < 0. \end{aligned} \tag{14}$$

The implementation of M is also illustrated in Fig.3. Let the following conditions be satisfied:

1) the controller is designed such that $0 < \nu_c + \rho_p < \infty$ and $0 < \rho_c + \nu_p < \infty$, with $\rho_c > 0$ and $\rho_c \nu_c < \frac{1}{4}$;
2) $0 \le \left|\frac{dT_1(t)}{dt}\right| \le d_1 < 1$, $0 \le \left|\frac{dT_2(t)}{dt}\right| \le d_2 < 1$;
3) the holder at the controller side yields

$$u_c(t) = \frac{1}{(1+\delta\sigma_o)\sqrt{1+d_2}} \upsilon_r(t_k), \quad (15)$$

$$\text{for } t \in \Big[t_k + T_2(t_k), t_{k+1} + T_2(t_{k+1})\Big), \forall k.$$

If the event time t_k is explicitly determined by the triggering condition

$$\|\tilde{e}_p(t)\|_2 = \delta\sigma_o \|y_p(t)\|_2, \text{ when } \|y_p(t)\|_2 \ge \frac{\Delta_{min}}{M_{11}} \quad (16)$$

then the control system is finite-gain $\mathcal{L}_2$ stable from $\omega(t)$ to $y_p(t)$.

Proof: Since the controller h_c is IF-OFP$(\nu_c, \rho_c)^m$, such that

$$\dot{V}_c \le u_c^T(t) y_c(t) - \rho_c y_c^T(t) y_c(t) - \nu_c u_c^T(t) u_c(t) \quad (17)$$

with $\rho_c > 0$ we can get

$$\begin{aligned}\dot{V}_c &\le -\rho_c \|y_c(t)\|_2^2 + \|u_c(t)\|_2 \|y_c(t)\|_2 + |\nu_c| \|u_c(t)\|_2^2 \\ &= -\frac{1}{2\rho_c}\big(\|u_c(t)\|_2 - \rho_c \|y_c(t)\|_2\big)^2 + \frac{1}{2\rho_c}\|u_c(t)\|_2^2 \\ &\quad - \frac{\rho_c}{2}\|y_c(t)\|_2^2 + |\nu_c| \|u_c(t)\|_2^2 \\ &\le \big(\frac{1}{2\rho_c} + |\nu_c|\big)\|u_c(t)\|_2^2 - \frac{\rho_c}{2}\|y_c(t)\|_2^2,\end{aligned} \quad (18)$$

integrating both sides of the inequality (18) from t_0 to t, we can get

$$\begin{aligned}\Delta V_c = V_c(t) - V_c(t_0) &\le \big(\frac{1}{2\rho_c} + |\nu_c|\big)\int_{t_0}^t \|u_c(t)\|_2^2 dt \\ &\quad - \frac{\rho_c}{2}\int_{t_0}^t \|y_c(t)\|_2^2 dt.\end{aligned} \quad (19)$$

Since

$$\hat{\upsilon}_r(t) = \begin{cases} 0, \text{ for } t \in \Big(t_k + T_2(t_k),\ t_{k+1} + T_2(t_{k+1})\Big),\ \forall k \\ \upsilon_r(t_k),\ \text{for } t = t_k + T_2(t_k),\ \forall k, \end{cases} \quad (20)$$

and $u_c(t) = \frac{1}{(1+\delta\sigma_o)\sqrt{1+d_2}}\upsilon_r(t_k)$, for $t \in \Big[t_k + T_2(t_k), t_{k+1} + T_2(t_{k+1})\Big)$, we can further obtain

$$\begin{aligned}&\int_{t_0}^t \|u_c(t)\|_2^2 dt \\ &= \sum_{k=0}^N \big[t_{k+1} - t_k + T_2(t_{k+1}) - T_2(t_k)\big] \frac{\|\upsilon_r(t_k)\|_2^2}{(1+\delta\sigma_o)^2(1+d_2)} \\ &\le \sum_{k=0}^N \frac{(1+d_2)(t_{k+1}-t_k)}{(1+d_2)} \frac{\|\upsilon_r(t_k)\|_2^2}{(1+\delta\sigma_o)^2} \\ &= \sum_{k=0}^N (t_{k+1}-t_k)\frac{\|\upsilon_r(t_k)\|_2^2}{(1+\delta\sigma_o)^2}.\end{aligned} \quad (21)$$

Note that the dead-band control actually guarantees that

$$\|\upsilon_r(t) - \upsilon_r(t_k)\|_2 \le \delta\sigma_o \|\upsilon_r(t)\|_2, \quad \text{for } t \in [t_k, t_{k+1}], \forall k, \quad (22)$$

also note that $\|\upsilon_r(t) - \upsilon_r(t_k)\|_2 \ge \|\upsilon_r(t_k)\|_2 - \|\upsilon_r(t)\|_2$, we can conclude that

$$\|\upsilon_r(t_k)\|_2 \le (1+\delta\sigma_o)\|\upsilon_r(t)\|_2, \text{ for } t \in [t_k, t_{k+1}],\ \forall k, \quad (23)$$

thus

$$\begin{aligned}\int_{t_0}^t \|u_c(t)\|_2 &\le \sum_{k=0}^N \big(t_{k+1} - t_k\big)\frac{\|\upsilon_r(t_k)\|_2^2}{(1+\delta\sigma_o)^2} \\ &= \sum_{k=0}^N \int_{t_k}^{t_{k+1}} \frac{\|\upsilon_r(t_k)\|_2^2}{(1+\delta\sigma_o)^2} dt \\ &\le \sum_{k=0}^N \int_{t_k}^{t_{k+1}} \|\upsilon_r(t)\|_2 dt = \int_{t_0}^t \|\upsilon_r(t)\|_2^2 dt.\end{aligned} \quad (24)$$

Since $0 \le \left|\frac{dT_1(t)}{dt}\right| \le d_1 < 1$, one could verify that

$$\int_{t_0}^t \|u_r(t)\|_2^2 dt \le \frac{1}{1-d_1}\int_{t_0}^t \|y_c(t)\|_2^2 dt,$$

so

$$-\int_{t_0}^t \|y_c(t)\|_2^2 dt \le -(1-d_1)\int_{t_0}^t \|u_r(t)\|_2^2 dt. \quad (25)$$

Substitute (24) and (25) into (19), we have

$$\begin{aligned}\Delta V_c &\le \big(\frac{1}{2\rho_c} + |\nu_c|\big)\int_{t_0}^t \|\upsilon_r(t)\|_2^2 dt \\ &\quad - \frac{\rho_c(1-d_1)}{2}\int_{t_0}^t \|u_r(t)\|_2^2 dt.\end{aligned} \quad (26)$$

Based on (13), we can get

$$\upsilon_r(t) = M_{11}\tilde{u}_c(t), \quad u_r(t) = M_{21}\tilde{u}_c(t) + M_{22}\tilde{y}_c(t), \quad (27)$$

substitute (27) and (14) into (26) we can get,

$$\Delta V_c \le \int_{t_0}^t \big(\tilde{u}_c^T(t)\tilde{y}_c(t) - \nu_c \tilde{u}_c^T(t)\tilde{u}_c(t) - \rho_c \tilde{y}_c^T(t)\tilde{y}_c(t)\big)dt, \quad (28)$$

which implies that the system $\widetilde{h}_c : \tilde{u}_c(t) \to \tilde{y}_c(t)$ is IF-OFP$(\nu_c, \rho_c)^m$.

According to Theorem 2, for the feedback interconnection of h_p and $\widetilde{h}_c$, if we schedule the transmission of the output measurement $y_p(t)$ according to the triggering condition (9), then the closed-loop system will be finite-gain $\mathcal{L}_2$ stable. Further more, because the growing rate on the threshold of the dead-band control is the same as the growing rate on the threshold of the triggering condition, one can conclude that the data transmission of $\upsilon_r(t)$ and the triggering process are actually synchronized. Thus, whenever a new output information of the plant is obtained, an updated signal $\upsilon_r(t)$ will be sent to the network controller. When $\|\upsilon_r(t)\|_2 < \Delta_{min}$, which could be considered as the case when the output of the plant reaches some safe region for practical application, then no more data transmission is needed. The proof is completed. ■

Remark 1: Based on the set-up shown in Proposition 1, one can see that system $\tilde{h}_c : \tilde{u}_c(t) \rightarrow \tilde{y}_c(t)$ is IF-OFP$(\nu_c, \rho_c)^m$ as the controller $h_c : u_c(t) \rightarrow y_c(t)$, so the networked control system can still be analyzed as a feedback interconnection of an IF-OFP$(\nu_p, \rho_p)^m$ plant with an IF-OFP$(\nu_c, \rho_c)^m$ subsystem $\tilde{h}_c$, and we can apply Theorem 2 to derive the triggering condition. One can further use the analysis shown in ([19], Proposition 1 and Proposition 3) to estimate the inter-event time.

Remark 2: Traditionally, NCSs are referred to the direct type remote control loops as shown in Fig.2. One may argue that in our set-up, we need a local controller at the plant side. But as illustrated in Fig.3, the local controller M only requires a direct output feedback loop from $\tilde{u}_c(t)$ to $\tilde{y}_c(t)$ with gain $\frac{M_{21}}{M_{22}}$, so in many cases, this should not be a problem regarding cost considerations and hardware limitations for the implementation of such a simple local controller. The tedious and complex control action computation can still be done at the network controller h_c.

Remark 3: In our proposed set-up, instead of obtaining a bound on the admissible networked induced delay based on the triggering condition or on the current state of the plant (see [5],[7],[8],[9],[18]), we consider delays both from the plant to the controller and from the controller to the plant, and we have shown that finite-gain $\mathcal{L}_2$ stability can be achieved with arbitrary constant delays or delays with bounded "jitter".

Remark 4: One should notice that the implementation of the local controller M at the plant side requires the knowledge of the network controller's passivity indices (ρ_c, ν_c), and the knowledge of the "jitter" of the network induced delay from the controller to the plant d_1. The implementation of the "holder" at the controller side requires the knowledge of the "jitter" of the network induced delays from the plant to the controller d_2, and also the information on the triggering threshold $\delta\sigma_o$.

V. Example

Example 1. Consider the IF-OFP system given by

$$\begin{aligned} \dot{x}_{p1}(t) &= -3x_{p1}^3(t) + x_{p1}(t)x_{p2}(t) \\ \dot{x}_{p2}(t) &= 0.2x_{p2}(t) + 2u_p(t) \\ y_p(t) &= x_{p2}(t), \end{aligned} \tag{29}$$

we can see that the system is ZSD but unstable. If we choose the storage function $V_p(x_p) = \frac{1}{4}x_{p2}^2(t)$, we can get

$$\dot{V}_p(x_p) = u_p(t)y_p(t) + 0.1y_p^2(t), \tag{30}$$

and in this case $\rho_p = -0.1$, $\nu_p = 0$.

If we consider an IF-OFP controller, which is given by

$$\begin{aligned} \dot{x}_c(t) &= -3x_c(t) + u_c(t) \\ y_c(t) &= 7x_c(t) + u_c(t), \end{aligned} \tag{31}$$

with storage function $V_c(x_c) = \frac{49}{26}x_c^2(t)$, we can get

$$\dot{V}_c(x_c) = u_c(t)y_c(t) - \frac{3}{13}y_c^2(t) - \frac{10}{13}u_c^2(t), \tag{32}$$

and in this case $\rho_c = \frac{3}{13}$, $\nu_c = \frac{10}{13}$. So we have $\rho_c + \nu_p > 0$ and $\nu_c + \rho_p > 0$.

If we choose $\alpha = \beta = 0.9$, then the triggering condition shown in Proposition 1 with $\delta = 1$ is given by

$$\|\tilde{e}_p(t)\|_2 = 0.3142\|y_p(t)\|_2, \forall t \geq 0. \tag{33}$$

Assume that the network induced delay from the plant to the controller is 0.5s, and the delay from the controller to the plant is 0.2s(since we assume constant delays in this case, $d_1 = d_2 = 0$). We can get

$$\begin{aligned} M_{11}^2 &= \frac{\frac{1}{4\rho_c} - \nu_c}{\frac{1}{2\rho_c} + |\nu_c|} = 0.1070, \quad M_{22}^2 = \frac{2}{1 - d_1} = 2, \\ M_{21}^2 &= \frac{1}{2(1 - d_1)\rho_c^2} = 9.3889, \quad M_{21}M_{22} < 0. \end{aligned} \tag{34}$$

Choose $M_{11} = 0.3271$, $M_{21} = 3.0641$, $M_{22} = -1.4142$, and note that the output of the holder in this case should be $u_c(t) = \frac{1}{(1+\delta\sigma_o)\sqrt{1+d_2}}\upsilon_r(t_k) = 0.7609\upsilon_r(t_k)$, for $t \in \big[t_k + T_2(t_k), t_{k+1} + T_2(t_{k+1})\big)$.

Assume that the external disturbance $\omega(t)$ is an uniformly distributed random signal on the interval $[0, 1]$. The simulation result is shown in Fig.4, where $\sigma(t)$ shows the evolution of $\frac{\|\tilde{e}_p(t)\|_2}{\|y_p(t)\|_2}$, $\{t_{k+1} - t_k\}$ shows the evolution of the inter-event time, x_{p1} and x_{p2} show the evolution of the states of the plant. Fig.5 shows the signal transimission process from $y_p(t)$ to $u_c(t)$.

Now assume that the network induced delay from the plant to the controller is increasing with rate 0.6($T_2(0) = 0.5s$), and the delay from the controller to the plant is increasing with rate 0.2($T_1(0) = 0.2s$), then in this case we have $d_1 = 0.2, d_2 = 0.6$. We can further get $M_{11} = 0.3271$, $M_{21} = 3.4258$, $M_{22} = -1.5811$, and the output of the holder in this case should be $u_c(t) = \frac{1}{(1+\delta\sigma_o)\sqrt{1+d_2}}\upsilon_r(t_k) = 0.6015\upsilon_r(t_k)$, for $t \in \big[t_k + T_2(t_k), t_{k+1} + T_2(t_{k+1})\big)$. The simulation results are shown in Fig.6 and Fig.7.

VI. Conclusion

In this paper, based on the dynamic output feedback based event-triggered control strategy shown in our companion paper [19], we propose a set-up for output feedback based event-triggered NCSs. The proposed set-up allows us to consider network induced delays both from the plant to the controller and from the controller to the plant. We have also shown that under the proposed scheme, finite-gain $\mathcal{L}_2$ stability can be achieved in the presence of arbitrary constant network induced delay or network induced delay with bounded jitter.

VII. Acknowledgments

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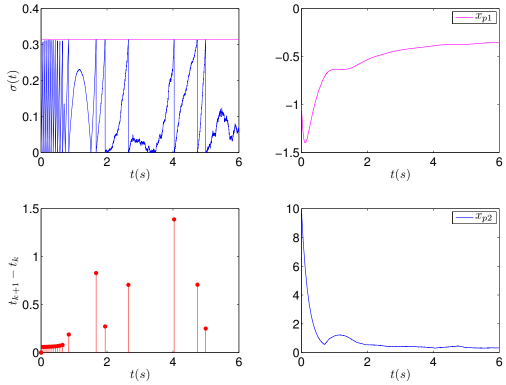

Fig. 4: simulation result of example 1 with constant network induced delay: triggering process and state evolution

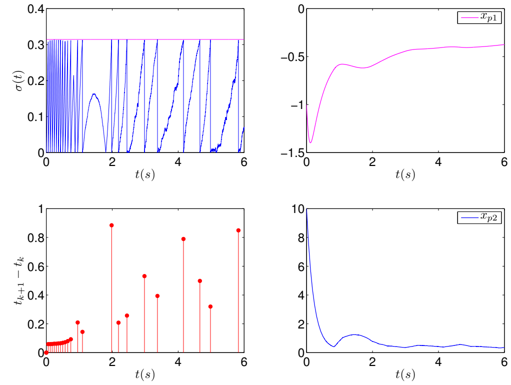

Fig. 6: simulation result of example 1 with time-varying network induced delay: triggering process and state evolution

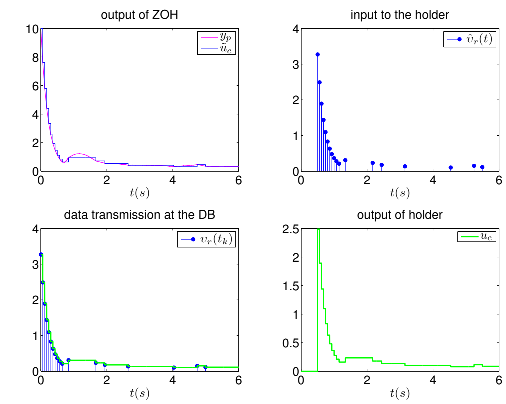

Fig. 5: simulation result of example 1 with constant network induced delay: signal transmission process from y_p to u_c

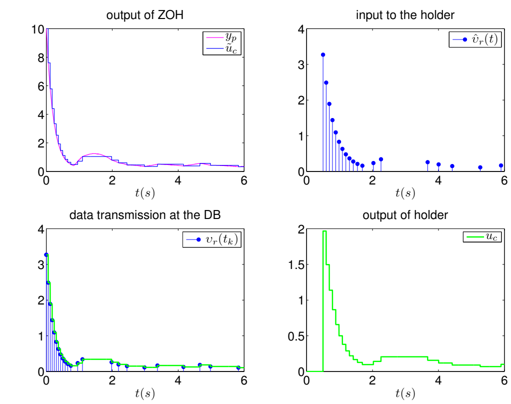

Fig. 7: simulation result of example 1 with time-varying network induced delay: signal transmission process from y_p to u_c

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