CNS-1329619: Robust team-triggered coordination for real-time control of networked cyber-physical systems

Event-triggered Control for Nonlinear Systems with Time-Varying Input Delay

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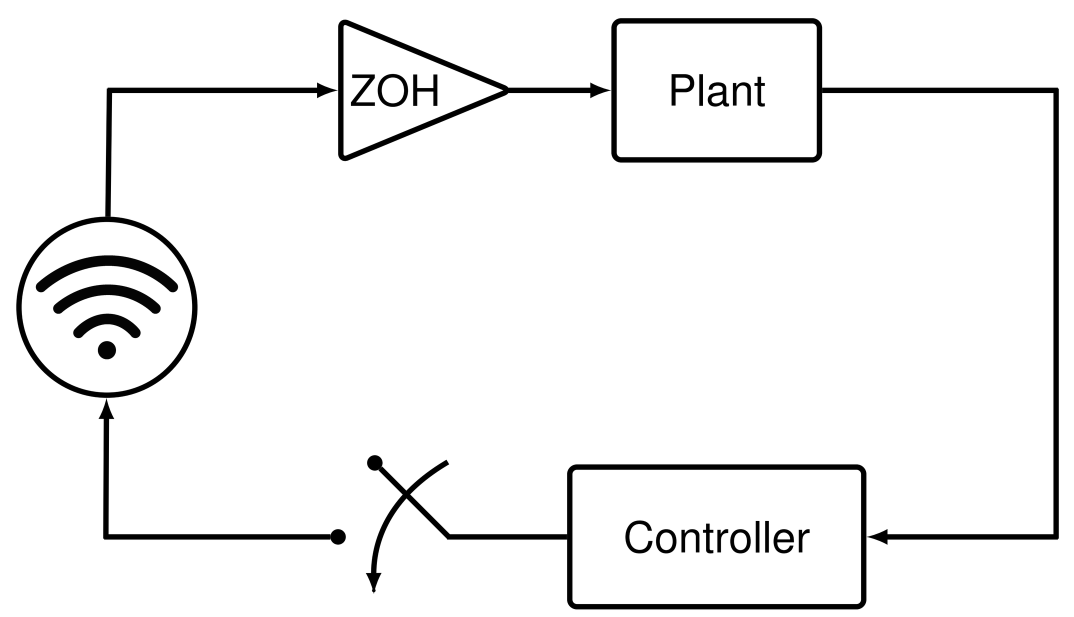
Overview

- ***Overarching project goal:*** opportunistic state-triggered aperiodic control for networked cyber-physical systems
- ***Time delay*** and ***bandwidth limitation*** widespread in real-world implementations of sensor-actuator networks
- For cyberphysical systems with ***nonlinear*** dynamics under time-varying ***input delays***, we address these limitations using predictor feedback (to compensate for time delay) and event-triggered control (to comply with limited bandwidth)
- ***Challenging*** due to opportunistic nature of event-triggering: controller "waits" until the system tends to become unstable and then updates the control accordingly, but if the control takes some time to reach the system, it may no longer be able to prevent the system from instability

Contributions

- ***Design*** of event-triggering controllers for wide class of nonlinear systems with arbitrarily large time delays using the method of predictor feedback
- ***Analysis*** of the proposed event-triggered control policy:
  - ***global asymptotic stability*** of the closed-loop system
  - uniform lower bound on the inter-event times (***no Zeno*** behavior)
  - ***exponential stability*** as well as, for linear systems, ***explicit expressions*** for design variables, convergence rate, and inter-event times
- Characterization of the ***trade-off*** between communication cost and convergence speed in event-triggered control for linear systems

Problem Statement



Consider general nonlinear dynamics

$$\dot{x}(t) = f(x(t), u(\phi(t))),$$

$\phi(t) = t - D(t)$ encodes known time-varying delay. ***Assumptions***:

- $\{u(t) \mid \phi(0) \le t \le 0\}$ is given and bounded
- system does not have finite escape time for any initial condition&bounded input
- ϕ is continuously differentiable and $\dot{\phi}(t) > 0$ for all $t \ge 0$
- there exist $M_0, M_1, m_2 > 0$ such that,

$$\forall t \ge 0 \qquad 0 < t - \phi(t) \le M_0 \quad \text{and} \quad m_2 \le \dot{\phi}(t) \le M_1$$

- there exists globally Lipschitz $K : \mathbb{R}^n \to \mathbb{R}$, $K(0) = 0$, such that

$$\dot{x}(t) = f(x(t), K(x(t)) + w(t)) \quad \text{is ISS with respect to } w$$

Design Objective

Design $\{(t_k, u(t_k))\}_{k=1}^{\infty}$ such that

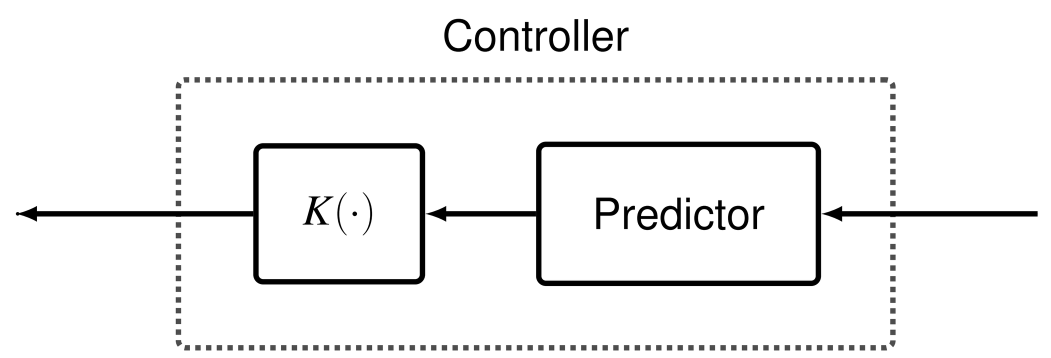
1. ***Event-triggered stabilization:*** the closed-loop system is globally asymptotically stable using

$$u(t) = u(t_k) \qquad t \in [t_k, t_{k+1}), k \in \mathbb{Z}_{\ge 0},$$

2. ***No Zeno behavior:*** $\lim_{k\to\infty} t_k = \infty$.

Event-Triggered Design

We propose the following structure for the controller:



For a complete design, we need to specify the ***predictor*** and ***triggering times***:

Design of Predictor [*Bekiaris-Liberis and Krstic, 2013*]

To compensate for the delay, the controller makes the following ***prediction of the future*** state of the plant,

$$p(t) = x(\phi^{-1}(t)) = x(t^+) + \int_{t^+}^{\sigma(t)} f\big(p(\phi(\tau)), u(\phi(\tau))\big)d\tau$$
$$= x(t^+) + \int_{\phi(t^+)}^{t} f(p(s), u(s))\frac{d\phi^{-1}(s)}{ds}ds, \qquad t \ge \phi(0),$$

where $t^+ = \max\{t, 0\}$.

- Integral only requires knowledge of the initial/current state of the plant and history of $u(t)$ and $p(t)$, which are ***both available to the controller***
- For general nonlinear vector fields f, prediction computed using ***numerical integration methods***

Design of Triggering Times

Let $S(x(t))$ be the storage/Lyapunov function for the ***delay-free system***. The Lyapunov function of the ***delayed system*** is

$$V(t) = S(x(t)) + \frac{2}{b}\int_0^{2L(t)} \frac{\rho(r)}{r}dr, \qquad L(t) = \sup_{t\le\tau\le\sigma(t)} |e^{b(\tau-t)}w(\phi(\tau))|,$$

and $b > 0$ is a design parameter. Then

$$\dot{V}(t) \le -\gamma(|x(t)|) - \rho(2L(t)) + \rho(2L_K|e(\phi(t))|),$$

(L_K is Lipschitz constant of K) so we design the triggering condition as

$$\rho(2L_K|e(\phi(t))|) \le \theta\gamma(|x(t)|) \Leftrightarrow |e(t)| \le \frac{\rho^{-1}(\theta\gamma(|p(t)|))}{2L_K}, \qquad \theta \in (0,1).$$

Control Analysis: Satisfaction of Design Goals

1. ***Event-triggered stabilization:***

Global Asymptotic Stability

There exists $\beta \in \mathcal{KL}$ such that for any $x(0) \in \mathbb{R}^n$ and bounded $\{u(t)\}_{t=\phi(0)}^{0}$,

$$|x(t)| + \sup_{\phi(t)\le\tau\le t} |u(\tau)| \le \beta\Big(|x(0)| + \sup_{\phi(0)\le\tau\le 0} |u(\tau)|, t\Big), \qquad t \ge 0.$$

2. ***No Zeno behavior:***

Uniform Lower Bound for the Inter-Event Times

$\boldsymbol{t_{k+1} - t_k \ge \delta}$ for all $k \ge 1$ where δ is the time that it takes for the solution of

$$\dot{r} = M_2(1+r)(L_f(1+L_K) + L_f L_K r),$$

to go from 0 to $\frac{1}{2L_{\gamma^{-1}\rho/\theta}L_K}$.

The Linear Case

If $\boldsymbol{f(x,u) = Ax + Bu}$, then ***the triggering condition simplifies*** to

$$|e(t)| \le \frac{\lambda_{\min}(Q)\sqrt{\theta}}{4|PB||K|}|p(t)|,$$

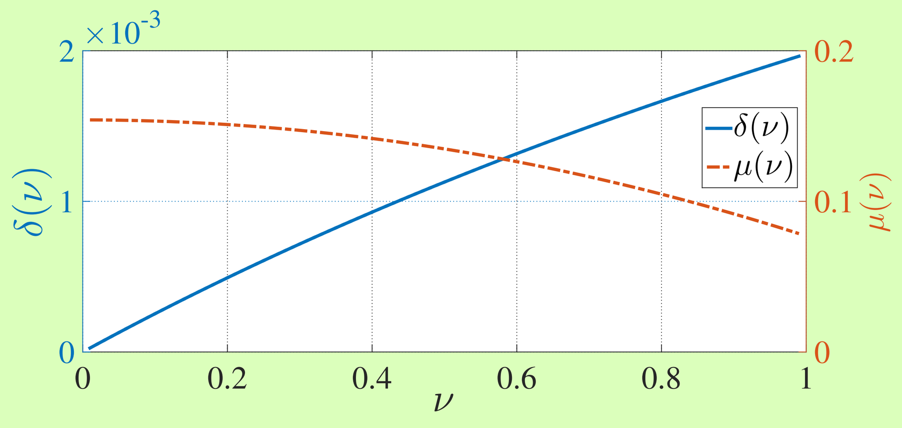
and we get ***exponential stability*** with rate $\mu = \frac{(2-\theta)\lambda_{\min}(Q)}{4\lambda_{\max}(P)}$.

Inter-Event Time vs Convergence Rate Trade-off

Let $\theta = \nu^2$, $Q = qI_n$. Then,

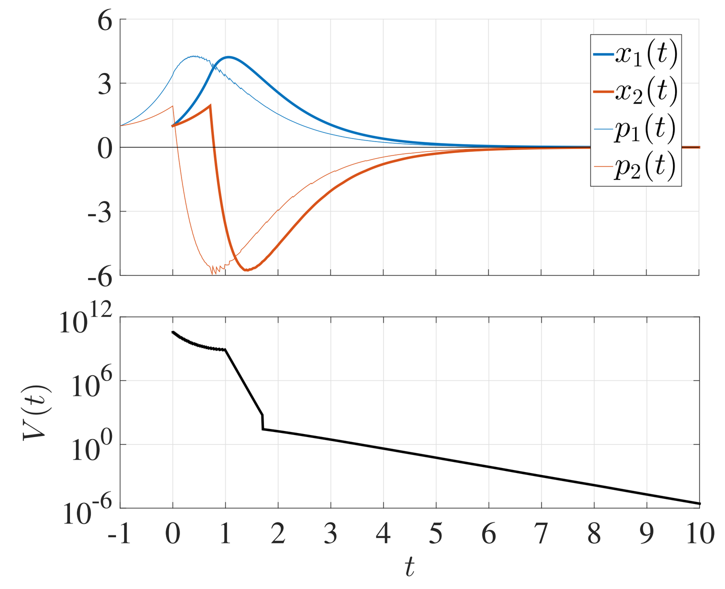
$$\delta(\nu) = \frac{1}{a-c}\ln\frac{c + \frac{\nu}{|P_1B||K|}a}{c + \frac{\nu}{|P_1B||K|}c}$$

$$\mu(\nu) = \frac{2-\nu^2}{4\lambda_{\max}(P_1)}$$

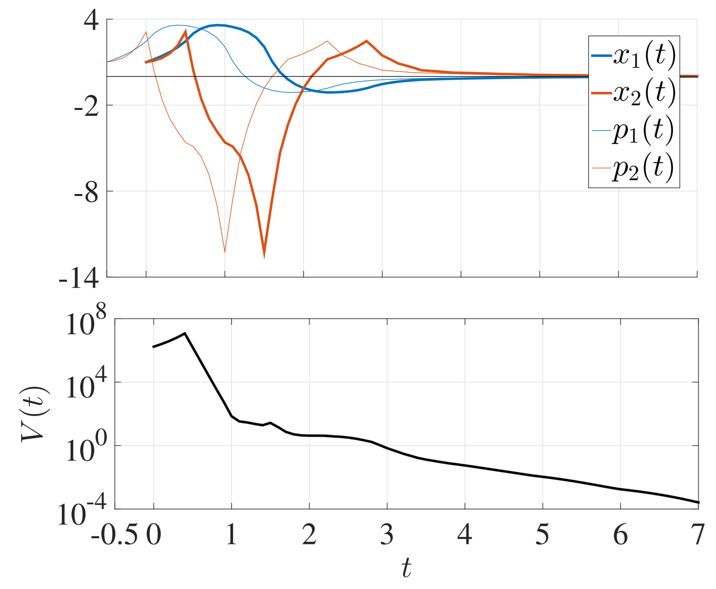


Simulations

Compliant System:



Non-compliant System:



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