Extending Passivity to Guarantee Properties in CPS Design

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Science of Integration for Cyber Physical Systems

Vanderbilt University
November 29-30, 2010

Fundamentals of Composition in Heterogeneous Systems

- Decoupling cross layer interaction.
  Achieving orthogonality of design concerns
  Develop passivity based tools
- Event-triggered and "any time" control in networked distributed systems with multiple controllers and processors

Role of Passivity and Symmetry

- How do we guarantee desirable properties in a network of heterogeneous systems which may change dynamically, and expand or contract?
- Can we start with a system and grow it in particular ways to preserve its properties?

Team & Topics

- Passivity in Dynamical Control Systems
  Continuous/Hybrid Framework PA, VG, BG
- Symmetry/Approx. Symmetry & Passivity
  Nonlinear/Graph Theory Framework BG, PA
- Networking, Information Processing, Control.
  Stochastic/Deterministic Framework VG, PA

  Baras (UM); Koutsoukos, Kottenstette (VU)

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- Next generation healthcare -- biomedical devices and systems engineering (wearable/implantable, minimally-invasive, bio-aware, bio-compatible, patient-specific, open, configurable, portable, universal point-of-care safety)
- Next generation energy systems (distributed, intermittent, renewable sources; shifting topology for generation, storage/transfer/transmission, distribution; smart loads, better control of dynamic demand-response; new sources/sinks: cars, buildings)
- Next generation environmental systems (in situ co-generation, multi-source energy harvesting, geo-thermal/ground-source heating and cooling; integrated environmental control: light, thermal, air- and water-quality, noise abatement, physical access)
- Next generation transportation (autonomous systems, energy-efficient, high-performance, multi-modal: air, automotive, rail, maritime systems, enhanced and affordable personal mobility and transport)
- Next generation manufacturing (flexible/configurable, multi-scale, interoperable line components, self-assembly, multi-process bio/chemical/mechanical engineering; precision next-generation - laser/thermal/EMF/bio/mechanical tooling and monitoring)
- Next generation agriculture (pervasive sensing, precision micro-climates/micro-cultures, pervasive animal health monitoring and veterinary medicine),
- Next generation water systems (atmospheric sources, reuse, quality sensing, exploration, hazard alerts)
- …

CPS Source: NSF

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It's a smart world

- **WHAT if there were two worlds, the real one and its digital reflection? The real one is strewn with sensors, picking up everything from movement to smell. The digital one, an edifice built of software, takes in all that information and automatically acts on it. If a door opens in the real world, so does its virtual equivalent. If the temperature in the room with the open door falls below a certain level, the digital world automatically turns on the heat.**
- **http://www.economist.com/node/17388368**

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It's a smart world

- **A Special Report on Smart Systems**
  - **A sea of sensors;**
  - **Making everydrop count;**
  - **Living on a platform;**
  - **Augmented business;**
  - **The IT payoff;**
  - **Your own private matrix;**
  - **Sensors and sensibilities;**
  - **Horror worlds.**

Economist November 6th, 2010

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Outline

- **Passivity & Dissipativity in Dynamical Systems**
  Definitions, Interconnections-Feedback, in Parallel.
  Conic Systems. Passivity Indices. Networked.
- **Passivity Indices-Cascade**
  Passivity Based Output Synchronization
  Event-Triggered Control
  Event-Triggered Control for Multi-Agent Systems
- **Switched Systems. Networked. Conic Systems**
- **Symmetry and Passivity**
- **Anytime Control Algorithms**

Definition of Passivity (CT)

- Consider a nonlinear dynamical system

$$\dot{x} = f(x,u)$$
$$y = h(x,u).$$

- This system is *passive* if there exists a continuous storage function *V(x) ≥ 0 (for all x)* such that

$$\int_{t_1}^{t_2} u^T(t)y(t)dt + V(x(t_1)) \geq V(x(t_2))$$

for all $t_2 \geq t_1$ and the input $u(t) \in U$.

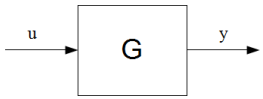
- When V(x) is continuously differentiable, this can be written:

$$u^T(t)y(t) \geq \dot{V}(x(t))$$

Definition of Lyapunov Stability

- Consider a nonlinear dynamical system

$$\dot{x} = f(x,u)$$
$$y = h(x,u)$$



- This system is *Lyapunov stable* if there exists a continuous *Lyapunov* function *V(x) (V(x) > 0 for all x≠0 and V(0)=0)* such that

$$V(x(t_1)) \geq V(x(t_2))$$

for all $t_2 \geq t_1$

Definition of Dissipativity (CT)

- This concept generalizes passivity to allow for an arbitrary supply rate *ω(u,y)*.
- A system is *dissipative* with respect to supply rate *ω(u,y)* if there exists a continuous storage function *V(x) ≥ 0* such that

$$\int_{t_1}^{t_2} \omega(u,y)dt \geq V(x(t_2)) - V(x(t_1))$$

for all t_1, t_2 and the input $u(t) \in U$.

- A special case of dissipativity is the QSR definition where the energy supply rate takes the following form:

$$\omega(u,y) = y^T Q y + 2y^T S u + u^T R u.$$

Examples of Passive Systems

- G1 is a passive system if the pole is positive (*a≥0*) but not passive for *a<0*

$$G_1(s) = \frac{1}{s+a}$$

- G2 is not passive for any *a* because of the negative gain

$$G_2(s) = \frac{-1}{s+a}$$

- Even a stable, non-minimum phase system can be non-passive if the phase shift is too large (G3)

$$G_3(s) = \frac{s+5}{s^2+2s+2}$$

- G3 would have been passive if the zero were closer to the origin (G4)

$$G_4(s) = \frac{s+0.5}{s^2+2s+2}$$

Definition of Passivity (DT)

- Passivity can be defined for discrete time systems as well. Consider a nonlinear discrete time system

$$x(k+1) = f(x(k), u(k))$$
$$y(k) = h(x(k), u(k)).$$

- This system is *passive* if there exists a continuous storage function *V(x) ≥ 0* such that

$$\sum_{k=k_1}^{k_2} u^T(k) y(k) \geq V(x(k_2)) - V(x(k_1))$$

for all k_1, k_2 and all inputs *u(k) ϵ U*.

Definition of Dissipativity (DT)

- The concept of dissipativity can apply to discrete time systems for an arbitrary supply rate *ω(u,y)*.
- A system is *dissipative* with respect to supply rate *ω(u,y)* if there exists a continuous storage function *V(x) ≥ 0* such that

$$\sum_{k=k_1}^{k_2} \omega(u,y) \geq V(x(k_2)) - V(x(k_1))$$

for all k_1, k_2 and the input *u(k) ϵ U*.

- A special case of dissipativity is the QSR definition where the energy supply rate takes the following form:

$$\omega(u,y) = y^T Q y + 2 y^T S u + u^T R u.$$

LMI Formulation

- There are computational methods to find storage functions. For LTI passive systems, can always assume quadratic storage function

$$V(x) = \frac{1}{2} x^T P x \quad \text{where} \quad P = P^T > 0.$$

- For continuous-time system this leads to the following LMI

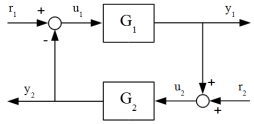
$$\begin{bmatrix} A^T P + PA & PB - C^T \\ B^T P - C & -D - D^T \end{bmatrix} \leq 0$$

- In discrete-time the LMI becomes the following

$$\begin{bmatrix} A^T PA - P & A^T PB - C^T \\ B^T PA - C & B^T PB - D - D^T \end{bmatrix} \leq 0$$

Interconnections of Passive Systems

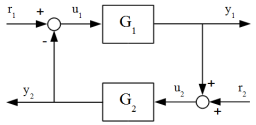
- One of the real strengths of passivity is in interconnected systems. Passive systems are stable and passivity is preserved in many practical interconnections.
- For example, the negative feedback interconnection of two passive systems is passive.



- If $u_1 \rightarrow y_1$ and $u_2 \rightarrow y_2$ are passive then the mapping $r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \rightarrow y = \begin{bmatrix} y_1 \\ y2 \end{bmatrix}$ is passive

Interconnections of Stable Systems

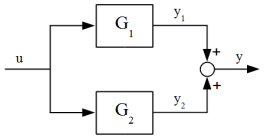
- Compared with passive systems, the feedback interconnection of two stable systems is not always stable



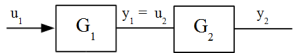
- One notable special case is the small gain theorem where if G_1 and G_2 are finite-gain L_2 stable with gains γ_1 and γ_2 then the interconnection is stable if $\gamma_1\gamma_2 < 1$.
- Both Passivity theory and the small gain theorem are special cases of larger frameworks including the conic systems theory and the passivity index theory discussed later.

Other Interconnections

- The parallel interconnection of two passive systems is still passive



- However, this isn't true for the series connection of two systems



- For example, the series connection of any two systems that have 90°of phase shift have a combined phase shift of 180°

Definitions for Continuous Time Systems

Passive	$u^T y \geq \dot{V}$
Lossless	$u^T y = \dot{V}$
Strictly Passive	$u^T y \geq \dot{V} + \psi(x)$
Strictly Output Passive	$u^T y \geq \dot{V} + \varepsilon y^T y$
Strictly Input Passive	$u^T y \geq \dot{V} + \delta u^T u$

- Note that $V(x)$ and $\Psi(x)$ are positive semidefinite and continuously differentiable. These equations hold for all times, inputs, and states.

Stability of Passive Systems

- Passive systems are Lyapunov stable
- Strictly passive systems ($\psi(x) \geq 0$) are asymptotically stable

$$u^T y \geq \dot{V}(x) + \psi(x)$$

- Output strictly passive systems ($\delta \geq 0$) are L_2 stable

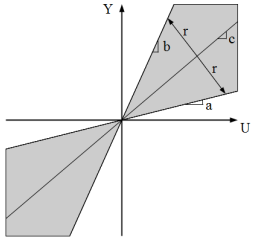
$$u^T y \geq \dot{V}(x) + \delta y^T y$$

- The following results hold in feedback
  - Two passive systems → passive and stable loop
  - Passive system and a strictly passive system → asymptotically stable loop
  - Two output strictly passive systems → L_2 stable loop
  - Two input strictly passive systems ($\varepsilon \geq 0$) → L_2 stable loop

$$u^T y \geq \dot{V}(x) + \varepsilon y^T y$$

Conic Systems

- A conic system is one whose input-output behavior is constrained to lie in a cone of the *U x Y* inner product space

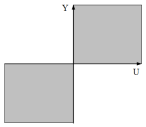


- A system is conic if the following dissipative inequality holds for all $t_2 \geq t_1$

$$\int_{t_1}^{t_2} \left[(1 + \frac{a}{b}) u^T y - a u^T u - \frac{1}{b} y^T y \right] dt \geq V(x(t_2)) - V(x(t_1))$$

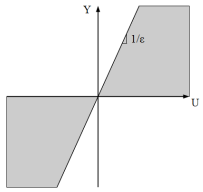
Passivity and Conic Systems

- Passive systems are a special case of conic systems where the cone is constrained to the first and third quadrants



Ex: The output strictly passive system *G(s)* is constrained to the following cone

$$G(s) = \frac{1}{s + \varepsilon}$$



Note: The upper bound on the cone indicates the level of stability

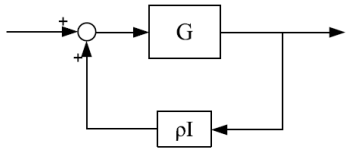
Passivity Indices

- Conic systems and the passivity indices capture the same information about a system

- A passivity index measures the level of passivity in a system
- Two indices are required to characterize the level of passivity in a system
  1. The first measures the level of stability of a system
  2. The second measures the extent of the minimum phase property in a system
- They are independent in the sense that knowing one provides no information about the other
- Each has a simple physical interpretation

Output Feedback Passivity Index

The output feedback passivity index (OFP) is the largest gain that can be put in positive feedback with a system such that the interconnected system is passive.

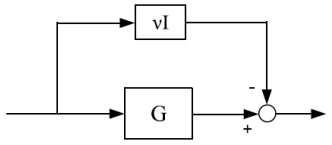


Equivalent to the following dissipative inequality holding for G

$$\int_{t_1}^{t_2} u^T y dt \geq V(x(t_2)) - V(x(t_1)) + \rho \int_{t_1}^{t_2} y^T y dt$$

Input Feed-Forward Passivity Index

The input feed-forward passivity index (IFP) is the largest gain that can be put in a negative parallel interconnection with a system such that the interconnected system is passive.

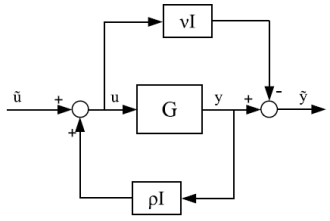


Equivalent to the following dissipative inequality holding for G

$$\int_{t_1}^{t_2} u^T y dt \geq V(x(t_2)) - V(x(t_1)) + \nu \int_{t_1}^{t_2} u^T u dt$$

Simultaneous Indices

When applying both indices they have the physical interpretation as in the block diagram

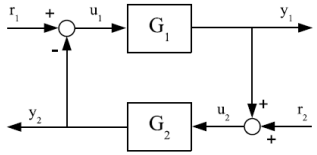


Equivalent to the following dissipative inequality holding for G

$$(1+\rho\nu) \int_{t_1}^{t_2} u^T y dt \geq V(x(t_2)) - V(x(t_1)) + \rho \int_{t_1}^{t_2} y^T y dt + \nu \int_{t_1}^{t_2} u^T u dt$$

Stability

We can assess the stability of an interconnection using the indices for G_1 and G_2



G_1 has indices ρ_1 and ν_1

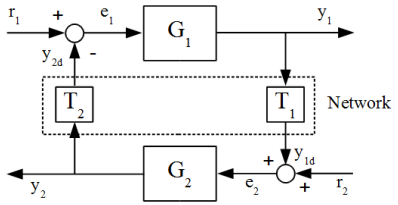
G_2 has indices ρ_2 and ν_2

The interconnection is L_2 stable if the following matrix is positive definite

$$\begin{bmatrix} (\rho_1+\nu_2)I & \frac{1}{2}(\rho_2\nu_2 - \rho_1\nu_1)I \\ \frac{1}{2}(\rho_2\nu_2 - \rho_1\nu_1)I & (\rho_2+\nu_1)I \end{bmatrix} > 0$$

Networked Systems

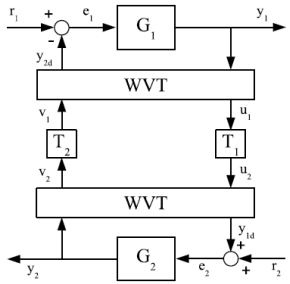
- Motivating Problem: The feedback interconnection of two passive systems is passive and stable. However, when the two are interconnected over a delayed network the result is not passive so stability is no longer guaranteed. How do we recover stability?



The systems G_1 and G_2 are interconnected over a network with time delays T_1 and T_2

Stability of Networked Passive Systems

- One solution to interconnecting passive systems over a delayed network is to add an interface between the systems and the network
- The wave variable transformation forces the interconnection to meet the small gain theorem. Stability is guaranteed for arbitrarily large time delays
- The WVT is defined below



$$\begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \frac{1}{\sqrt{2b}} \begin{bmatrix} bI & I \\ bI & -I \end{bmatrix} \begin{bmatrix} y_1 \\ y_{2d} \end{bmatrix} \qquad \begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \frac{1}{\sqrt{2b}} \begin{bmatrix} bI & I \\ bI & -I \end{bmatrix} \begin{bmatrix} y_{1d} \\ y_2 \end{bmatrix}$$

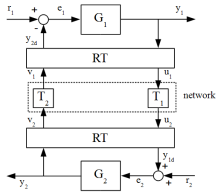
Stability of Networked Conic Systems

- This approach can be generalized to conic systems with a new transformation
- The rotational transformation (RT) can turn any conic system into an L_2 stable system with an appropriate rotation θ

$$\begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} \cos(\theta)I & \sin(\theta)I \\ -\sin(\theta)I & \cos(\theta)I \end{bmatrix} \begin{bmatrix} y_{2d} \\ y_1 \end{bmatrix}$$

$$\begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} \cos(\theta)I & -\sin(\theta)I \\ \sin(\theta)I & \cos(\theta)I \end{bmatrix} \begin{bmatrix} y_2 \\ y_{1d} \end{bmatrix}$$

- This approach decouples the control design from the network design
- First, G_2 can be designed using traditional conic systems theory
- The upper RT pre-stabilizes G_1 (in the L_2 sense) to tolerate network delays
- The lower RT inverts the rotation to preserve the conic sector of G_1



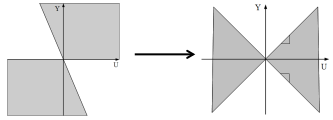
Example

- Consider the unstable conic system G_1 (has indices ρ=-2, v=0)
- A conic controller G_2 can be designed that directly stabilizes G_1 (stabilizes when K_i>0 and K_p>2)

$$G_1: \quad \dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad y = \begin{bmatrix} 1 & 1 \end{bmatrix} x$$

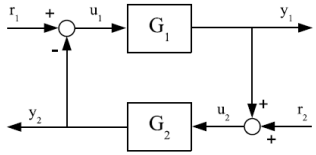
$$G_2: \quad \dot{x} = .01x + u \qquad y = K_i x + K_p$$

- Now when the two are interconnected over a network, the transformation needs to be designed to compensate for delay
- It can be analytically verified that the rotation stabilizes for θ in the range (63.6°, 90°)
- The rotation makes the unstable G_1 appear to the network to be an L_2 stable system



Dissipativity for Switched Systems

- Dissipativity and passivity are strong tools for analysis and synthesis of interconnected systems—For example, the feedback of two passive systems is passive and stable



- Applying the traditional definitions of dissipativity and passivity (single storage function) to switched systems gives rather conservative results
- We can relax this restriction by using multiple storage functions and multiple supply rates

Dissipativity for Switched Systems

- Recently dissipativity has been defined for switched systems using multiple storage functions. Consider a nonlinear switched system with switching signal σ

$$\dot{x} = f_\sigma(x,u)$$
$$y = h_\sigma(x,u)$$

A *switched system* is *dissipative* if it meets the following conditions

1. Each subsystem *i* is dissipative w.r.t. ω_i^i when active:
$$\int_{t_1}^{t_2} \omega_i^i(u,y)dt \geq V_i(x(t_2)) - V_i(x(t_1))$$
2. Each subsystem *i* is dissipative w.r.t. ω_j^i when inactive:
$$\int_{t_1}^{t_2} \omega_j^i(u,y,x,t)dt \geq V_j(x(t_2)) - V_j(x(t_1))$$
3. There exists an input *u* so that the dissipative supply rate ω_j^i is integrable on the infinite time interval.

Passive Switched Systems

- A *switched system* is *passive* when

1. Each subsystem *i* is passive when on:
$$u^T y \geq \frac{dV_i}{dt}$$
2. Each subsystem *i* is dissipative when off:
$$\omega_i(u,y) \geq \frac{dV_i}{dt}$$

Passivity for Switched Systems

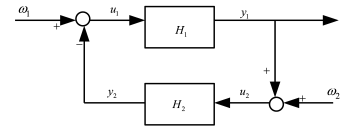
- The definition of passivity is a special case of dissipativity for switched systems where the first condition is replaced by:

1. Each subsystem *i* is passive when active:
$$\int_{t_1}^{t_2} u^T(t)y(t)dt \geq V_i(x(t_2)) - V_i(x(t_1))$$

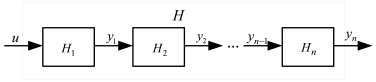
- The notion of passivity for switched systems implies Lyapunov stability when the input *u(t)* is zero and each V_i is positive definite
- It can be shown that passivity is preserved in feedback and in parallel as expected from traditional passivity theory

A Passivity Measure Of Systems In Cascade Based On Passivity Indices

- **Problem Statement**

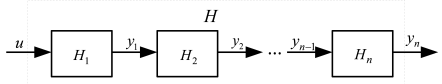


• The negative feedback interconnection of two passive systems is still passive.



• Will the cascade interconnection of several passive systems still be passive?

A Passivity Measure Of Systems In Cascade Based On Passivity Indices: main results (1)



Each H_i is OFP(ρ_i) such that $\dot{V}_i \leq -\rho_i y_i^T y_i + u_i^T y_i$ with $\rho_i > 0$, if for some $\nu > 0$, such that

Secant Criterion

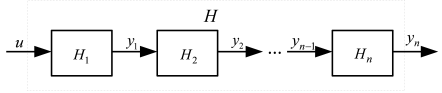
$$\nu > \frac{cos(\frac{\pi}{n+1})^{n+1}}{\rho_1 \rho_2 \cdots \rho_n}, \; n \geq 2$$

then cascade system admits a storage function of the form

$$V = \sum_{i=1}^{n} d_i V_i, \; d_i > 0$$

and the cascade interconnection is IFP(-ν).

A Passivity Measure Of Systems In Cascade Based On Passivity Indices: main results (2)



Each H_i is IFP(ν_i) such that $\dot{V}_i \leq -\nu_i u_i^T u_i + u_i^T y_i$ with $\nu_i > 0$, if for some $\rho > 0$, such that

Secant Criterion

$$\rho > \frac{cos(\frac{\pi}{n+1})^{n+1}}{\nu_1 \nu_2 \cdots \nu_n}, \; n \geq 2$$

then cascade system admits a storage function of the form

$$V = \sum_{i=1}^{n} d_i V_i, \; d_i > 0$$

and the cascade interconnection is OFP(-ρ).

A Passivity Measure Of Systems In Cascade Based On Passivity Indices: main results (3)

Each H_i is IF-OFP(ν_i,ρ_i) such that $\dot{V}_i \leq u_i^T y_i - \rho_i y_i^T y_i - \nu_i u_i^T u_i$ with $\nu_i, \rho_i \in \mathbb{R}$. If for some $\hat{\rho}, \hat{\nu} \in \mathbb{R}$ and

$$A = \begin{bmatrix} -\nu_1 + \hat{\nu} & \frac{1}{2} & 0 & \cdots & -\frac{1}{2} \\ \frac{1}{2} & -\nu_2 - \rho_1 & \frac{1}{2} & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{2} & -\nu_n - \rho_{n-1} & \frac{1}{2} \\ -\frac{1}{2} & 0 & \cdots & \frac{1}{2} & -\rho_n + \hat{\rho} \end{bmatrix},$$

such that $-A$ is quasi-dominant,
then cascade system admits a storage function of the form

$$V = \sum_{i=1}^{n} d_i V_i, \; d_i > 0, \; n \geq 2$$

and the cascade interconnection is IF-OFP($\hat{\nu}$,$\hat{\rho}$).

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A Passivity Measure Of Systems In Cascade Based On Passivity Indices: Example

Consider a cascade of three dissipative systems H_1, H_2 and H_3 given by

$$H_1: \begin{cases} \dot{x}_1(t) = \frac{1}{20}x_1(t) + u_1(t) \\ y_1(t) = x_1(t) + 5u_1(t)\ , \end{cases} \qquad H_2: \begin{cases} \dot{x}_2(t) = -\frac{1}{6}x_2(t) + u_2(t) \\ y_2(t) = x_2(t) + 6u_2(t) \end{cases}$$

$$H_3: \begin{cases} \dot{x}_{31}(t) = x_{32}(t) \\ \dot{x}_{32}(t) = -0.5x_{31}^3(t) + 0.5x_{32}(t) + 2u_3(t) \\ y_3(t) = x_{32}(t) + u_3(t) \end{cases}$$

One an verify that

H_1: with $V_1 = x_1^2$, $\nu_1 = 7.5$, $\rho_1 = -0.1$; H_2: with $V_2 = \frac{1}{6}x_2^2$, $\nu_2 = 4$, $\rho_2 = \frac{1}{18}$;

H_3: with $V_3 = \frac{1}{8}x_{31}^4 + \frac{1}{2}x_{32}^2$, $\nu_3 = 1.5$, $\rho_3 = -0.5$

In this case

$$A = \begin{bmatrix} -7.5+\hat{\nu} & 0.5 & 0 & -0.5 \\ 0.5 & -3.9 & 0.5 & 0 \\ 0 & 0.5 & -1.556 & 0.5 \\ -0.5 & 0 & 0.5 & 0.5+\hat{\rho} \end{bmatrix}$$

If we choose $V = \sum_{i=1}^{3} V_i$ then with $\hat{\nu} = 0$, $\hat{\rho} = -1.5$ $-A$ is quasi-dominant. The cascade system is OFP(-1.5).

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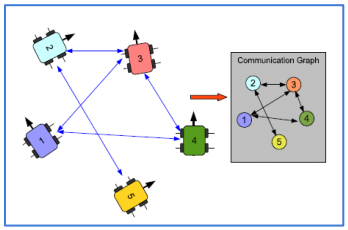
Passivity-Based Output Synchronization With Application To Output Synchronization of Networked Euler-Lagrange Systems Subject to Nonholonomic Constraints



- Problem Statement
- We have a group of mobile agents and agent's motion is subject to nonholonomic constraints;
- Agent communicates each other through networks;
- How to achieve output synchronization among them (velocity, heading direction, etc.)

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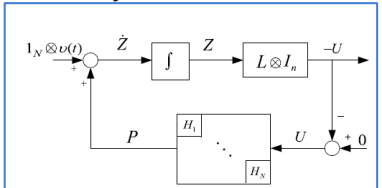
Passivity-Based Output Synchronization With Application To Output Synchronization of Networked Euler-Lagrange Systems Subject to Nonholonomic Constraints: Main Results



By input-output linearized an Euler-Langrange systems subject to nonholonomic constraints and some variables transformation

$$\frac{d}{dt}\frac{\partial L(q,\dot{q})}{\partial \dot{q}} - \frac{\partial L(q,\dot{q})}{\partial q} = A^T(q)\lambda + Q$$

$A(q)\dot{q} = 0$,

we can get

$$H_i: \begin{cases} \dot{\xi}_i(t) = -\alpha_i\xi_i(t) + u_i(t) \\ p_i(t) = \xi_i(t) \end{cases}$$

- Assume that the underlying communication graph is balanced and strongly connected;
- Output synchronization is based on the passivity analysis of the closed-loop system;

- From P to -U is passive;
- From U to P is passive;

⇩

- The closed-loop is passive.
- Use scattering transformation to deal with communication delay.

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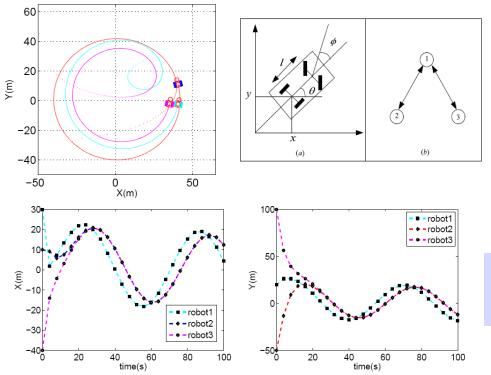
Passivity-Based Output Synchronization With Application To Output Synchronization of Networked Euler-Lagrange Systems Subject to Nonholonomic Constraints: Example

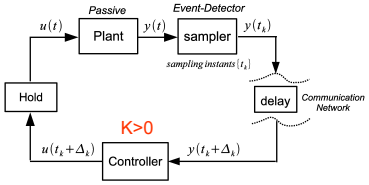


Wheeled mobile vehicle can be modeled as EL system subject to nonholonomic constraints.

Synchronization of three wheeled mobile robots.

Event-Triggered Real-Time Scheduling For Stabilization Of Passive/Output Feedback Passive Systems

• Problem Statement



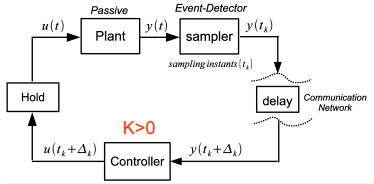
A sequence of measurements
$y(t_0),\ y(t_1),\ \ldots, y(t_k)$
corresponds to a sequence of actuation updates
$u(t_0+\Delta_0),\ u(t_1+\Delta_1),\ \ldots, u(t_k+\Delta_k)$.

• We assume the plant is *passive*, which indicates a positive gain K>0 could stabilize it if it is also ZSO.

• When should we sample the output of the plant?

• How often should we sample the output of the plant?

Event-Triggered Real-Time Scheduling For Stabilization Of Passive/Output Feedback Passive Systems: main results(1)



since $u(t) = -Ky(t_k)$ for $t \in [t_k, t_{k+1})$
define $e(t) = y(t) - y(t_k)$

$$\begin{aligned}\dot{V}(x) &\le u(t)^T y(t) = -Ky(t_k)^T y(t) \\ &= -K(y(t) - e(t))^T y(t) \\ &= -Ky(t)^T y(t) + Ke(t)^T y(t) \\ &\le K\|e(t)\|_2 \|y(t)\|_2 - K\|y(t)\|_2^2 \\ t &\in [t_i, t_{i+1}),\ \forall i.\end{aligned}$$

For no delay case

• When should we sample the output of the plant?

Triggering Condition

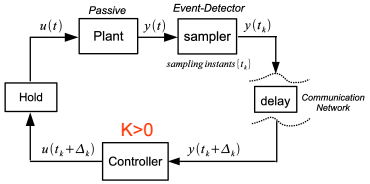
whenever $\|e(t)\|_2 > \|y(t)\|_2$

• How often should we sample the output of the plant?

Inter-sampling Time

$t_{k+1} - t_k \ge \tau = \frac{1}{\gamma L \zeta} \ln\left(\frac{2+2\zeta}{2+\zeta}\right)$
where $\gamma, L, \zeta > 0$
are parameters related to plant dynamics

Event-Triggered Real-Time Scheduling For Stabilization Of Passive/Output Feedback Passive Systems: main results(2)



For non-trivial delay case

• When should we sample the output of the plant?

Triggering Condition

whenever $\|\tilde{e}(t)\|_2 = \hat{\sigma}\|y(t_k)\|_2,\ \hat{\sigma} \in (0, 0.5)$

Inter-sampling Time

there exist $\varepsilon_i > 0$ and $\eta > 0$, such that for $\Delta_i \in [0, \varepsilon_i]$ is lower bounded by $\eta + \Delta_i$, where
with $0.5 > \hat{\sigma} > \tilde{\sigma} > 0$

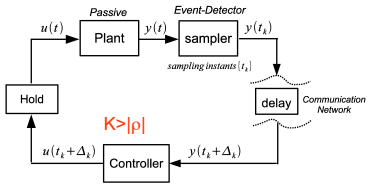
$\eta = \frac{1}{\gamma L(2+\zeta)} \ln\left(\frac{\frac{1+\zeta}{2+\zeta}+\hat{\sigma}}{\frac{1+\zeta}{2+\zeta}+\tilde{\sigma}}\right)$

$\varepsilon_i^- \ge \frac{1}{(2+\zeta)\gamma L} \ln(\tilde{\sigma}+1)$

$\varepsilon_i^+ = \frac{1}{\gamma L(2+\zeta)} \ln\left(\frac{\frac{1+\zeta}{2+\zeta}+0.5}{\frac{1+\zeta}{2+\zeta}+\hat{\sigma}}\right)$

$\varepsilon_i = \min\{\varepsilon_i^-, \varepsilon_i^+\}$

Event-Triggered Real-Time Scheduling For Stabilization Of Passive/Output Feedback Passive Systems: main results(3)



When the plant is OFP(-|ρ|)

$\dot{V}(x) \le u^T y - \rho y^T y, \quad \rho < 0$

example: $G(s) = \frac{1}{s-5}$,
in this case: $\rho = -5$

For the no delay case, the triggering condition becomes $\|e(t)\|_2 > \|K^{-1}(K - |\rho|I)\|_2 \|y(t)\|_2,\ \forall t \ge 0$ and the triggering condition for the non-trivial delay case can be derived in a similar way to the case when the plant is passive.

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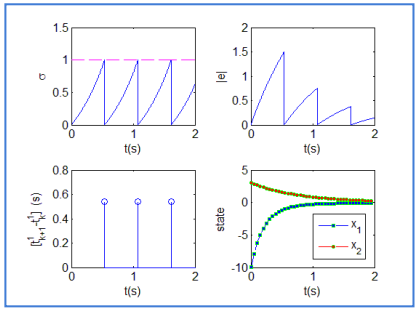
Event-Triggered Real-Time Scheduling For Stabilization Of Passive/Output Feedback Passive Systems: Example

Consider the linear passive system given by

$$\dot{x}_1(t) = -5x_1(t) - x_2(t)$$
$$\dot{x}_2(t) = -x_2(t) + u(t)$$
$$y(t) = x_2(t),$$

An estimate of the lower bound of the inter-event time $\tau \geq 0.5390s$, which is verifed through simulation.



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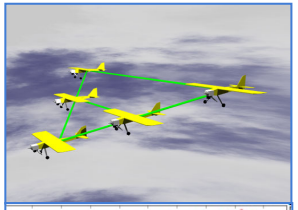
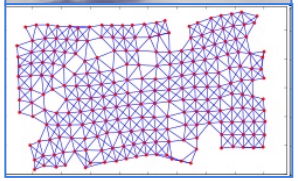
Event-Triggered Cooperative Control For Multi-Agent Systems Based On Passivity Analysis

• Problem Statement

The system considered consists of N passive agents: $\dot{V}_i(x_i) \leq u_i^T(t)y_i(t)$;

the interconnection between agents is described by a graph:
$u_i(t) = -\sum_{j\in\mathcal{N}_i}\left(y_i(t_k^i) - y_j(t_{k'}^j)\right)$
for $t \in [t_k^i, t_{k+1}^i]$;





• The goal is to achieve output synchronization among all the interconnected agents.

• When should an agent broadcast its output information to its neighbors?

• How often does this happen?

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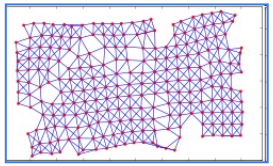
Event-Triggered Cooperative Control For Multi-Agent Systems Based On Passivity Analysis: Main Results(1)

• When should an agent broadcast its output information to its neighbors?



Note that for a strongly connected graph, a(G)>0

Assume that the underlying communication graph is balanced and strongly connected, the transmission delay is trivial, then a broadcast is triggered whenever:
$\|e_i(t)\|_2 > \frac{a(G)}{2|\mathcal{N}_i|}\|y_i(t)\|_2,\ \forall i,\ \forall t \geq 0.$
where $a(G)$ is the algebraic connectivity of the underlying communication graph; $|\mathcal{N}_i|$ is number of neighbors of agent i;
$e_i(t) = y_i(t) - y_i(t_k^i)$, for $t \in [t_k^i, t_{k+1}^i]$, $\forall k$;

• How often does this happen?

$t_{k+1}^i - t_k^i > \tau_i = \frac{1}{\gamma_i L_i \zeta_i}\ln\left(1 + \frac{1}{\Lambda}\frac{\sigma_m}{1+\sigma_m}\|y_i(t_k^i)\|_2\right)$, where $\sigma_m = \frac{a(G)}{2|\mathcal{N}_i|}$

$\Lambda = \frac{\gamma_i L_i \zeta_i \|y_i(t_k^i)\|_2 + \gamma_i L_i \|\sum_{j\in\mathcal{N}_i}[y_i(t_k^i) - y_j(t_{k'}^j)]\|_2}{\gamma_i L_i \zeta_i}$

γ_i, L_i, ζ_i are parameters related to system's dynamics.

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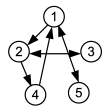
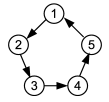
Event-Triggered Cooperative Control For Multi-Agent Systems Based On Passivity Analysis: Main Results(2)

• Since the triggering condition is given by:

$\|e_i(t)\|_2 > \frac{a(G)}{2|\mathcal{N}_i|}\|y_i(t)\|_2,\ \forall i,\ \forall t \geq 0,$
so an intuitive way to reduce the communication frequency among agents is by increasing the ratio $\frac{a(G)}{2|\mathcal{N}_i|}$: improving the connectivity of the communication graph while reducing the number of neighbors.

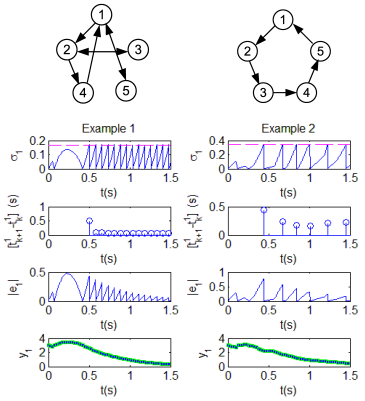


The algebraic connectivity for the graph is $a(G) = 0.6753$.
The corresponding triggering condition for each agent is given by
$\|e_i(t)\|_2 > 0.1688\|y_i(t)\|_2, \forall t \geq 0,\ i = 1, 2$
$\|e_i(t)\|_2 > 0.3376\|y_i(t)\|_2, \forall t \geq 0,\ i = 3, 4, 5.$



The algebraic connectivity for the graph is $a(G) = 0.6910$.
The corresponding triggering condition for each agent is given by
$\|e_i(t)\|_2 > 0.3455\|y_i(t)\|_2, \forall t \geq 0,\ i = 1, 2, \ldots, 5.$

Event-Triggered Cooperative Control For Multi-Agent Systems Based On Passivity Analysis: Example



Symmetry & Passivity

-Symmetry: A basic feature of shapes and graphs

-Symmetry in dynamical systems: Identical dynamics of subsystems; Same characterizations of information structure

-Approximate Symmetry

-Why Symmetry?

Decompose into lower dimensional systems

Construct symmetric large-scale systems

Symmetry & Passivity

-Linear systems $\dot{x}(t) = Ax(t) + Bu(t)$

$AT = TA, TB = BS$ $\quad e.g. T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

-Nonlinear systems $\dot{x}_i = f_i(x_i) + g_i(x_i)u_i \quad y_i = h_i(x_i)$

-Vector field equivalence

Two vector fields, g_1 and g_2 are equivalent, denoted by $g_1 \sim g_2$ if there exists a diffeomorphism $\psi : M \mapsto M$ such that

$$\psi_* \circ g_1 = g_2$$

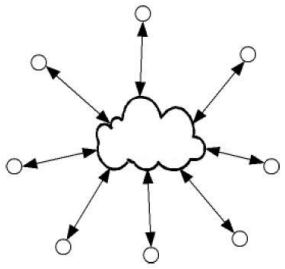
where

$$\psi_* g = T\psi \circ g \circ \psi^{-1}$$

-A general idea: system dynamics are invariant under transformations of coordinates.

Simple Examples

Star-shaped Symmetry



$$u = u_e - \tilde{H}y$$

$$\tilde{H} = \begin{bmatrix} H & b & \cdots & b \\ c & h & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c & 0 & \cdots & h \end{bmatrix}$$

$$u_0 = u_{e0} - Hy_0 - by_1 - \cdots - by_m$$

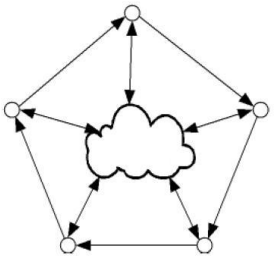
$$u_1 = u_{e1} - cy_0 - hy_1$$

$$\vdots$$

$$u_m = u_{em} - cy_0 - hy_m$$

Simple Examples

Cyclic Symmetry



$$u = u_e - \tilde{H}y$$

$$\tilde{H} = \begin{bmatrix} H & b & \cdots & b \\ c & & & \\ \vdots & & \tilde{h} & \\ c & & & \end{bmatrix}$$

$$\tilde{h} = circ([v_1, v_2, \ldots, v_m])$$

$$u_0 = u_{e0} - Hy_0 - by_1 - \cdots - by_m$$

$$u_1 = u_{e1} - cy_0 - v_1 y_1 - v_2 y_2 - \cdots - v_m y_m$$

$$\vdots$$

$$u_m = u_{em} - cy_0 - v_2 y_1 - v_3 y_2 - \cdots - v_1 y_m$$

Problem

-A system is (Q, S, R) – dissipative if there exists a positive semi-definite storage function V(x) and a specific supply rate $\omega(u, y)$ such that the following inequality holds

where $V(x(0)) + \int_0^T \omega(u(t), y(t))dt \geq V(x(T))$

$$\omega(u, y) = y^T Qy + 2y^T Su + u^T Ru$$

-Nonlinear symmetric distributed systems with dissipativity

$$\dot{x}_i = f_i(x_i) + g_i(x_i)u_i \qquad y_i = h_i(x_i)$$

Feedback interconnections

$$u_i = u_{ei} - \sum_{j=1}^{N} H_{ij} y_j \qquad H = [H_{ij}] \quad \Rightarrow \quad u = u_e - Hy$$

Main Result (1)

Theorem (Star-shaped Symmetry)

Consider a (Q, S, R) – dissipative system extended by *m* star-shaped symmetric (q, s, r) – dissipative subsystems. The whole system is finite gain input-out stable if

$$m < \min\left(\frac{\underline{\sigma}(\hat{Q})}{\bar{\sigma}(c^T rc + \beta(\hat{q} - b^T Rb)^{-1}\beta^T)}, \frac{\hat{q}}{b^T Rb}\right)$$

where

$$\hat{Q} = -H^T RH + SH + H^T S^T - Q > 0$$

$$\hat{q} = -h^T rh + sh + h^T s^T - q > 0$$

$$\beta = Sb + c^T s^T - H^T Rb - c^T rh$$

$$\tilde{H} = \begin{bmatrix} H & b & \cdots & b \\ c & h & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c & 0 & \cdots & h \end{bmatrix}$$

Main Result (2)

Theorem (Cyclic Symmetry)

Consider a (Q, S, R) – dissipative system extended by *m* cyclic symmetric (q, s, r) – dissipative subsystems. The whole system is finite gain input-out stable if

$$m < \min\left(\frac{\underline{\sigma}(\hat{Q})}{\bar{\sigma}(c^T rc + \beta_m \Lambda^{-1} \beta_m{}^T)}, \frac{-r\sigma(\tilde{h})\overline{\sigma(\tilde{h})} + s(\sigma(\tilde{h}) + \overline{\sigma(\tilde{h})}) - q}{b^T Rb}\right)$$

where

$$\tilde{h} = circ([v_1, v_2, \ldots, v_m])$$

$$\sigma(\tilde{h}) = \sum_{j=1}^{m} v_j \lambda_i^j = \sum_{j=1}^{m} v_j e^{\frac{2\pi ij}{m}}$$

$$\tilde{H} = \begin{bmatrix} H & b & \cdots & b \\ c & & & \\ \vdots & & \tilde{h} & \\ c & & & \end{bmatrix}$$

Main Result (3)

(cont.)

$$\Lambda = -r\tilde{h}^T\tilde{h} + s(\tilde{h}^T + \tilde{h}) - q \otimes I_m - b^T Rb \otimes circ([1,1,\ldots,1])$$

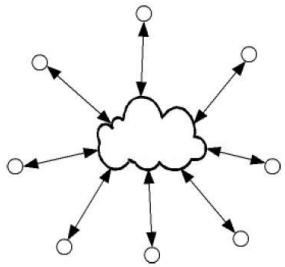
$$\beta = Sb + c^T s^T - H^T Rb - c^T r\tilde{h}$$

$$\beta_m = [\underbrace{\beta\beta\ldots\beta}_{m}]$$

the spectral characterization of $\tilde{h}$ should satisfy

$$\| \sigma(\tilde{h}) - \frac{s}{r} \| < \sqrt{\frac{s^2}{r^2} - \frac{q + mb^T Rb}{r}}$$

Simple Examples



$$u = u_e - \tilde{H}y$$

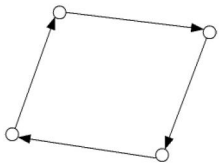
$$\tilde{H} = \underbrace{\begin{bmatrix} 0.9 & -0.8 & -0.8 & \cdots & -0.8 \\ -0.8 & 0.1 & 0 & \cdots & 0 \\ -0.8 & 0 & 0.1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -0.8 & 0 & 0 & \cdots & 0.1 \end{bmatrix}}_{m+1}$$

$$Q = q = -I, S = s = 0, R = r = \tfrac{1}{4} I$$

$$\Rightarrow \quad m < \min(3.11, 6.25) = 3.11$$

Remark: $(-I, 0, \alpha^2 I)$ – dissipative systems corresponding to systems with gain less or equal to α (here $\alpha = \frac{1}{2}$)

Simple Examples



$$u = u_e - \tilde{H}y \qquad q = 0, s = \tfrac{1}{2}, r = 1$$

$$\tilde{H} = \tilde{h} = \underbrace{\begin{bmatrix} 0.1 & 0.2 & 0 & \cdots & 0 \\ 0 & 0.1 & 0.2 & \cdots & 0 \\ 0 & 0 & 0.1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0.2 & 0 & 0 & \cdots & 0.1 \end{bmatrix}}_{m}$$

The cyclic symmetric system is stable if

$$|\sigma(\tilde{h}) - \frac{s}{r}\| = \| \sum_{j=0}^{m-1} v_j e^{\frac{2\pi i j}{m}} \| \le 0.3 < 0.5 = \sqrt{\frac{s^2}{r^2} - \frac{q}{r}}$$

The above stability condition is always satisfied. Also

$$m < \min(+\infty, +\infty)$$

Thus the system can be extended with infinite numbers of subsystems without losing stability.

Main Result (4)

Theorem (Star-shaped Symmetry for Passive Systems)
Consider a passive system extended by *m* star-shaped symmetric passive subsystems. The whole system is finite gain input-output stable if

$$m < \frac{\underline{\sigma}(\hat{Q})}{\sigma(\beta \hat{Q}^{-1} \beta^T)}$$

where

$$\hat{Q} = \frac{H + H^T}{2} > 0$$

$$\hat{q} = \frac{h + h^T}{2} > 0$$

$$\beta = \frac{b + c^T}{2}$$

$$\tilde{H} = \begin{bmatrix} H & b & \cdots & b \\ c & h & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c & 0 & \cdots & h \end{bmatrix}$$

Main Result (5)

Theorem (Cyclic Symmetry for Passive Systems)
Consider a passive system extended by *m* cyclic symmetric passive subsystems. The whole system is finite gain input-output stable if

where $m < \frac{\sigma(\hat{Q})}{\sigma(\beta_m \Lambda^{-1} \beta_m{}^T)}$

$$\hat{Q} = \frac{H + H^T}{2} > 0 \qquad \Lambda = \frac{\tilde{h} + \tilde{h}^T}{2} \qquad \tilde{H} = \begin{bmatrix} H & b & \cdots & b \\ c & & & \\ \vdots & & \tilde{h} & \\ c & & & \end{bmatrix}$$

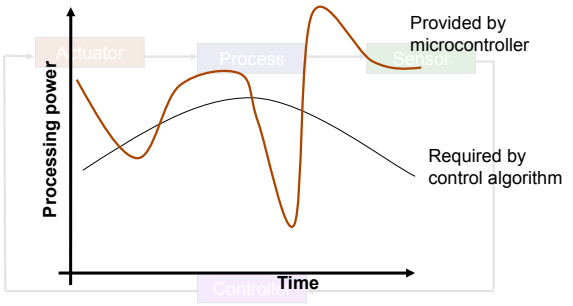
$$\beta = \frac{b + c^T}{2} \qquad \beta_m = \underbrace{[\beta\beta \ldots \beta]}_{m}$$

$$\sigma(\tilde{h}) = \sum_{j=1}^{m} v_j \lambda_i^j = \sum_{j=1}^{m} v_j e^{\frac{2\pi i j}{m}} \qquad \tilde{h} = circ([v_1, v_2, \ldots, v_m])$$

Some Recent Work

- M.J. McCourt, P.J. Antsaklis, "Control Design for Switched Systems Using Passivity Indices," *2010 American Control Conference*, pp. 2499-2504, Baltimore, MD, 2010.
- Han Yu, Panos J. Antsaklis, "Passivity-Based Output Synchronization of Networked Euler-Lagrange Systems Subject to Nonholonomic Constraints," *2010 American Control Conference*, pp. 208-213, Baltimore, MD, 2010.
- Han Yu, Panos J. Antsaklis, "Passivity and L2 Stability of Networked Dissipative Systems," *8th IEEE Intern. Conference on Control and Automation,* pp. 584-589, *Xiamen, China,* June 9-11, 2010.
- M.J. McCourt and P.J. Antsaklis, Connection Between the Passivity Index and Conic Systems, ISIS-2009-009, Dec 2009. (http://www.nd.edu/~isis/tech.html)
- Po Wu and P.J. Antsaklis, "Symmetry in the Design of Large-Scale Complex Control Systems: Some Initial Results using Dissipativity and Lyapunov Stability," *18th Med Conf on Control and Automation (MED'10)*, pp. 197-202, Marrakech, Morocco, 2010.
- H. Yu and P.J. Antsaklis, "Passivity Measure for Cascade Systems Based On The Analysis Of Passivity Indices," *CDC'10*, Atlanta, Georgia USA, 2010.
- M.J. McCourt and P.J. Antsaklis, "Stability of Networked Passive Switched Systems," *CDC'10*, Atlanta, Georgia USA, 2010.
- Han Yu, Feng Zhu, Panos J. Antsaklis, Event-Triggered Cooperative Control For Multi-Agent Systems Based On Passivity Analysis. Submitted to 2011 ACC.

Need to go beyond time triggered control



-In CPS, same microprocessor may execute many tasks; thus the processing power available to control algorithm may be time-varying, uncertain, and limited
- Two solutions: Event triggered control, and anytime algorithms

Anytime algorithms: directions

- Basic idea: Anytime algorithm based on using coarse model of the process to generate a control input; then progressively using more complicated models to refine the input

-Stability and performance analysis for stochastic models of execution time availability

-Algorithms for linear, constrained, and nonlinear plants

- This project: impact on passivity of such algorithms, interactions with processor scheduling, distributed control using anytime algorithms

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Fundamentals of Composition in Heterogeneous Systems Research Plans

- Stability, Safety, Performance
- Passivity indices; Discrete-time systems; Switched/Hybrid Systems

- Passivity based computational tools
  Computational methods; Optimization
- Control methods: event-triggered; adaptive; learning. Higher degrees of autonomy.

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