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Formal Synthesis for Traffic Control

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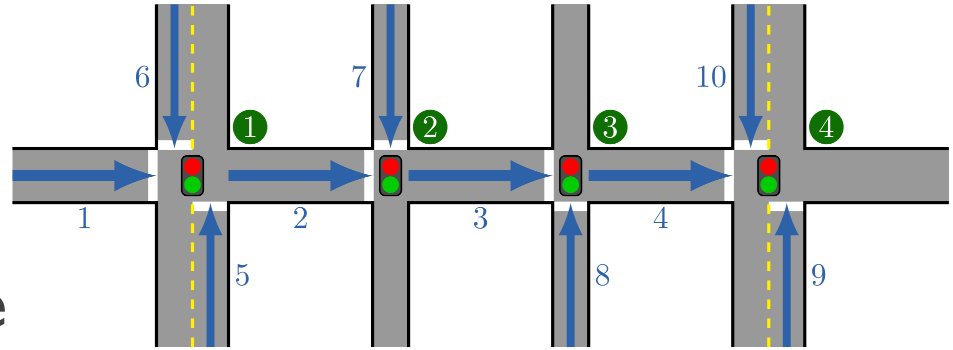
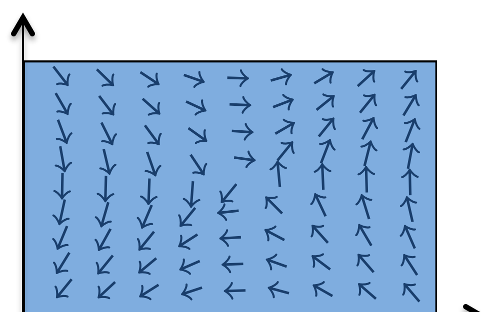
Objective: Traffic control to meet objectives expressed in temporal logic, e.g.,

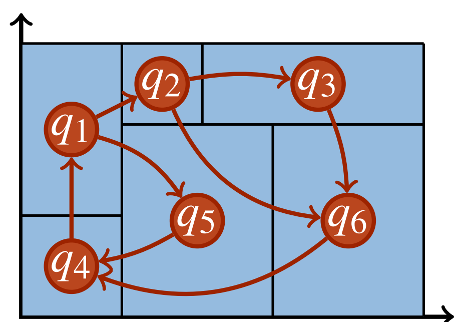
- eventually each link will have ≤30 vehicles
- upstream link will have low demand until downstream link is no longer congested
- each queue at a junction will be actuated at least once every two minutes.



Formal Synthesis Workflow



$$x^+ = F(x)$$



finite abstraction

formal methods

Outline:

1. Finite abstraction for formal methods

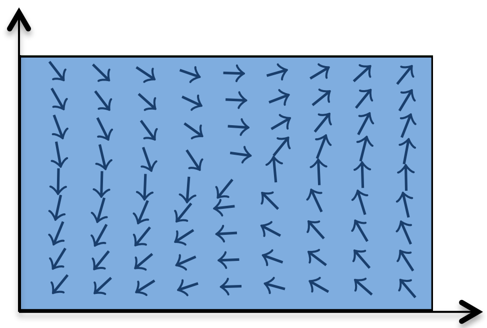
   Exploiting a "mixed monotonicity" property for scalability.

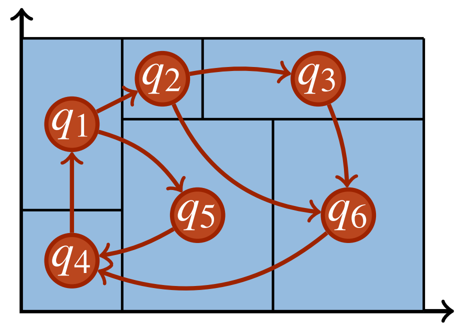
   Application to a macroscopic traffic flow model.

2. Compositional synthesis for large networks

   Decoupled synthesis for subnetworks with supply and demand contracts.

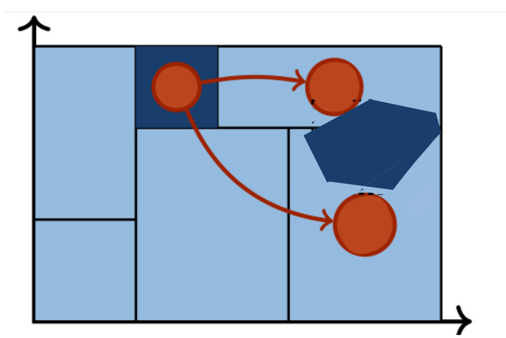
1. Finite Abstraction for Formal Methods





Capture the underlying dynamics with a finite set of symbols and transitions between them. Methods exist for classes of systems (Tabuada, Girard, Pappas, Reissig, Abate, Belta, and others…)

Example: polyhedral computations for piecewise affine systems (Belta et al.)



Monotonicity and Mixed Monotonicity

The discrete-time system:

$$x^+ = F(x) \quad x \in \mathcal{X}$$

is monotone if

$$x_1 \leq x_2 \quad \implies \quad F(x_1) \leq F(x_2)$$

with respect to a partial order (standard order in this talk).

Monotonicity offers strong dynamical properties [Hirsch, Smith, Angeli, Sontag] but is restrictive in practice.

Necessary and sufficient condition for monotonicity:

$$\frac{\partial F_i(x)}{\partial x_j} \geq 0 \quad \forall x \in \mathcal{X} \quad \forall i, j$$

Monotonicity and Mixed Monotonicity

$$x^+ = F(x) \quad x \in \mathcal{X}$$

is mixed monotone if there exists a "decomposition function"

$$f : \mathcal{X} \times \mathcal{X} \to \mathcal{X}$$

such that

$$f(x, x) = F(x)$$
$$x_1 \leq x_2 \;\Rightarrow\; f(x_1, y) \leq f(x_2, y)$$
$$y_1 \leq y_2 \;\Rightarrow\; f(x, y_2) \leq f(x, y_1).$$

A sufficient condition for mixed monotonicity:

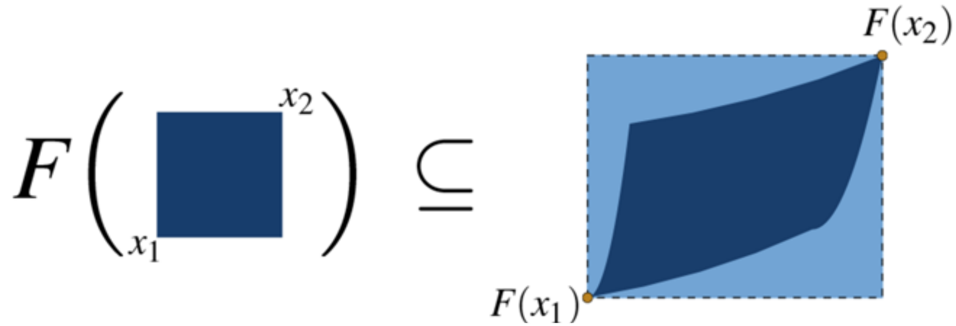
$$\exists \delta_{ij} \in \{-1, 1\} \quad \text{s.t.} \quad \delta_{ij} \frac{\partial F_i(x)}{\partial x_j} \geq 0 \quad \forall i, j$$

Decomposition function: $F_i(\cdots, \cancel{x_j}^{\,y_j}, \cdots)$ if $\delta_{ij} = -1$

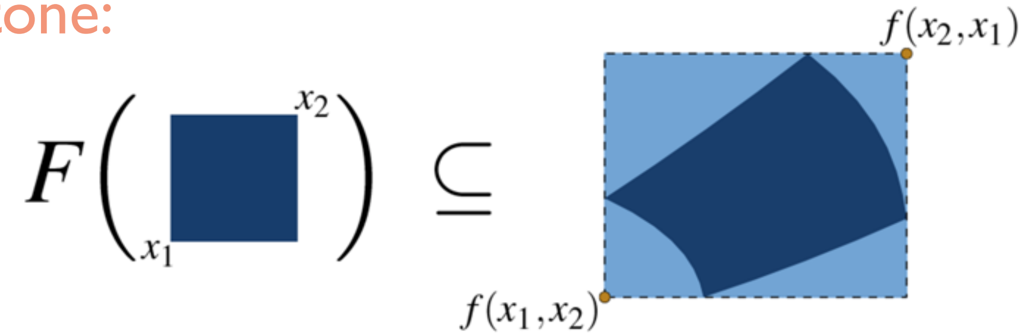
Mixed Monotonicity Allows Scalable Finite Abstraction

Two function evaluations tightly bound the one-step reach set:

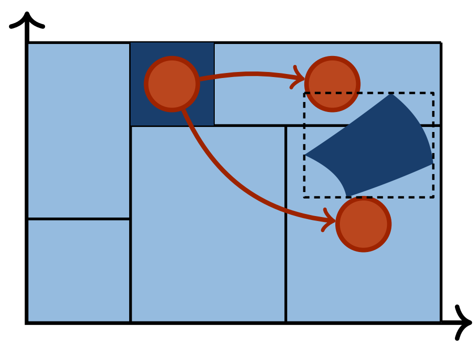
Monotone:

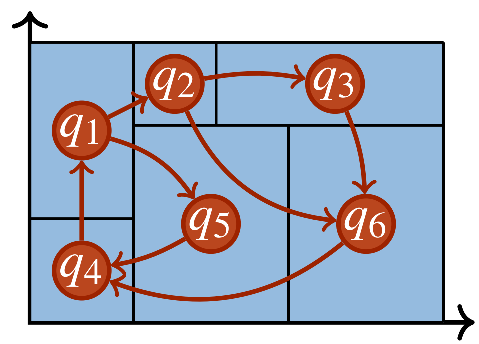


Mixed Monotone:

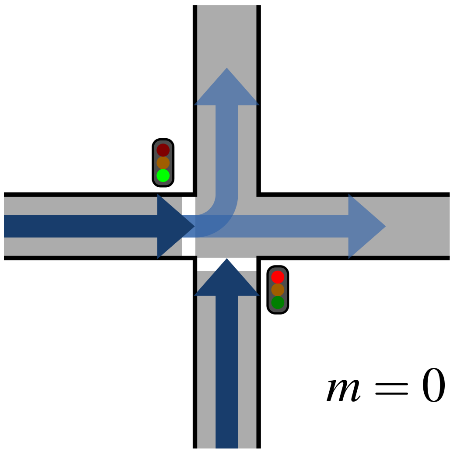


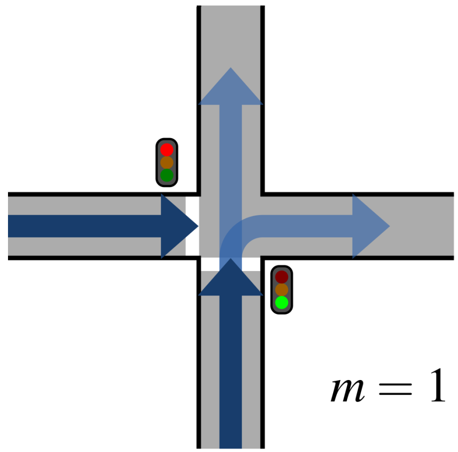
This allows a scalable abstraction algorithm:

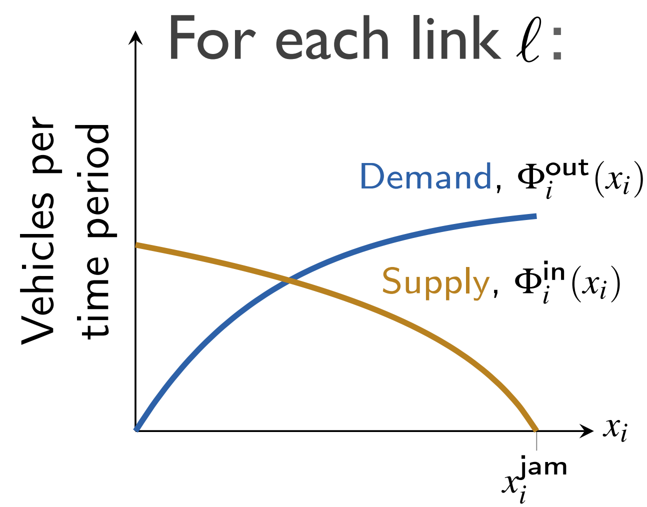




Traffic Flow: a Macroscopic Model







$$x_\ell^+ = x_\ell + f_\ell^{\text{in}}(x) - f_\ell^{\text{out}}(x) =: F_\ell(x)$$

Outgoing links:

$$f_k^{\text{in}}(x, m) = \sum_{\ell \in \text{in}} \beta_{\ell k} f_\ell^{\text{out}}(x, m)$$

$\beta_{\ell k}$ → turn ratio

Incoming links:

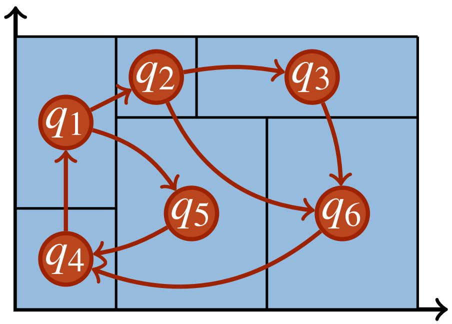
$$f_\ell^{\text{out}}(x, m) = s_\ell(m) \min\left\{ \Phi_\ell^{\text{out}}(x_\ell), \min_{k \in \text{out}} \frac{1}{\beta_{\ell k}} \Phi_k^{\text{in}}(x_k) \right\}$$

$s_\ell(m) \in \{0, 1\}$

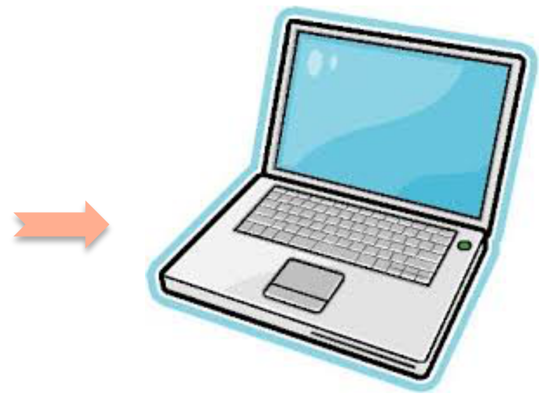
Traffic Flow is Mixed Monotone

$$\delta_{ij}\frac{\partial F_i(x)}{\partial x_j} \geq 0 \qquad \delta_{ij} = \begin{cases} -1 & \text{if } i \text{ and } j \text{ share tail node} \\ +1 & \text{otherwise} \end{cases}$$

Apply abstraction algorithm and add signaling states to transition model

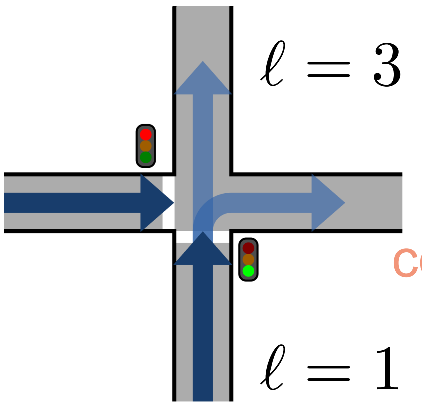


finite abstraction



formal methods

Note: Standard monotonicity breaks down at splits

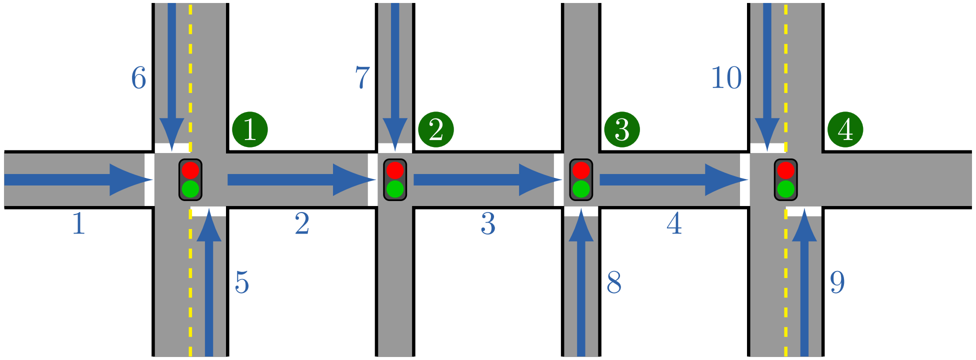


$$f_1^{\text{out}} = \frac{1}{\beta_{12}}\Phi_2^{\text{in}}(x_2)$$

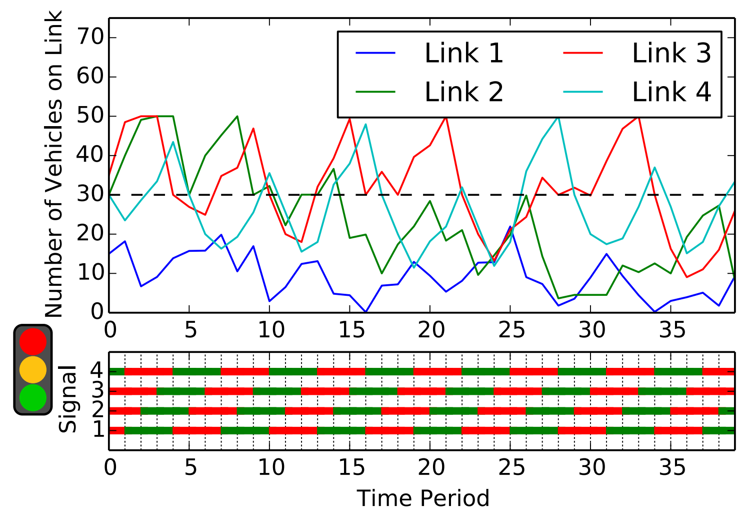
$$f_3^{\text{in}} = \beta_{13} f_1^{\text{out}} = \frac{\beta_{13}}{\beta_{12}}\Phi_2^{\text{in}}(x_2)$$

$$\Rightarrow \quad \delta_{32} = -1$$

Example: Signal Control for a Corridor



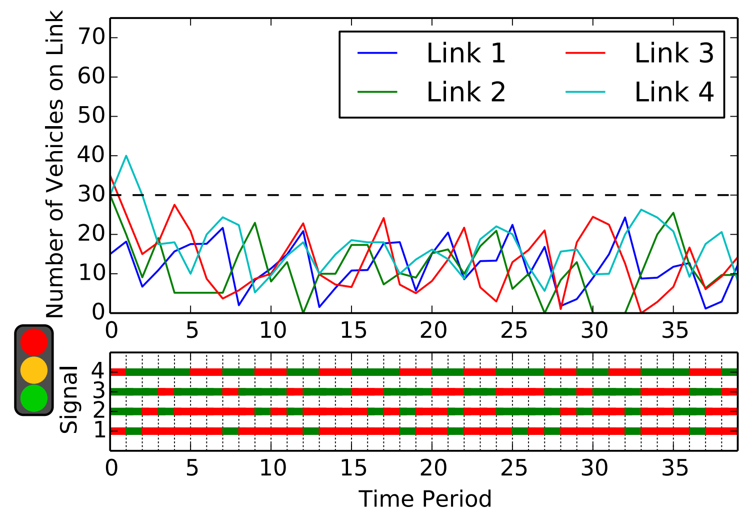
Naïve offset optimal policy



Temporal Logic Specifications:

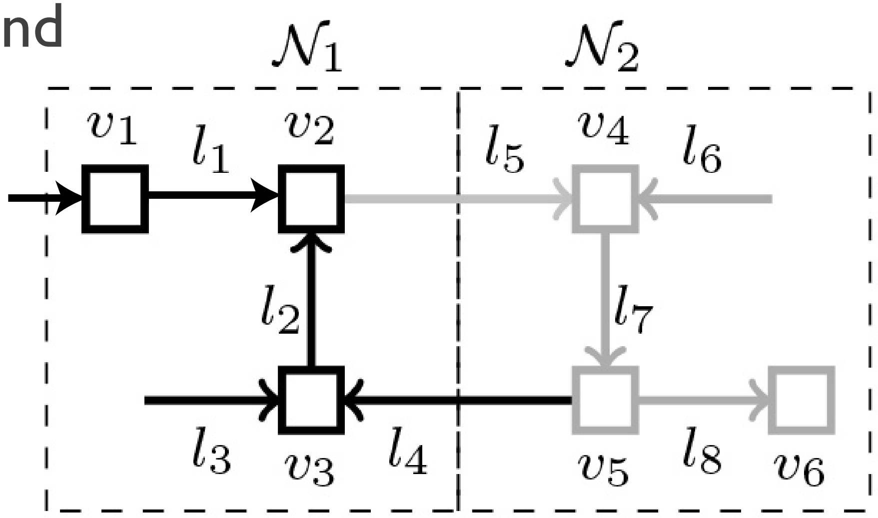
- Each signal actuates cross street traffic infinitely often
- Eventually, links 1, 2, 3, and 4 have fewer than 30 vehicles each
- The signal at junction 4 must actuate cross street traffic for at least two sequential time-steps

Correct-by-design policy



2. Compositional Synthesis for Large Networks

- "Contracts" between neighboring subnetworks to limit demand and guarantee adequate supply



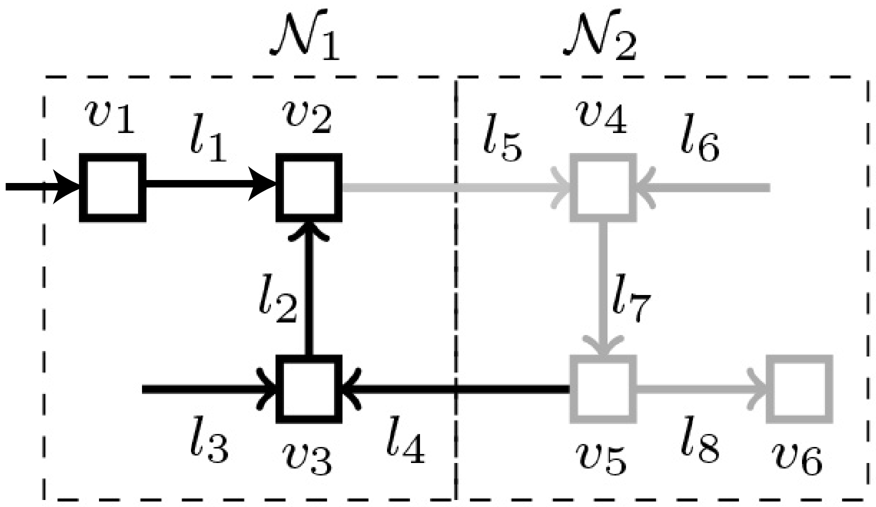
- Neighbors' promises allow decoupled subnetwork models with set valued maps

- Augment temporal logic specifications with own promises and synthesize controller for each subnetwork

$$\phi_i^{\text{new}} = \phi_i^{\text{original}} \wedge \underbrace{\phi_i^{\text{supply}} \wedge \phi_i^{\text{demand}}}_{\text{promises to neighbors}} \quad i = 1, 2, \ldots$$

Neighbors' Promises Allow Decoupled Models

Subnetwork 2 promises a minimum supply of $\sigma_2^{\text{contract}}$ on link 5 and to limit its demand on link 4 by $\delta_4^{\text{contract}}$ vehicles per period.



Decoupled subnet 1 model:

$$f_2^{\text{out}}(x) = \min\left\{\Phi_2^{\text{out}}(x_2), \frac{1}{\beta_{25}}\Phi_5^{\text{in}}(x_5)\right\}$$

$$\in \min\left\{\Phi_2^{\text{out}}(x_2), \frac{1}{\beta_{25}}\sigma\right\}, \;\; \sigma \in [\sigma_2^{\text{contract}}, \sigma_2^{\text{best}}]$$

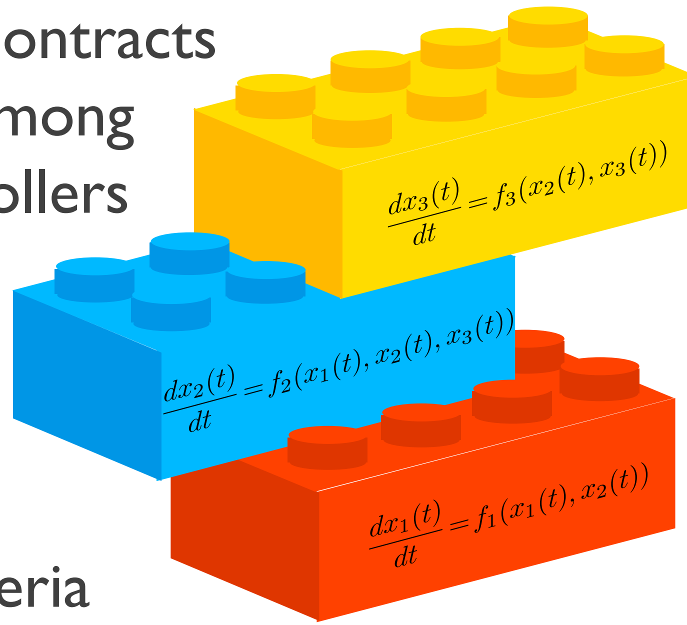
$$f_4^{\text{in}}(x) = \beta_{74}\min\left\{\Phi_7^{\text{out}}(x_7), \frac{1}{\beta_{78}}\Phi_8^{\text{in}}(x_8), \frac{1}{\beta_{74}}\Phi_4^{\text{in}}(x_4)\right\}$$

$$\in \beta_{74}\min\left\{\Phi_4^{\text{in}}(x_4), \delta\right\}, \;\; \delta \in [0, \delta_4^{\text{contract}}]$$

Ongoing Research

Compositional synthesis:

less conservative contracts and cooperation among subnetwork controllers rather than fully decentralized control



Probabilistic guarantees:

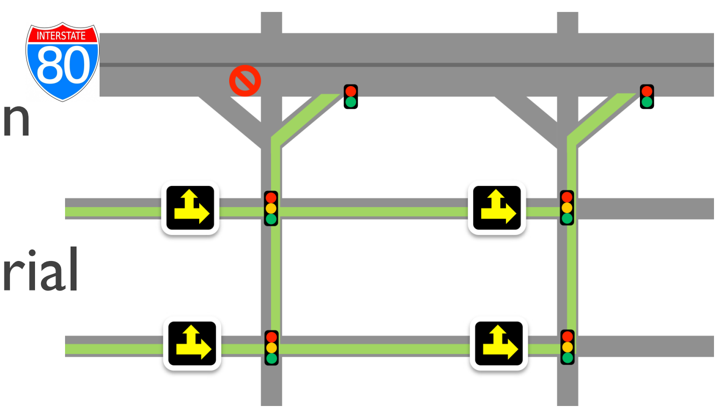
exploit demand statistics to find transition probabilities and guarantee satisfaction with high probability

Optimality:

add optimality criteria to specifications, e.g., minimize travel time, minimize spatial variations in traffic density, maximize throughput

Coordinated onramp metering / arterial signaling

and validation with hybrid freeway/arterial simulation



Publications This Year

Coogan, Gol, Arcak and Belta "Traffic network control from temporal logic specifications" – IEEE Trans. Control Network Systems, in press

Kim, Arcak and Seshia "Compositional controller synthesis for vehicular traffic networks" – CDC 2015

Saddraddini and Belta "Robust temporal logic model predictive control" – Allerton Conference, 2015

Coogan, Gol, Arcak and Belta "Controlling a network of signalized intersections from temporal logic specifications" – ACC 2015

Coogan and Arcak "Efficient finite abstraction of mixed monotone systems" – HSCC 2015 (Best Student Paper Award)

Saddraddini and Belta "A provably correct MPC approach to safety control of urban traffic networks" – submitted

Coogan, Arcak and Belta "Formal methods for control of transportation networks" – submitted