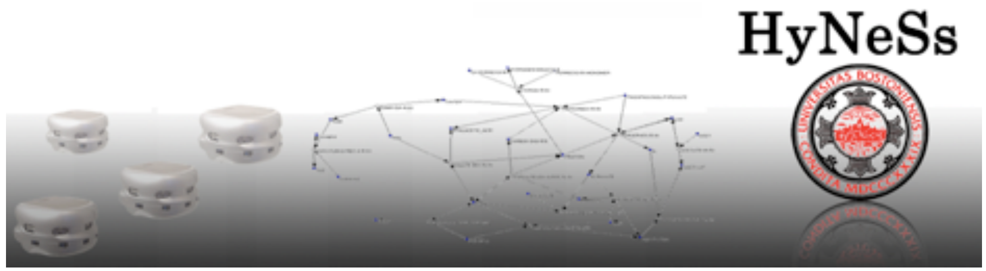
Hybrid and Networked Systems Lab



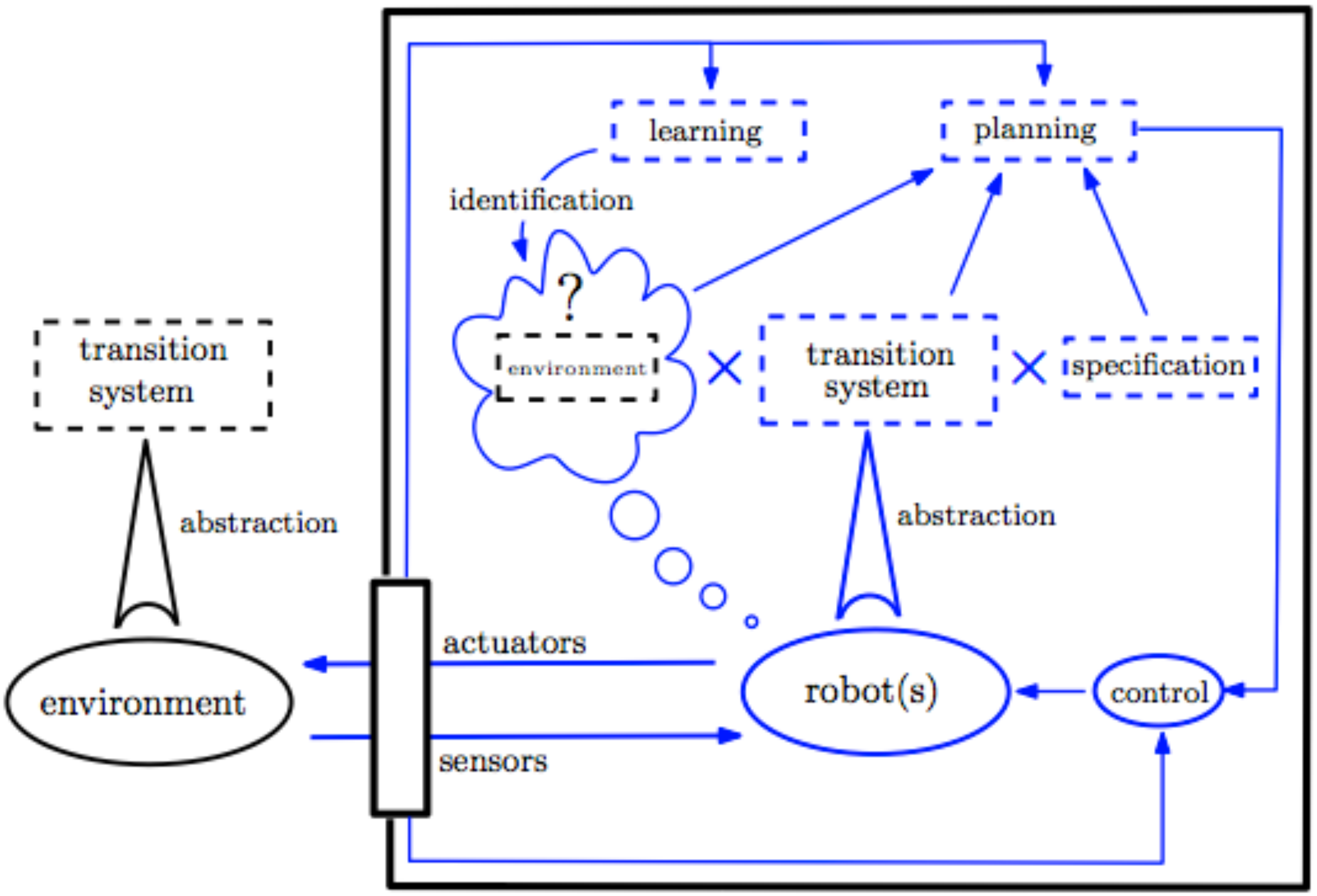
Formal Synthesis with Learning of Environmental Dynamics

Calin Belta

Mechanical Engineering and Systems Engineering
Boston University

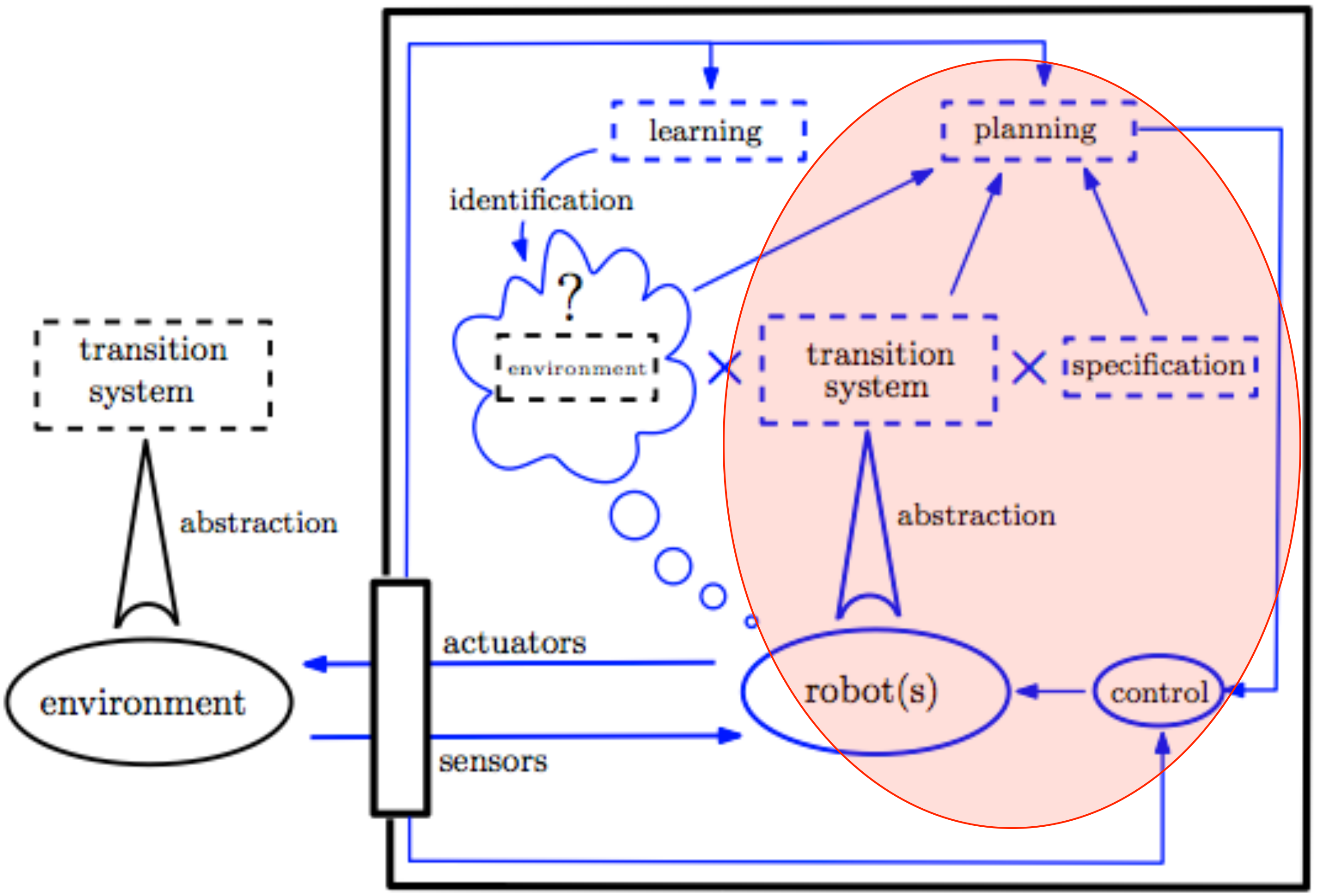
CPS: Medium: Collaborative Research: Efficient Control Synthesis and Learning in Distributed Cyber-Physical Systems

Big picture

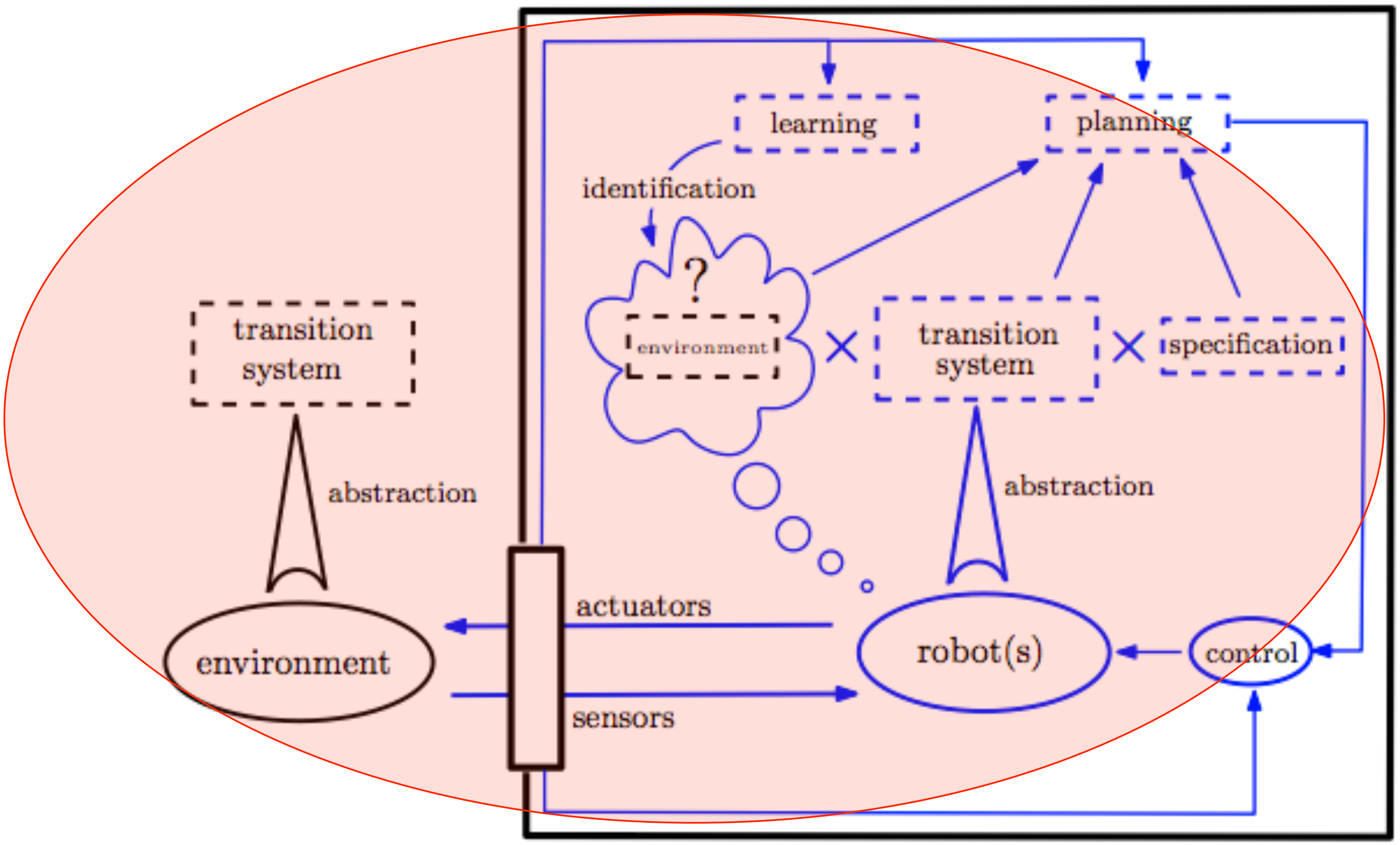


learning
planning
identification
?
environment
transition system
specification
transition system
abstraction
abstraction
actuators
environment
robot(s)
control
sensors

1. Formal Synthesis



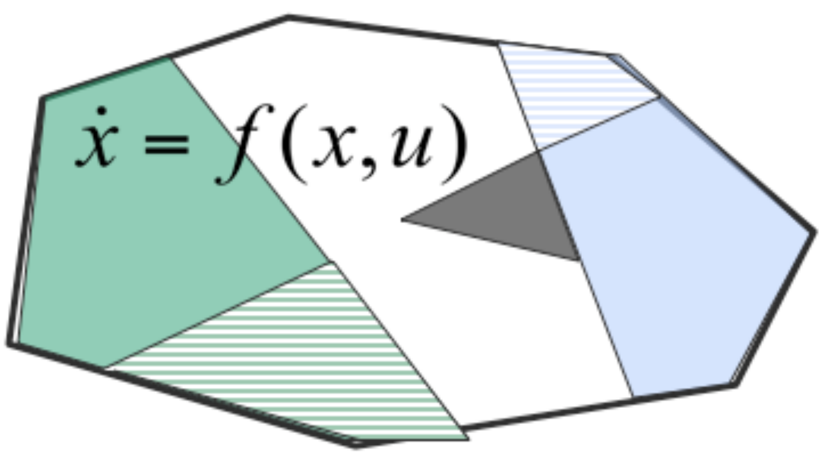
2. Formal Synthesis with Learning of Environmental Dynamics



Conservative Formal Synthesis for Dynamical Systems

Problem Formulation

Given a control system and a temporal logic specification over a set of regions, **find** initial states and feedback control strategies such that all the trajectories of the closed loop system satisfy the specification.

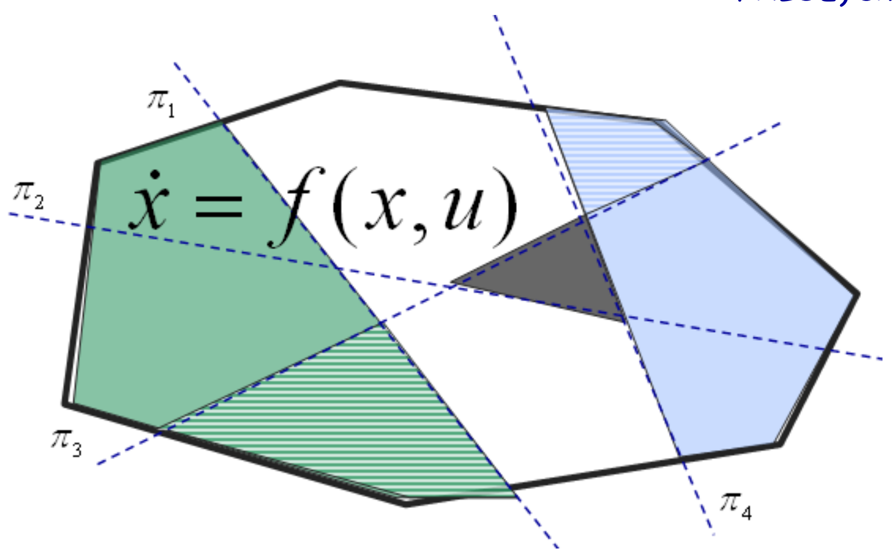


↔ “Avoid the grey region for all times. Visit the blue region, then the green region, and then keep surveying the striped blue and green regions, in this order.”

Conservative Formal Synthesis for Dynamical Systems

Approach

"(pi2 = TRUE and pi4 = FALSE and pi3 = FALSE) should never happen. Then pi4 = TRUE and then pi1 = TRUE should happen. After that, (pi3 = TRUE and pi4 = TRUE) and then (pi1 = TRUE and pi3 = FALSE) should occur infinitely often."



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Conservative Formal Synthesis for Dynamical Systems

Approach

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$$\Diamond(\pi_4 \wedge \Diamond(\pi_1 \wedge \Diamond$$
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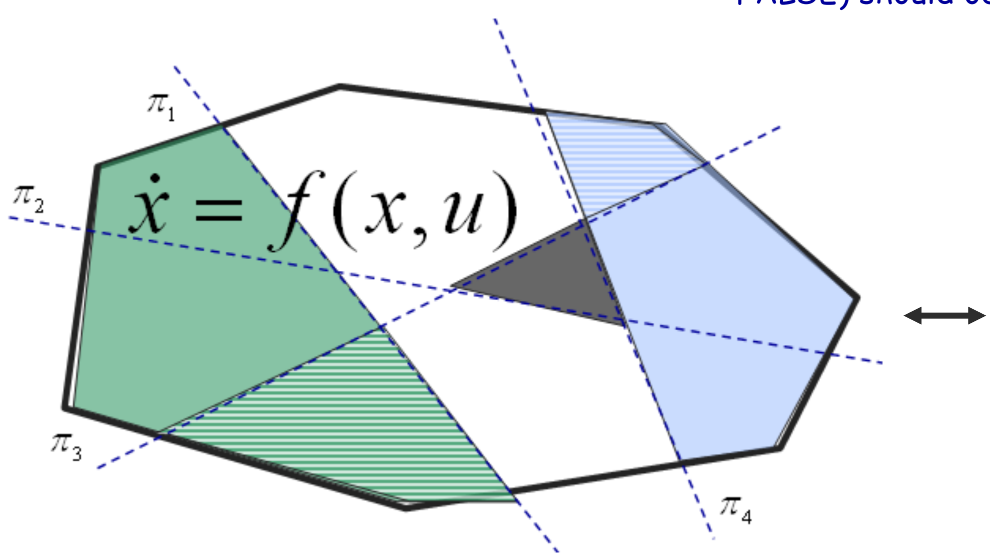

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Conservative Formal Synthesis for Dynamical Systems

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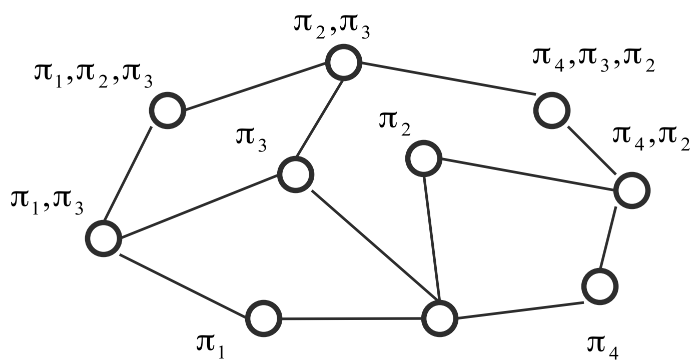

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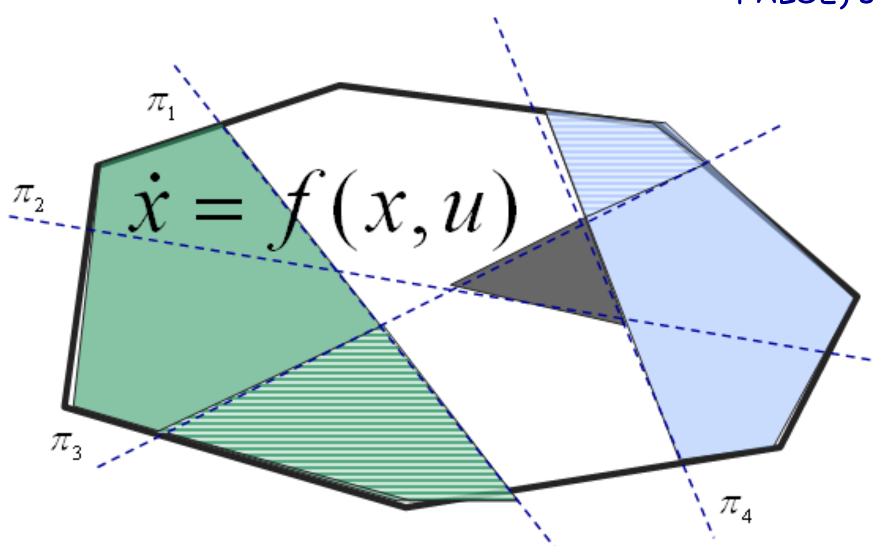

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Conservative Formal Synthesis for Dynamical Systems

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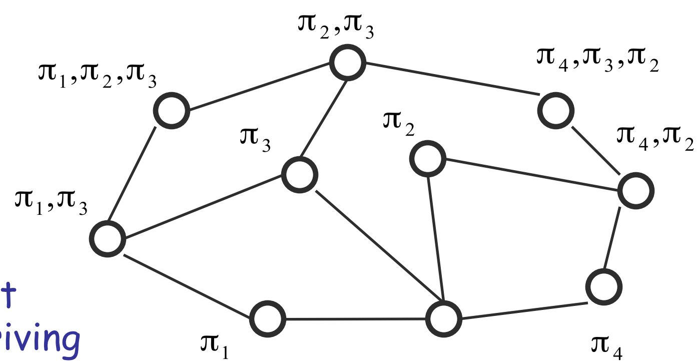

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In each region construct feedback controllers driving all states in finite time to a subset of facets (including the empty set – controller making the region an invariant) – **this is only possible for simple dynamics and partitions**

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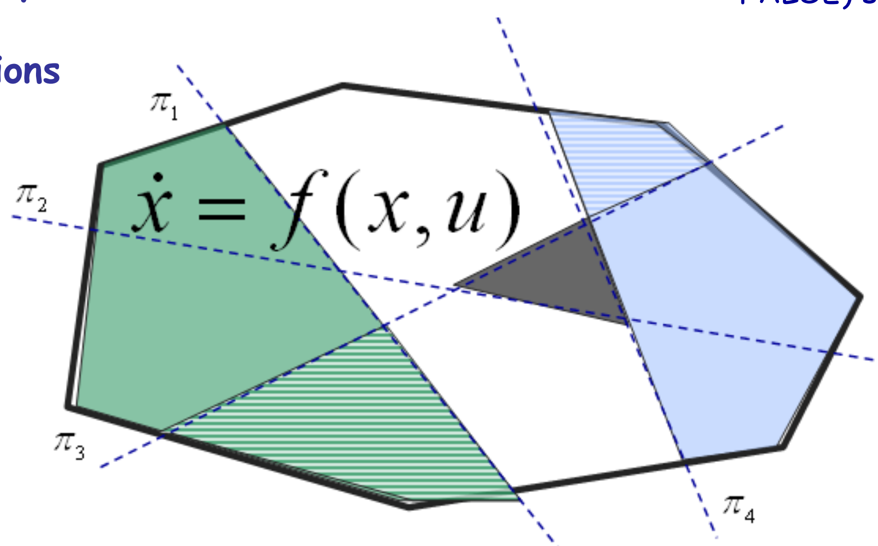

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Conservative Formal Synthesis for Dynamical Systems

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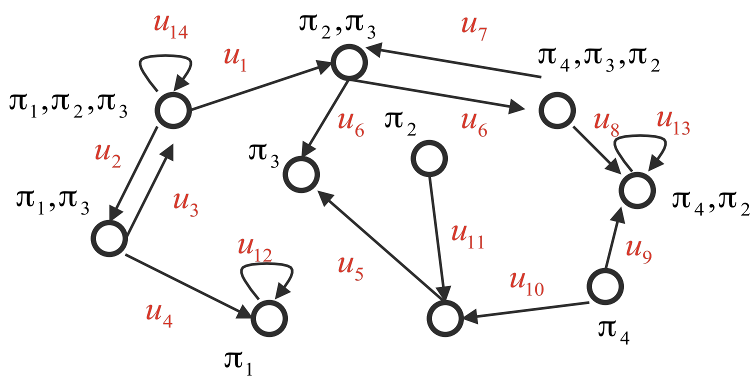

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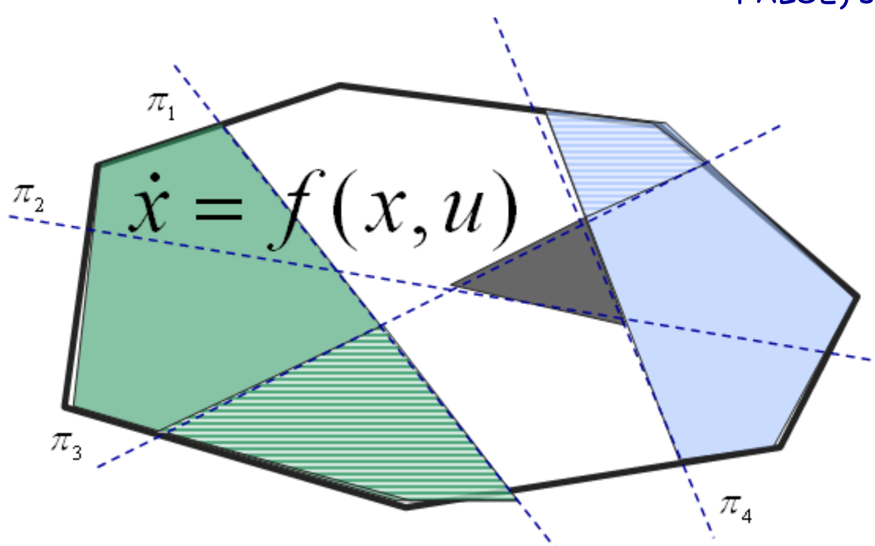

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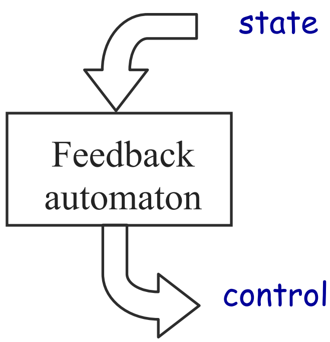

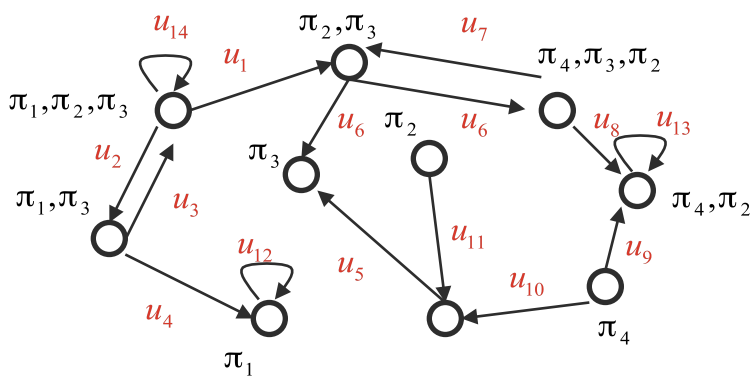

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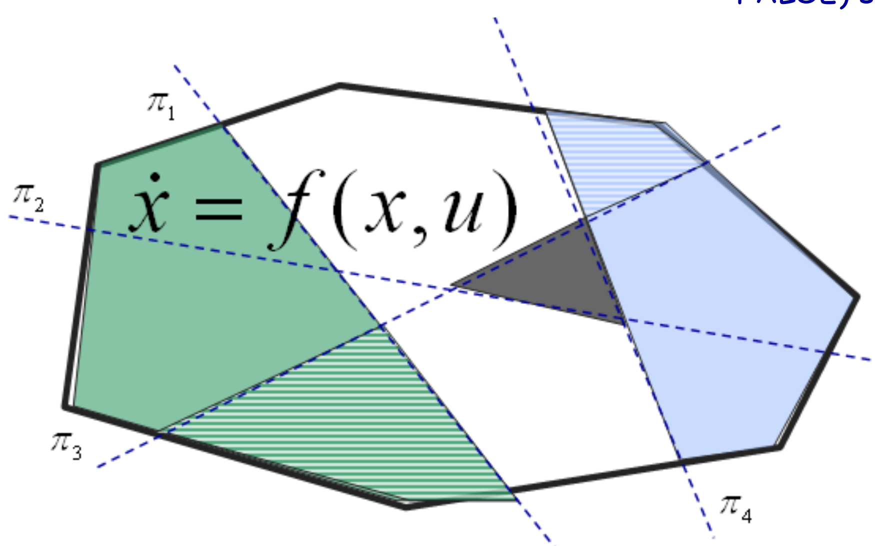

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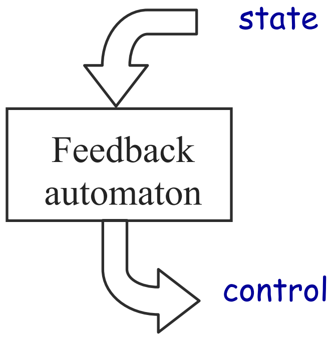

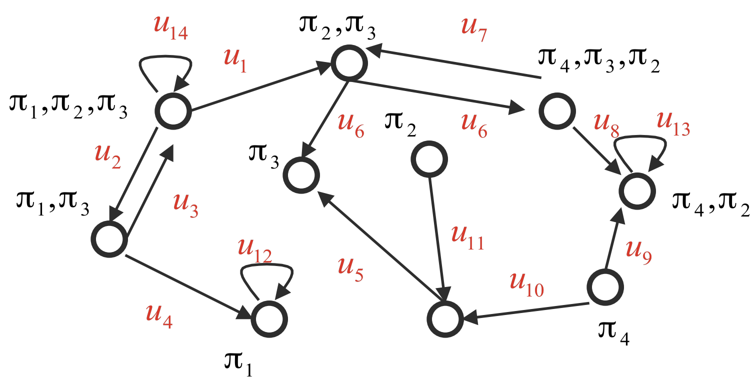

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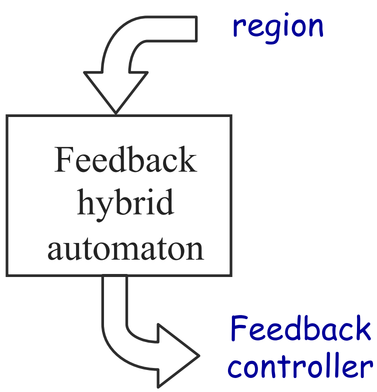
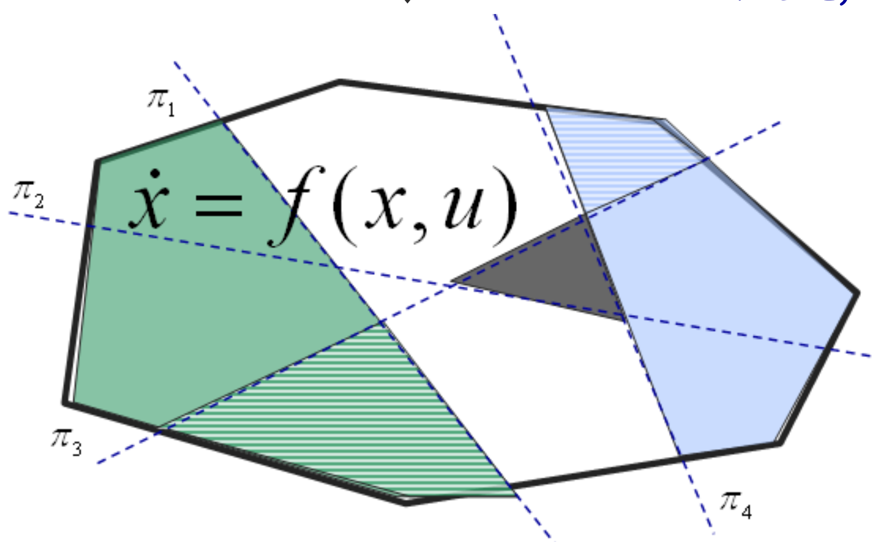
Refinement

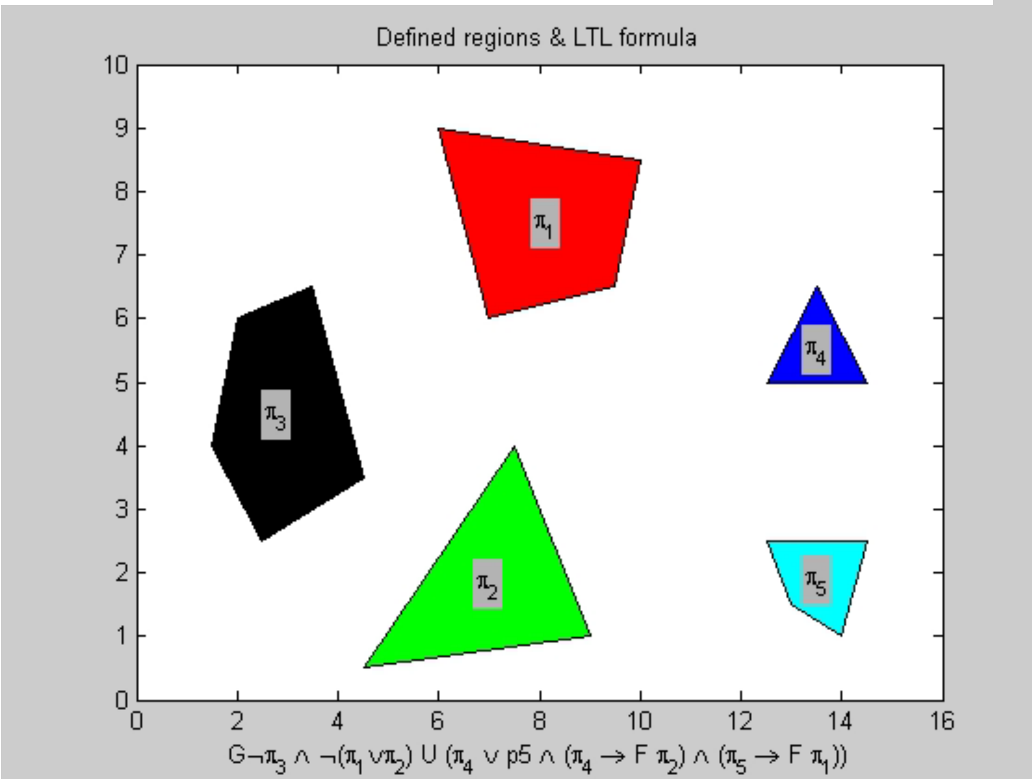


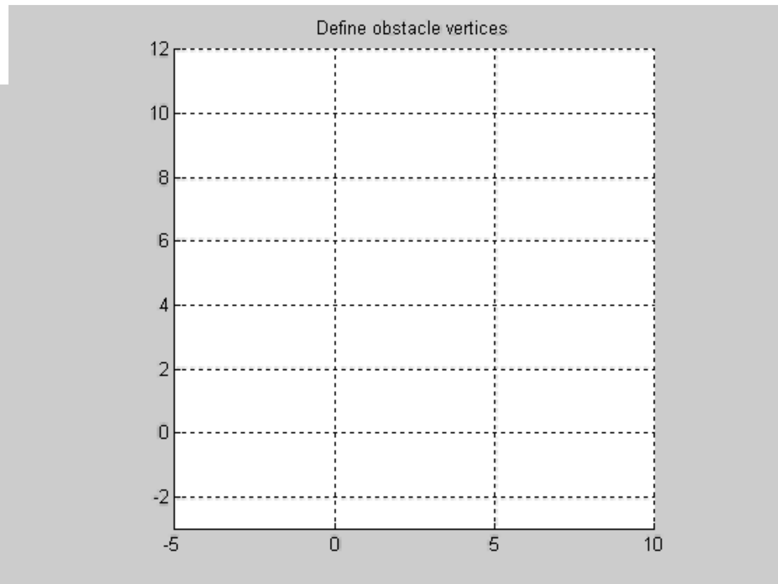


"Avoid the grey region for all times. Visit the blue region, then the green region, and then keep surveying the striped blue and green regions, in this order."

Conservative Formal Synthesis for Dynamical Systems

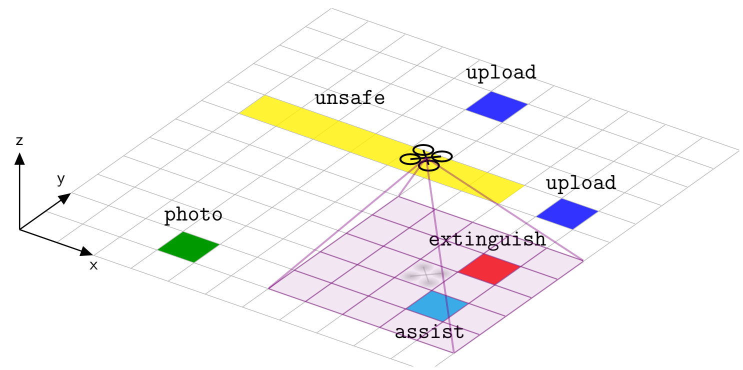
"Always avoid black. Avoid red and green until blue or cyan are reached. If blue is reached then eventually visit green. If cyan is reached then eventually visit red."

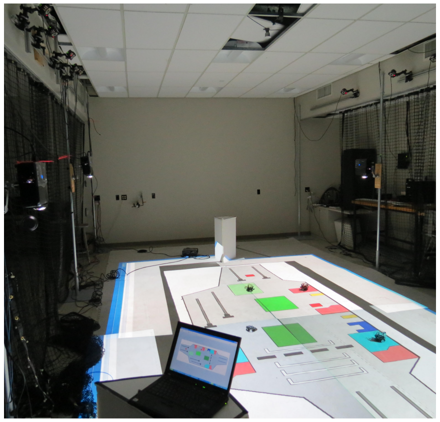




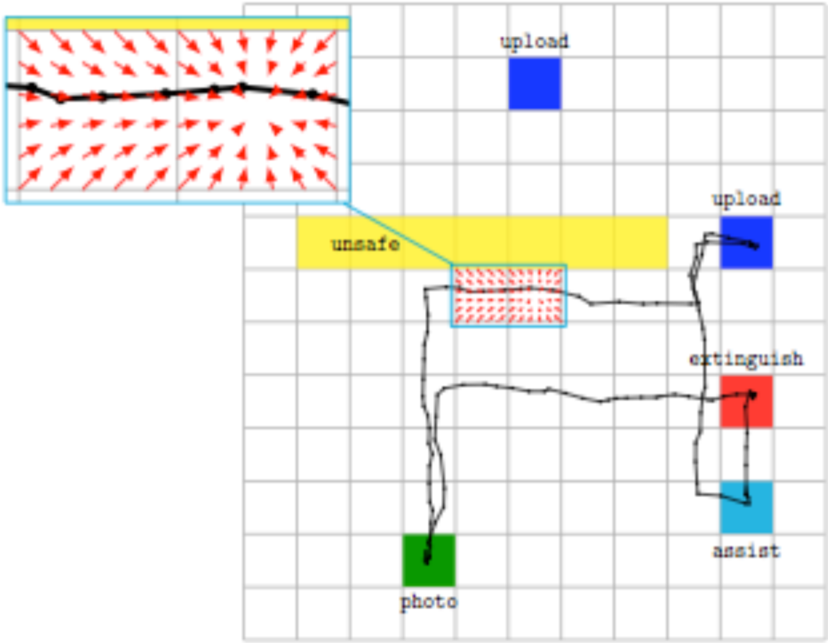
Conservative Formal Synthesis for Dynamical Systems

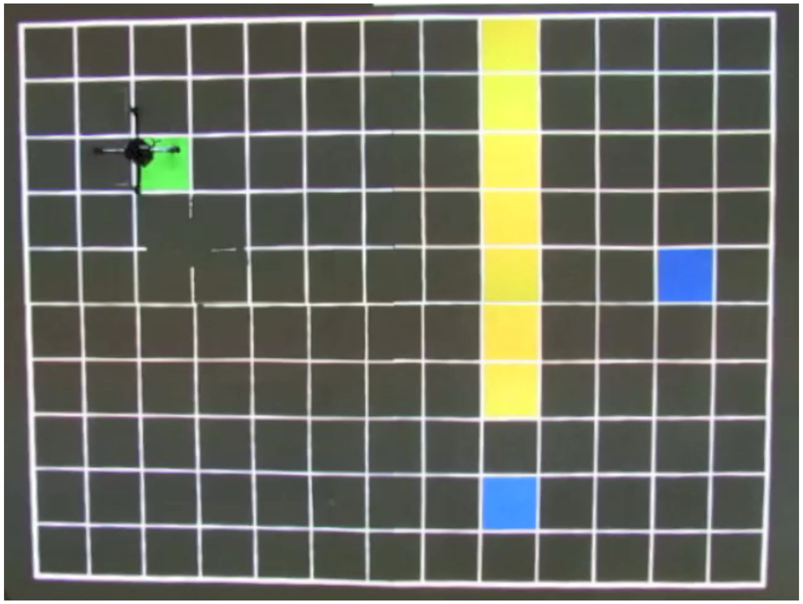
Global Spec: "Keep taking photos and upload current photo before taking another photo."
Local Spec: "Unsafe regions should always be avoided. If fires are detected, then they should be extinguished. If survivors are detected, then they should be provided medical assistance. If both fires and survivors are detected locally, priority should be given to the survivors."





A potential, Lyapunov-like function is used to make sure that local decisions do not affect global correctness (analogy to terminal constraints in MPC)

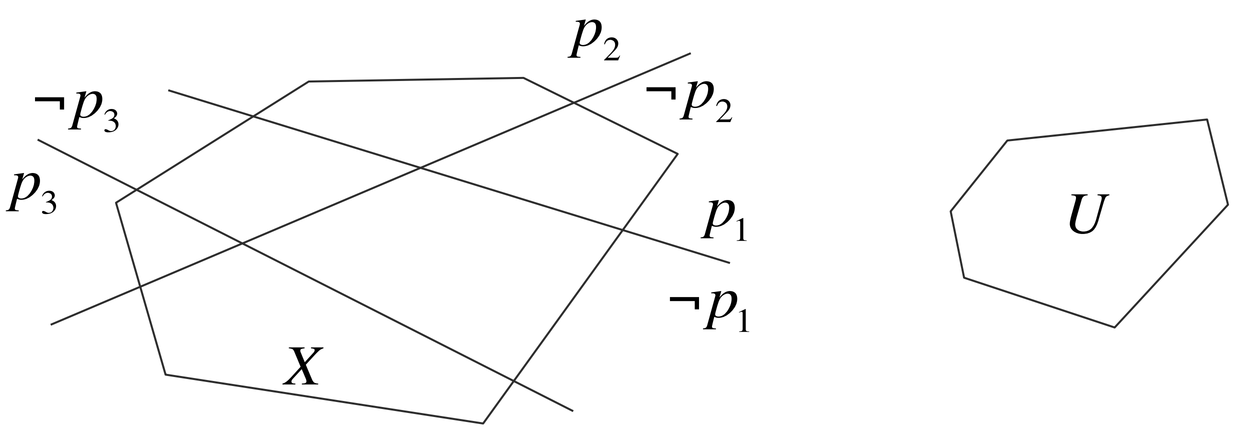




A. Ulusoy, M. Marrazzo, C. Belta, RSS 2013

Language-guided synthesis of control strategies

$x_{k+1} = Ax_k + Bu_k, x_k \in X, u_k \in U$ $\quad$ X, U polytopes



Problem Formulation: Find $X_0 \subseteq X$ and a state-feedback control strategy such that all trajectories of the closed loop system originating at X_0 satisfy an LTL formula ϕ over the linear predicates p_i

Language-guided synthesis of control strategies

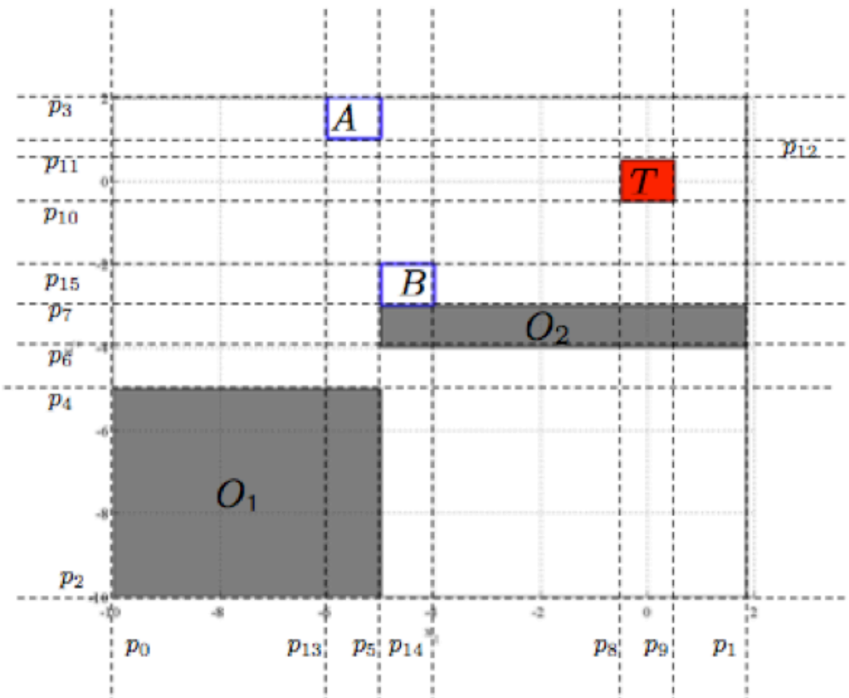
Example

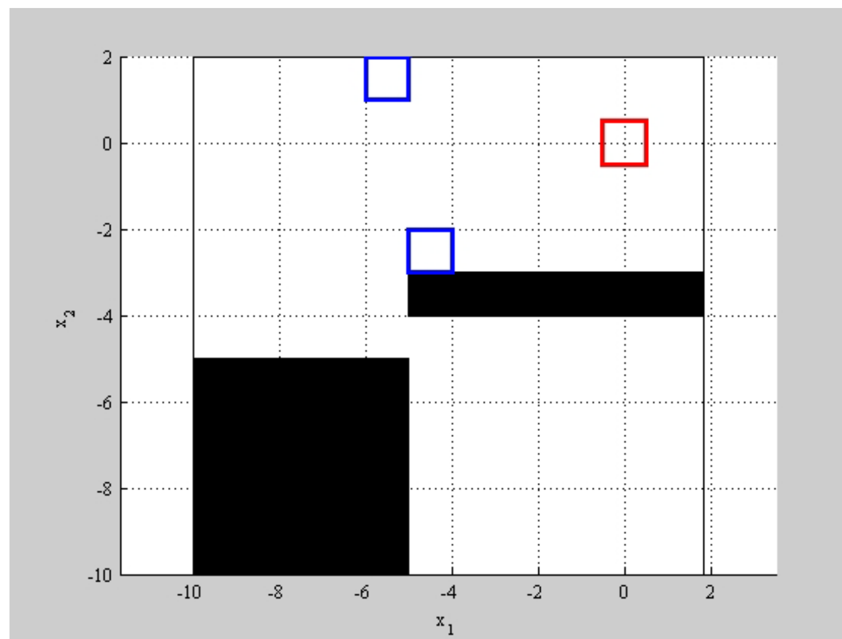
$$x_{k+1} = Ax_k + Bu_k, \quad x_k \in \mathbb{X},\ u_k \in \mathbb{U}$$

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$$

"Visit region A or region B before reaching the target while always avoiding the obstacles"

$$\Phi_2 = ((p_0 \wedge p_1 \wedge p_2 \wedge p_3 \wedge \neg(p_4 \wedge p_5) \wedge \neg(\neg p_5 \wedge \neg p_6 \wedge p_7))\ \mathscr{U}\ (\neg p_8 \wedge p_9 \wedge \neg p_{10} \wedge p_{11})) \wedge (\neg(\neg p_8 \wedge p_9 \wedge \neg p_{10} \wedge p_{11})\ \mathscr{U}\ ((p_5 \wedge \neg p_{12} \wedge \neg p_{13}) \vee (\neg p_5 \wedge \neg p_7 \wedge p_{14} \wedge p_{15})))$$

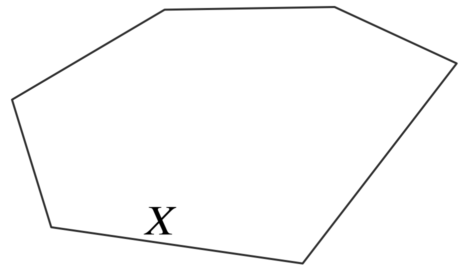


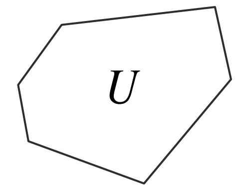


E. Aydin Gol, M. Lazar, C. Belta, HSCC 2012

Optimal temporal logic control

$$x_{k+1} = Ax_k + Bu_k, \quad x_k \in \mathbb{X},\ u_k \in \mathbb{U},$$





Initial state: x_0

Reference trajectories:

$x_0^r, x_1^r \ldots$

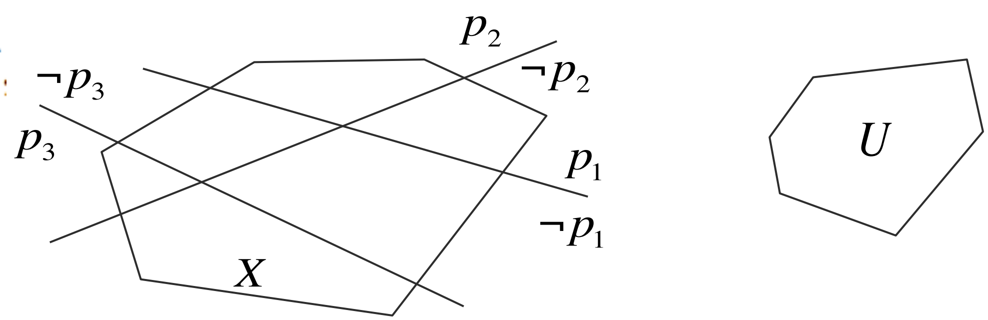
$u_0^r, u_1^r, \ldots$

Observation horizon : N

$$C(x_k, \mathbf{u}_k) = (x_{k+N} - x_{k+N}^r)^\top L_N (x_{k+N} - x_{k+N}^r) + \sum_{i=0}^{N-1} \left\{ (x_{k+i} - x_{k+i}^r)^\top L (x_{k+i} - x_{k+i}^r) + (u_{k+i} - u_{k+i}^r)^\top R (u_{k+i} - u_{k+i}^r) \right\},$$

Optimal temporal logic control

$$x_{k+1} = Ax_k + Bu_k, \quad x_k \in \mathbb{X},\ u_k \in \mathbb{U},$$



Initial state: x_0

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$$\begin{aligned} C(x_k, \mathbf{u}_k) = {} & (x_{k+N} - x_{k+N}^r)^\top L_N (x_{k+N} - x_{k+N}^r) \\ & + \sum_{i=0}^{N-1} \big\{ (x_{k+i} - x_{k+i}^r)^\top L (x_{k+i} - x_{k+i}^r) \\ & \quad + (u_{k+i} - u_{k+i}^r)^\top R (u_{k+i} - u_{k+i}^r) \big\}, \end{aligned}$$

Syntactically co-safe LTL formula over linear predicates p_i

Problem Formulation: Find an optimal state-feedback control strategy such that the trajectory originating at x_0 satisfies the formula.

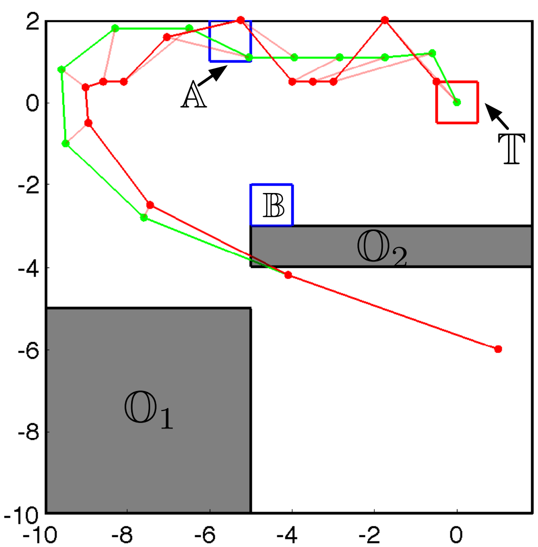
Optimal temporal logic control

Example

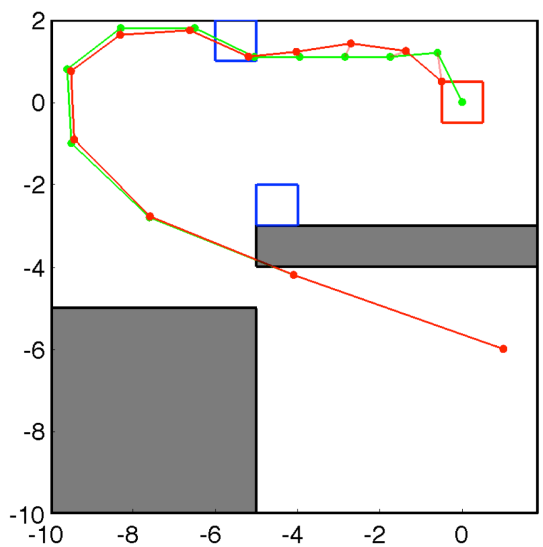
$$x_{k+1} = Ax_k + Bu_k, \quad x_k \in \mathbb{X},\ u_k \in \mathbb{U}$$

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$$

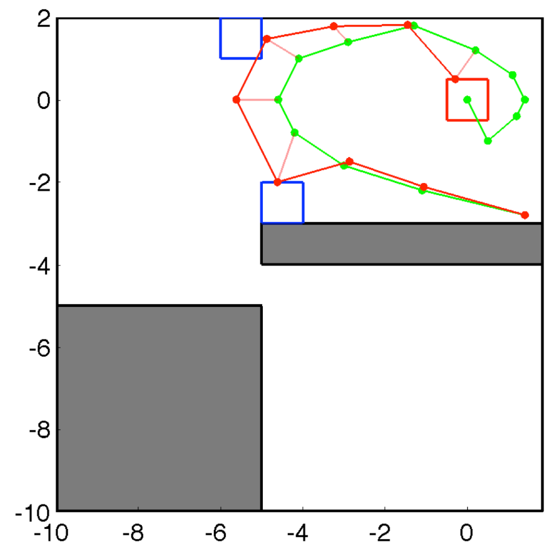
"Visit region A or region B before reaching the target while always avoiding the obstacles"



N = 2
total cost = 29.688



N = 4
total cost = 0.886



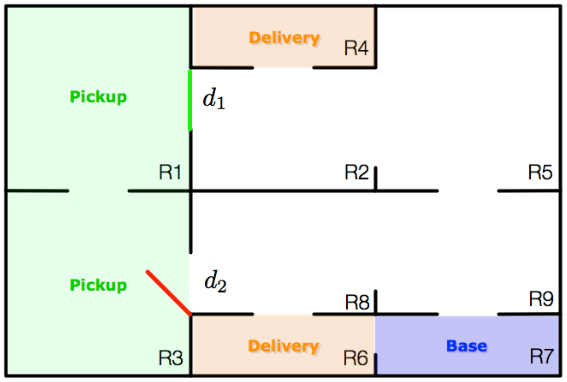
N = 6
total cost = 5.12

Reference trajectory
Controlled trajectory

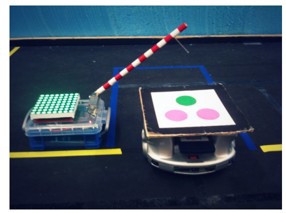
Reference trajectory violates the specification

E. Aydin Gol, M. Lazar, C. Belta, HSCC 2013

Temporal Logic Control with Learning of Environmental Events



Environment with **known** topology



Environmental events driven by **unknown** MC processes

Find an optimal policy such that (1) the robot mission is accomplished, and (2) the expected time in between consecutive satisfactions of the optimizing task is minimized.

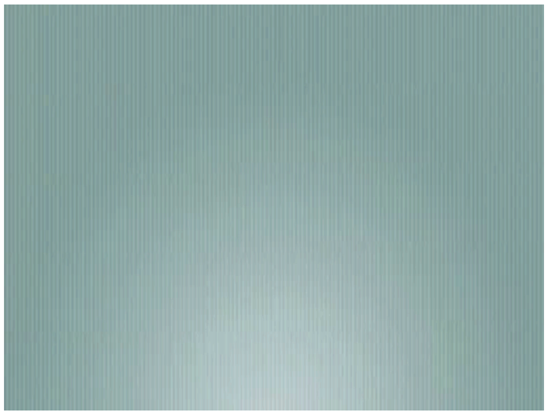
Mission:

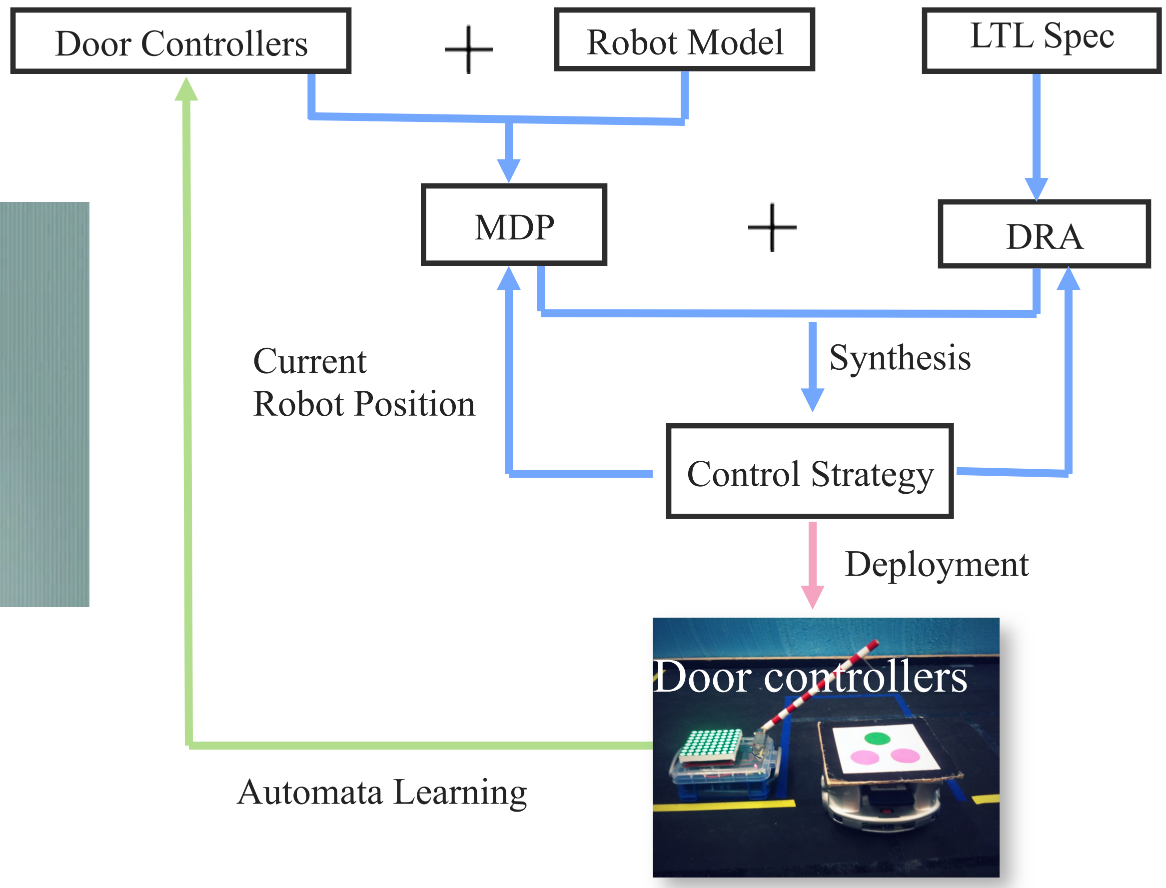
"Always eventually go back to Base, perform Pickup repeatedly, make sure Pickup and Delivery are executed in between two consecutive visits to base"

$$\phi = \overbrace{\mathsf{GF}(\mathsf{Base})}^{\text{Optimizing task}} \wedge \mathsf{GF}(\mathsf{Pickup}) \wedge \mathsf{G}(\mathsf{Base} \rightarrow \mathsf{Base}\ \mathsf{U}\ ((\neg \mathsf{Base})\ \mathsf{U}\ \mathsf{Pickup})) \wedge$$
$$\mathsf{G}(\mathsf{Pickup} \rightarrow ((\neg \mathsf{Base})\ \mathsf{U}\ \mathsf{Delivery}))$$

Temporal Logic Control with Learning of Environmental Events

Approach:





Assumption : the environmental events generate a specific subclass of omega-regular language, called stochastic strictly k-local language, which (1) can be learned from positive samples, and (2) is rich enough to represent interesting door behavior.

Theorem : When time goes to infinity, the door controller approximation almost surely generates a language that is equal to the language generated by the real door controller.

Y. Chen, J. Tumova, C. Belta, ICRA 2012, IJRR 2013

J. Heinz, "String extension learning, 2010

Acknowledgements

Boston University

Ebru Aydin Gol

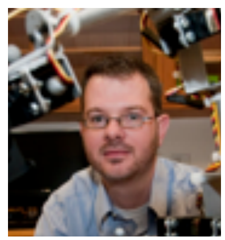
Yushan Chen

Dennis Ding
(now at UTRC)

Jana Tumova
(now at MIT)

Alphan Ulusoy

University of Delaware



Bert Tanner

Jeff Heinz

CPS: Medium: Collaborative Research: Efficient Control Synthesis and Learning in Distributed Cyber-Physical Systems