Graphical Model for Cyber-Physical Systems

Generalization from Electric Grids

Yang Weng, Advisors: Marija Ilić, Fellow, IEEE, Rohit Negi

Carnegie Mellon University

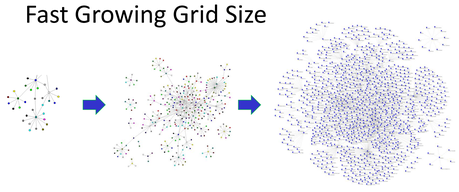
Motivation: Human Machine Interfaces

How to Make Cyber-Physical Power Grid Smart? **What is Special for Smart Grid?**

Smart Grid (Wikipedia) → Modernized electrical physical grid → Info., comm., and comp. tech.

Distributed Generator Plants | Vehicle Charging/ Discharging | Various Demand Management Software

Fast Growing Grid Size



Problems 1:

Unconventional Characteristics → Grid Architecture Shift → **Proper Stochastic Modeling !!** (to maintain robustness)

Problems 2:

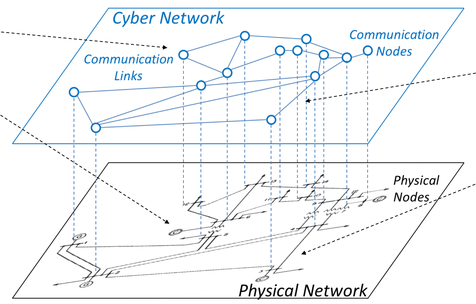
Traditional Approach → Hard to estimate Low Voltage Users and New diverse resources → **Scalable Distributed Inference !!**

Unifying CPS Design Across Domains: Single Layer Graphical Modeling

Graphical Model Description → (Electric Power) Grid *Example*

1) Vertices → (power) grid states → random variables

2) Edges → state variables' interaction → physical Laws (i.e. Kirchhoff laws)



$$z_i = h_i(\boldsymbol{v}) + u_i$$

Measurement ↑ State ↑ Noise ↑

3) Joint density of random variables
- probabilistic states
- subject to the constraints
  - imposed by the physical law

4) Weighted Least Square (WLS) → Maximum a posteriori probability (MAP)

$$\min_{\boldsymbol{v}} J_p(\boldsymbol{v}) = \sum_{i=1}^{m} \left| \frac{z_i - h_i(\boldsymbol{v})}{\sigma_i} \right|^p \quad \Rightarrow \quad \max_{\boldsymbol{v}} p(\boldsymbol{v}|\boldsymbol{z}) = \frac{p(\boldsymbol{v})p(\boldsymbol{z}|\boldsymbol{v})}{p(\boldsymbol{z})}$$

Big Data ✓ ✓

$$p(\boldsymbol{z}|\boldsymbol{v}) \sim \exp\left\{ -\sum_i (z_i - h_i(\boldsymbol{v}))^2/\sigma_i^2 \right\}$$

Probability Density Function

- i.e. Power Flow

$$p(z_i^{pf}|\boldsymbol{v}) \sim \exp\left\{ -\sum_i \left| z_i - (v_s - v_t)Y_{st}^* v_s^* \right|^2 \right\}$$

5) Conduct state (of interest) marginalization (estimation)
- Can be done **fast** and **distributedly !!**

Efficient Distributed Inference: State Estimation

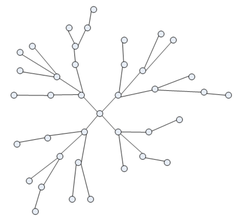
Scalable Estimation Algorithm for Tree Networks - Exact

- A Belief Propagation (BP) (Prob. Estimation)
  - Binary Variables

$$p(v_1) = \sum_{v_2,v_3,v_4} p(v_1, v_2, v_3, v_4)$$

8 Summations → exponentially

Scalable !!



  - **Structure** Information

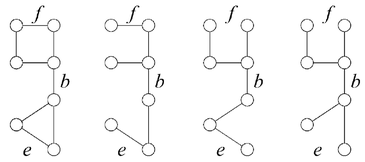
$$p(v_1) = \sum_{v_2} p(v_1|v_2) \sum_{v_3} p(v_2|v_3) \sum_{v_4} p(v_4)p(v_3|v_4)$$

Belief → linearly

6 Summations

  - BP algorithm

$$M_{s\to t}(v_t) \leftarrow \sum_{v_s} \phi_s(v_s)\phi_{st}(v_s, v_t) \prod_{k\in\mathcal{N}(s),\ k\neq t} M_{k\to s}(v_s)$$

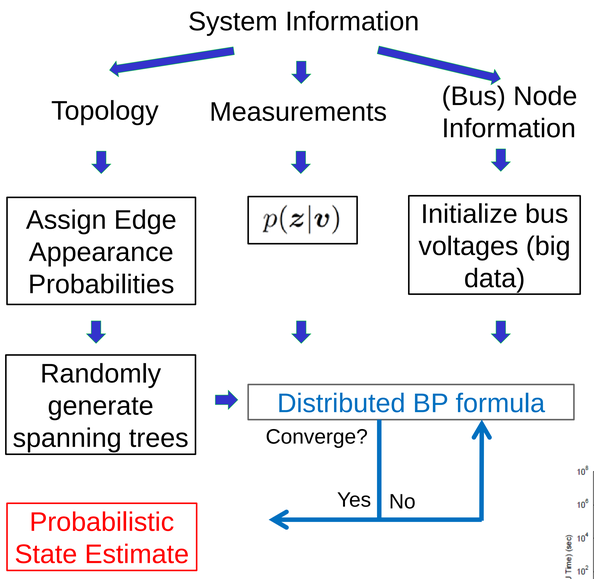


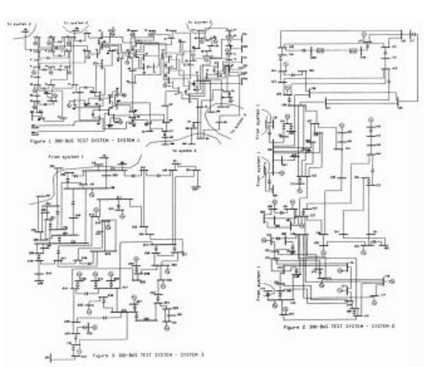
Extension to Mesh Network

- Idea: **Decompose** the Network into Trees.

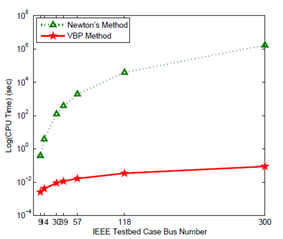
$$M_{t\to s}^{n+1}(v_s) = \alpha \sum_{v_t} \left\{ \exp\left(\frac{\theta_{st}(v_s, v_t)}{\rho_{st}} + \theta_t(v_t) \right) \frac{\prod_{k\in\mathcal{N}(t)\setminus s} [M_{k\to t}^n(v_t)]^{\rho_{kt}}}{[M_{s\to t}^n(v_t)]^{(1-\rho_{ts})}} \right\}, \quad \rho_{st} = \frac{\text{No. of spanning trees with edge (s,t)}}{\text{No. of all spanning trees}}$$

Sustainable CPS Designs

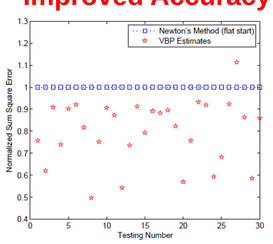




Scalable



Improved Accuracy



CPS Education

Multi-disciplinary Education:

- **Connect Computer Science with Electrical Power Grids**

- **Reference**
- [1] - YangWeng, Rohit Negi and Marija Ilic, **[Ranking First of All Submissions]**, "Graphical Model for State Estimation in Electric Power Systems", IEEE SmartGridComm Symposium (SGC), Vancouver, Canada, Oct., 2013.
- [2] - Yang Weng, Rohit Negi and Marija Ilic, **[Best Paper Award]**, "A Search Method for Obtaining Initial Guesses for Smart Grid State Estimation", IEEE SmartGridComm Symposium (SGC), Tainan, Taiwan, Nov., 2012.

Electric Energy Systems Group | ABB | NSF

Electrical & Computer ENGINEERING