NRI: Information-Theoretic Trajectory Optimization for Motion Planning and Control with Applications to Space Proximity Operations

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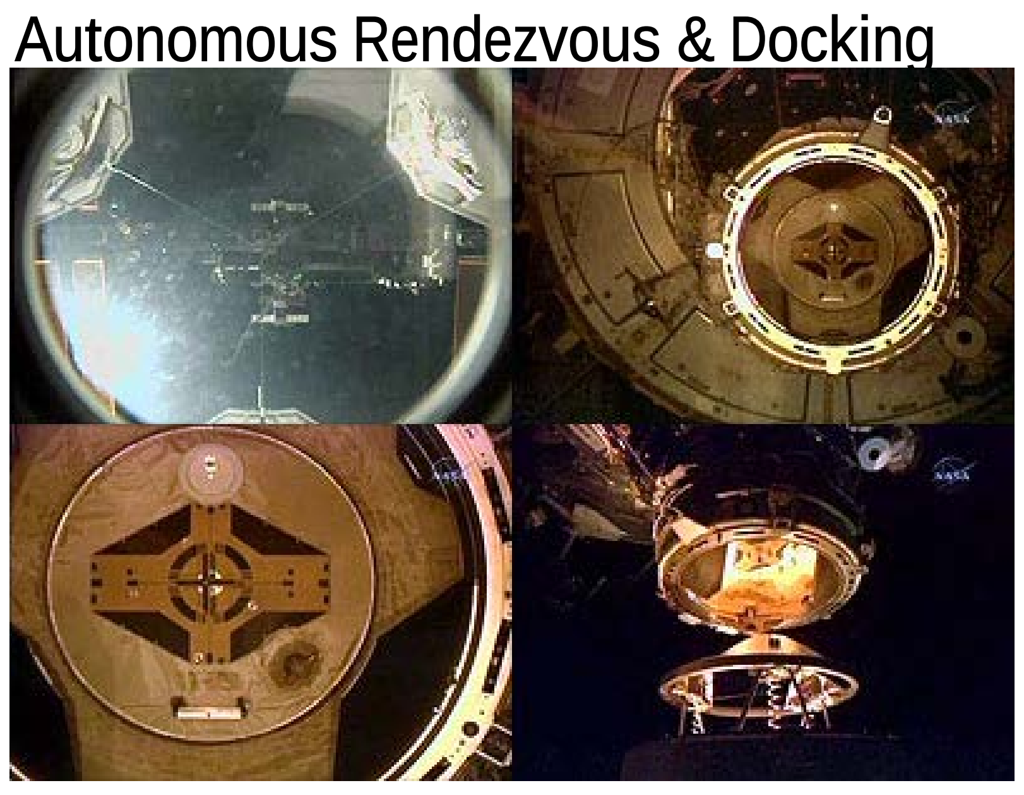
Motivation

Autonomous Proximity Operations (ProxOps) in Space

NASA and USAF have identified proximity robotics operations such as autonomous rendezvous and docking as crucial technologies.

- Autonomous Rendezvous & Docking



- Space Station Resupply / Structural Integrity Inspection

Approach Outline

$$dx = f(x,u)dt + C(x)dw$$

$$dy = h(x,u)dt + dv$$

The problem is formulated in the context of *continuous time stochastic optimal control.*

Trajectory optimization using Stochastic Optimal Feedback Control

- Stochastic Differential Dynamic Programming (DDP).
- Probabilistic inference using Sparse Spectrum Gaussian Process regression for uncertainty representation and real time probabilistic inference.

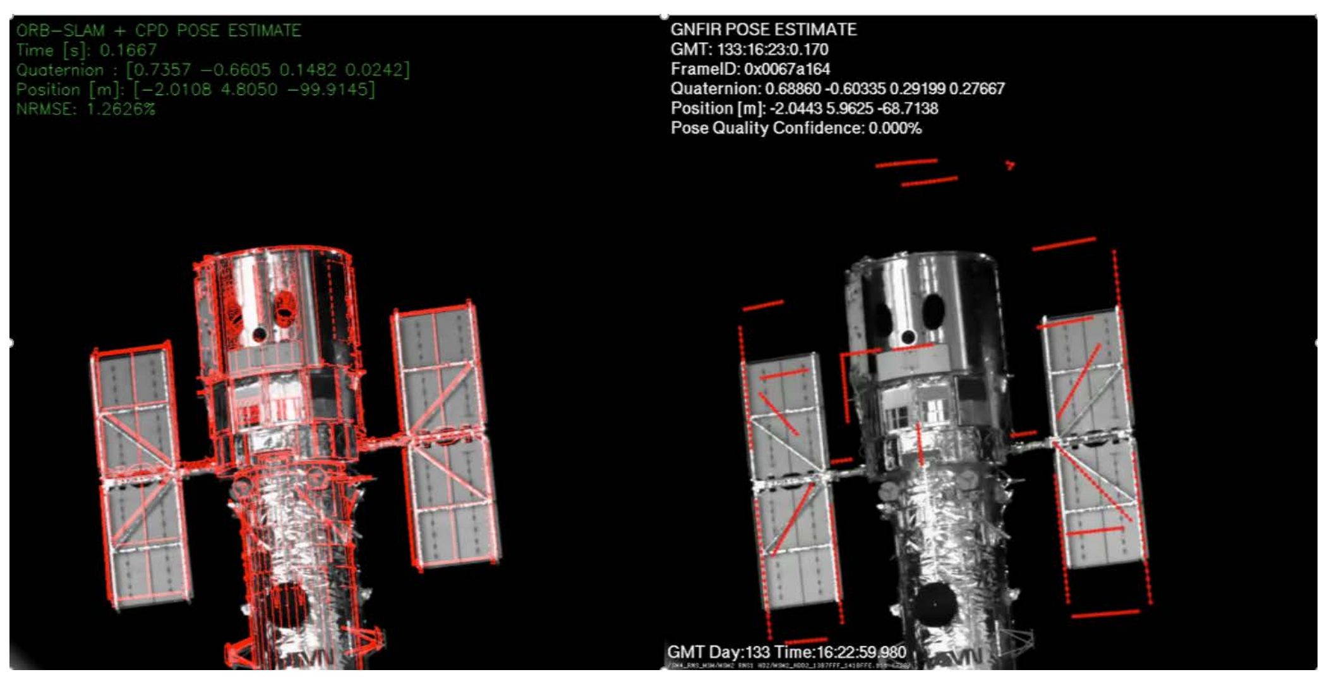
Information Theoretic Relative Navigation and Guidance in Orbit

- Exploit the structure of ProxOps (orbit constraint, Lie manifold) and apply information theoretic algorithms to this problem.

Results

Vision-Based Relative Navigation in Orbit

- Investigate SLAM solutions for RelNav problem in orbit
- ORB-SLAM based on BRIEF binary descriptor -> More efficient than SIFT
- Automates loop closure with BoW pose recognition
- Applied to NASA STS-125, Hubble Space Telescope (HST) Servicing Mission



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ORB−SLAM + CPD POSE ESTIMATE
Time [s]: 0.1667
Quaternion : [0.7357 −0.6605 0.1482 0.0242]
Position [m]: [−2.0108 4.8050 −99.9145]
NRMSE: 1.2626%
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GNFIR POSE ESTIMATE
GMT: 133:16:23:0.170
FrameID: 0x0067a164
Quaternion: 0.68860 -0.60335 0.29199 0.27667
Position [m]: -2.0443 5.9625 -68.7138
Pose Quality Confidence: 0.000%
```

Results

Probabilistic Trajectory Optimization using Sparse Spectrum Gaussian Processes

- Takes into account explicit model uncertainties using Sparse Spectrum Gaussian processes (GPs).
- Trajectory optimization in belief spaces.
- No a-priori control policy parameterization.
- Scales to high-dimensional control problems.
- Computational efficiency for real time inference.

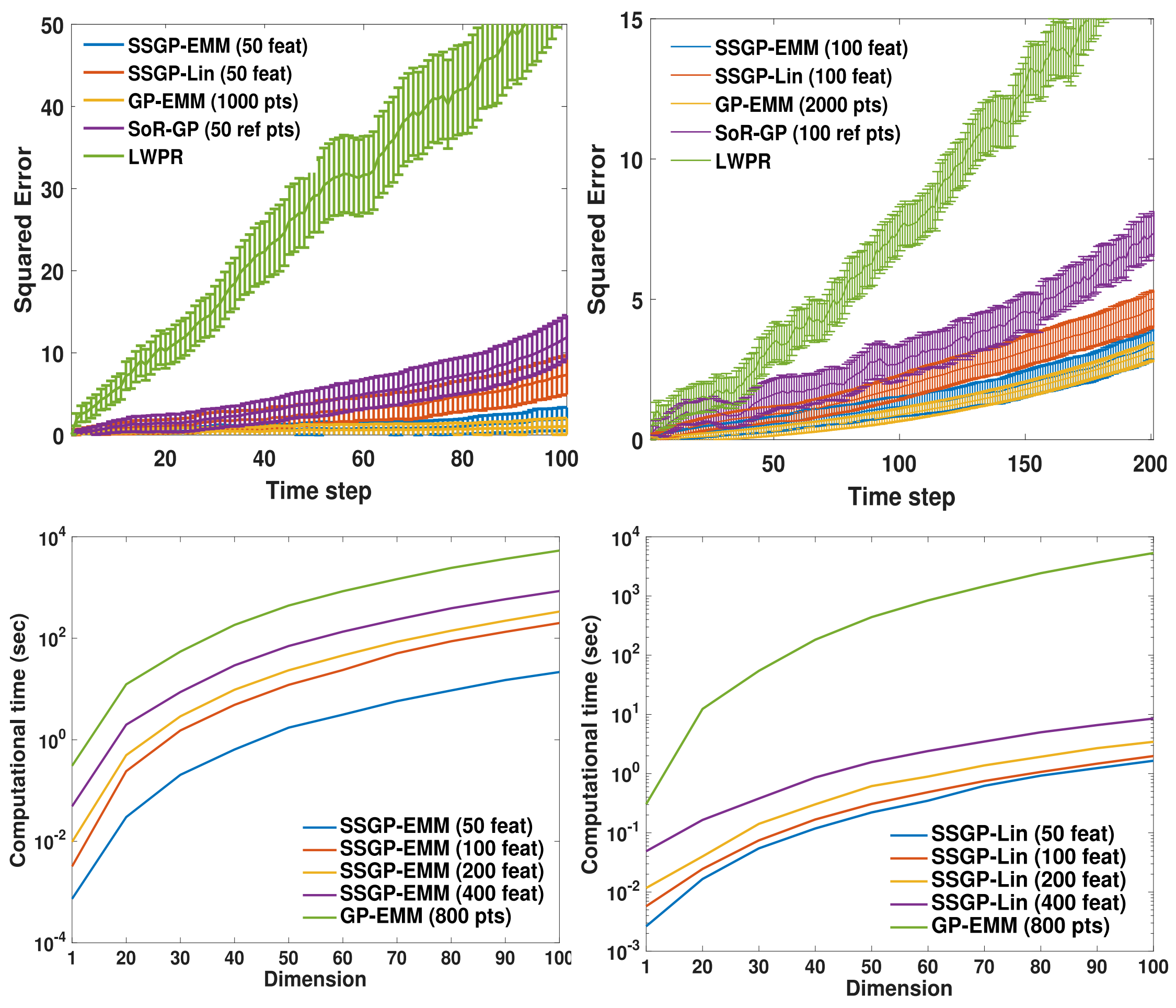
$$\Delta \mathbf{x} | \tilde{\mathbf{X}}, \Delta \mathbf{X}, \tilde{\mathbf{x}} \sim \mathcal{N}(\mathbf{w}^{\mathsf{T}} \phi, \sigma_n^2 (1 + \boldsymbol{\phi}^{\mathsf{T}} \mathbf{A}^{-1} \boldsymbol{\phi}))$$

$$\phi_{\boldsymbol{\omega}}(\tilde{\mathbf{x}}) = \frac{\sigma_f}{\sqrt{r}} [\ \cos(\boldsymbol{\omega}^{\mathsf{T}} \tilde{\mathbf{x}}) \quad \sin(\boldsymbol{\omega}^{\mathsf{T}} \tilde{\mathbf{x}})\ ]^{\mathsf{T}}$$

Local approximation of the belief dynamics $\mathbf{z}_k = \begin{bmatrix} \mu_k \\ \mathrm{vec}(\boldsymbol{\Sigma}_k) \end{bmatrix}$

$$\delta \mathbf{z}_{k+1} = \begin{bmatrix} \frac{\partial \mu_{k+1}}{\partial \mu_k} & \frac{\partial \mu_{k+1}}{\partial \boldsymbol{\Sigma}_k} \\ \frac{\partial \boldsymbol{\Sigma}_{k+1}}{\partial \mu_k} & \frac{\boldsymbol{\Sigma}_{k+1}}{\partial \boldsymbol{\Sigma}_k} \end{bmatrix} \delta \mathbf{z}_k + \begin{bmatrix} \frac{\partial \mu_{k+1}}{\partial \mathbf{u}_k} \\ \frac{\partial \boldsymbol{\Sigma}_{k+1}}{\partial \mathbf{u}_k} \end{bmatrix} \delta \mathbf{u}_k$$

Comparison w.r.t Uncertainty Propagation and Computational Efficiency



Spacecraft Robotic Manipulator Analysis and Control using Dual Quaternions

- Provide unified framework enabling kinematic and dynamic analysis of robotic manipulators on rigid bodies using dual quaternions (DQs).
- Dual quaternions capture the combined rotational and translational motion.

$$\boldsymbol{q}_{\mathrm{B/A}} = q_{\mathrm{B/A}} + \epsilon \frac{1}{2} q_{\mathrm{B/A}} r^{\mathrm{B}}_{\mathrm{B/A}}$$

$$\boldsymbol{q}_{\mathrm{B/A}} = q_{\mathrm{B/A}} + \epsilon \frac{1}{2} r^{\mathrm{A}}_{\mathrm{B/A}} q_{\mathrm{B/A}}$$

DQ Kinematics:

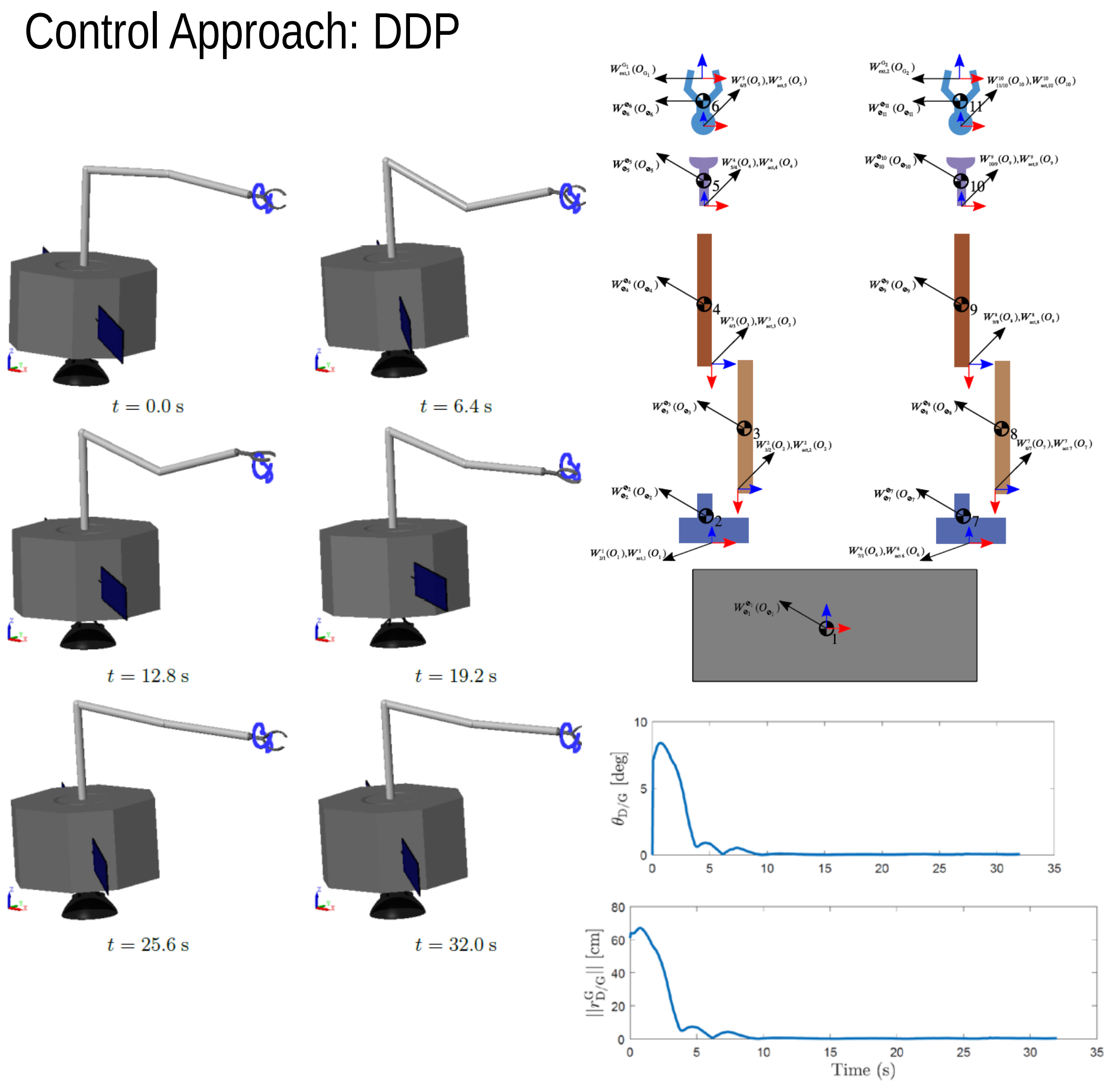
$$\dot{\boldsymbol{q}}_{\mathrm{X/Y}} = \frac{1}{2} \boldsymbol{q}_{\mathrm{X/Y}} \boldsymbol{\omega}^{\mathrm{X}}_{\mathrm{X/Y}} = \frac{1}{2} \boldsymbol{\omega}^{\mathrm{Y}}_{\mathrm{X/Y}} \boldsymbol{q}_{\mathrm{X/Y}}$$

$$\boldsymbol{\omega}^{\mathrm{X}}_{\mathrm{Y/Z}} = \boldsymbol{q}^*_{\mathrm{X/Y}} \boldsymbol{\omega}^{\mathrm{Y}}_{\mathrm{Y/Z}} \boldsymbol{q}_{\mathrm{X/Y}} = \omega^{\mathrm{X}}_{\mathrm{Y/Z}} + \epsilon (v^{\mathrm{X}}_{\mathrm{Y/Z}} + \omega^{\mathrm{X}}_{\mathrm{Y/Z}} \times r^{\mathrm{X}}_{\mathrm{X/Y}})$$

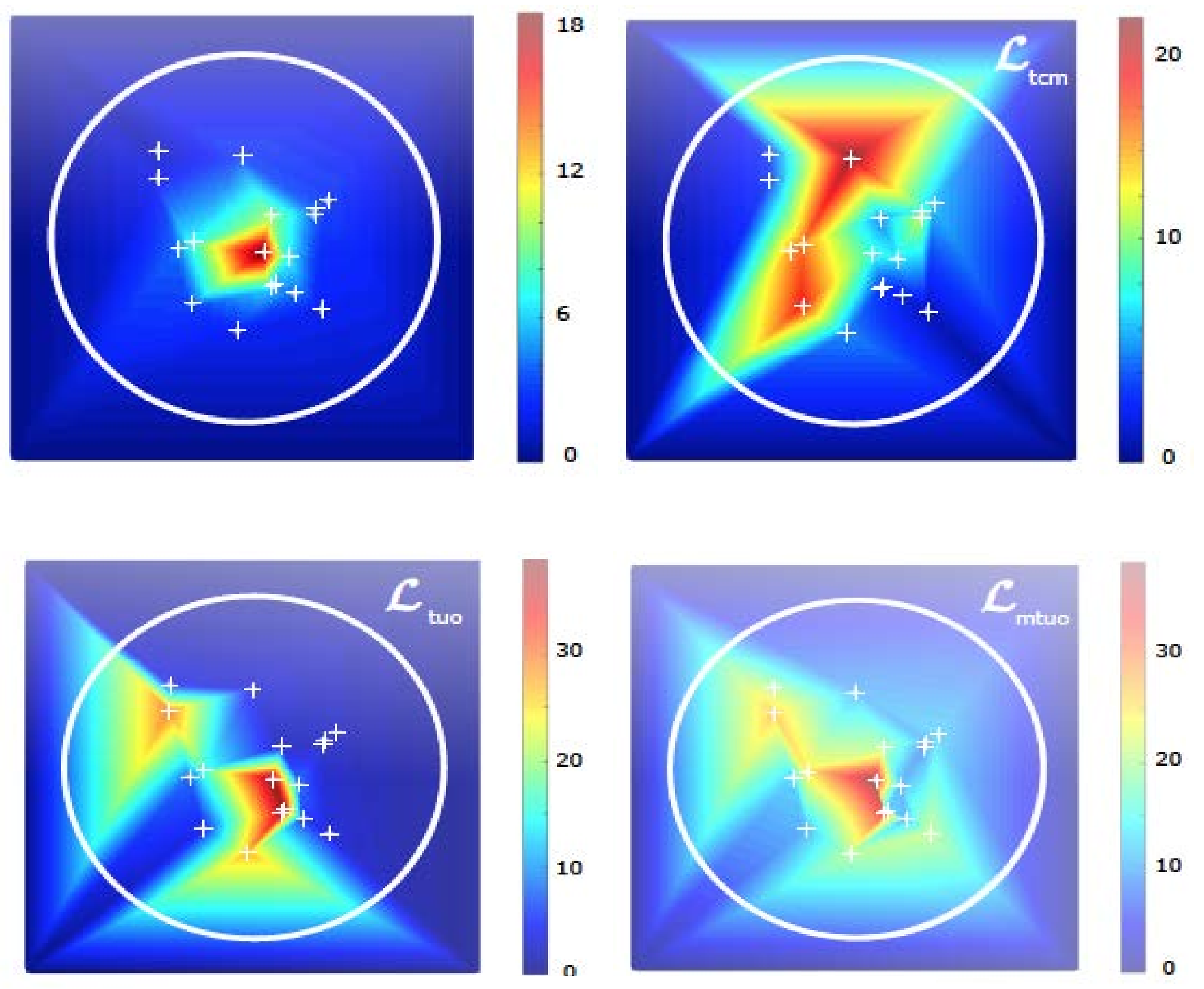
DQ Dynamics: Based on DQ form of Newton-Euler equations

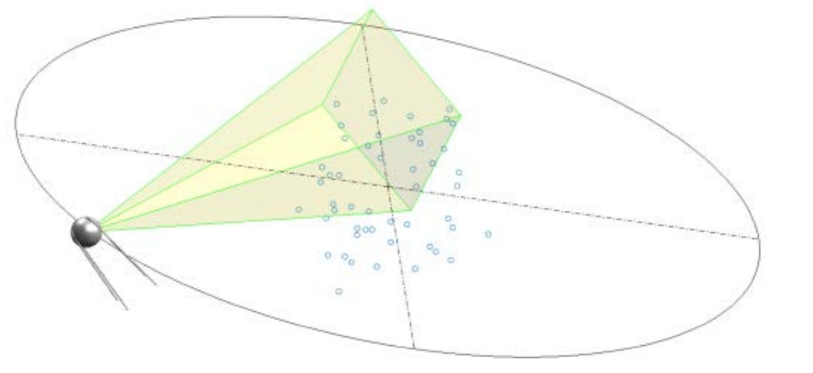
$$M_{\circ_{\mathrm{i}}} \star (\dot{\boldsymbol{\omega}}^{\circ_{\mathrm{i}}}_{\circ_{\mathrm{i}}/\mathrm{I}})^{\mathrm{s}} + \boldsymbol{\omega}^{\circ_{\mathrm{i}}}_{\circ_{\mathrm{i}}/\mathrm{I}} \times \left(M_{\circ_{\mathrm{i}}} \star (\boldsymbol{\omega}^{\circ_{\mathrm{i}}}_{\circ_{\mathrm{i}}/\mathrm{I}})^{\mathrm{s}}\right) = \boldsymbol{W}^{\circ_{\mathrm{i}}}_{\mathrm{i}}(O_{\circ_{\mathrm{i}}}),$$

Control Approach: DDP



Trajectory optimization using parameterized control policies



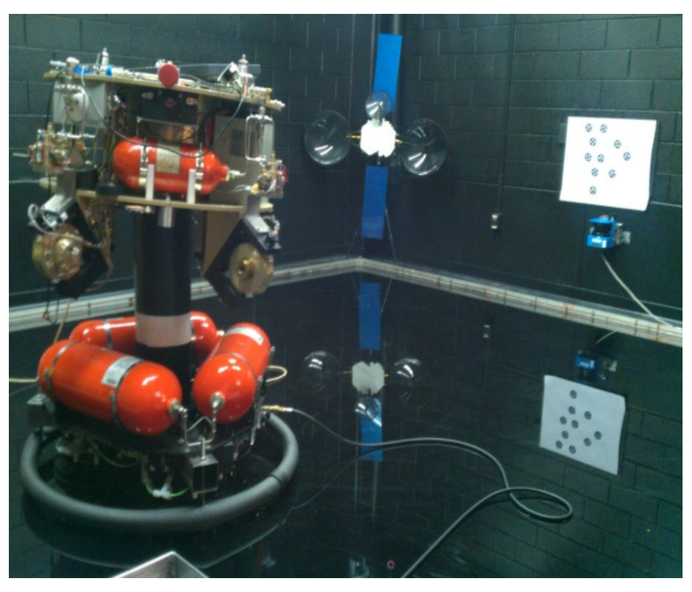


Landmark potential map

- Relative circumnavigation of target satellite
- Goal: maximize time allocated to landmark observation
- Three cost functions
  - Time under observation (TUO)
  - Trace of covariance matrix (TCM)
  - TUO and no. of different landmarks observed
- By jointly considering the planning, control and estimation it is possible to balance control actuation costs and localization uncertainty
- Extension to 3D case using C-W relative orbit equations

Experimental Validation

5DOF simulator for ARD



Control Room

Sensors

- 4 VSCMGs, 12 Cold Gas Thrusters
- IMU, 3 axis Rate Gyro, 3 axis Magnetometer
- CCD and 3D Range Camera
- VICON motion capture system

Ongoing and future work

- Ongoing: Performing experiments with Probabilistic Trajectory Optimization using GP in Belief Space.
- Future: Performing experiments using either sparse GPs or semi-parametric representations.

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