Logical Clustering and Learning For Time-Series Data

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- Problem: Want to group together "similar" traces (time series).
- New Problem: What does "similar" mean?
- Key Idea: Use Parametric Signal Temporal (PSTL) to design Features.

Toy Example

Task: Organize based on how successful the lane change at t = 10 was

PSTL Template

$$\varphi(\tau, a) = F\left(\text{lane\_change}(t) \land F_{(0,\tau]}(x(t) - x_{\text{ref}(t)}) > a\right)$$

Example Lane Change

Lexicographic Projections
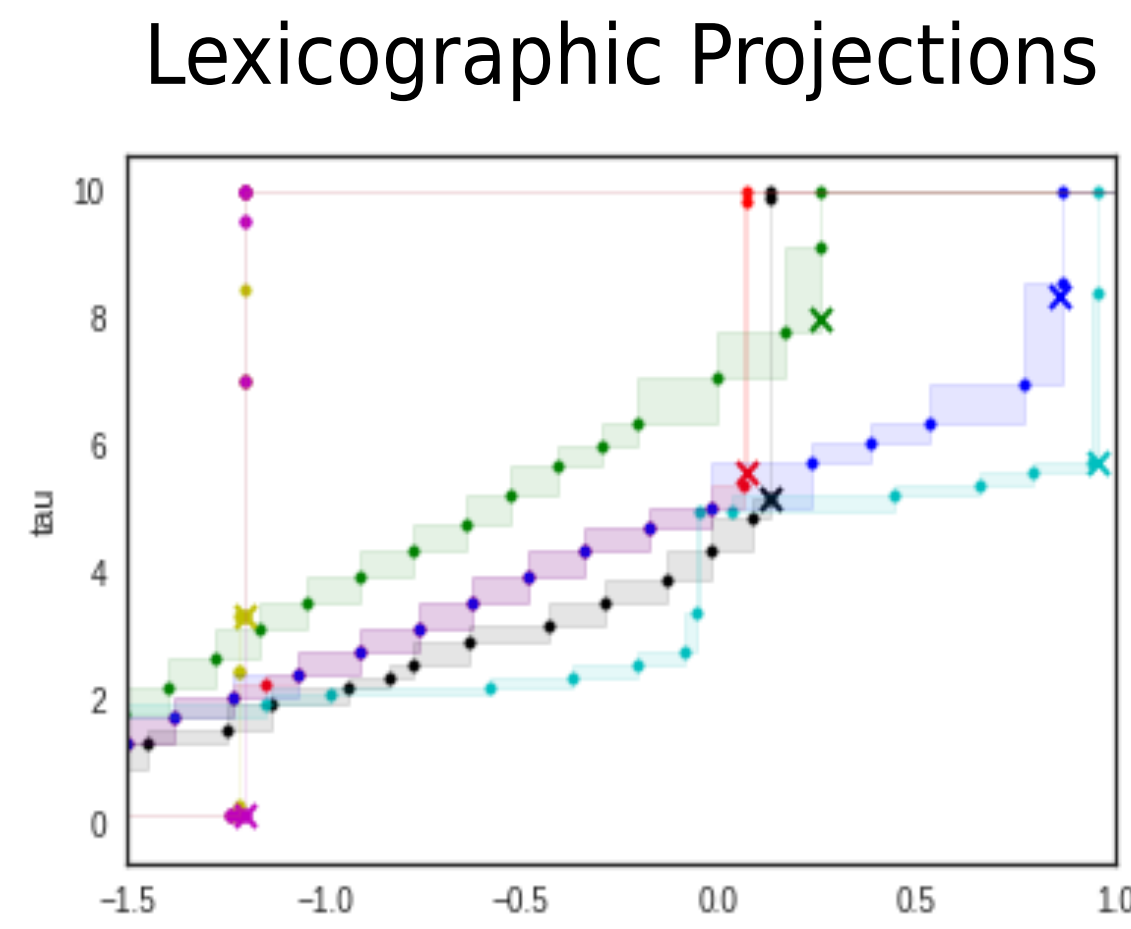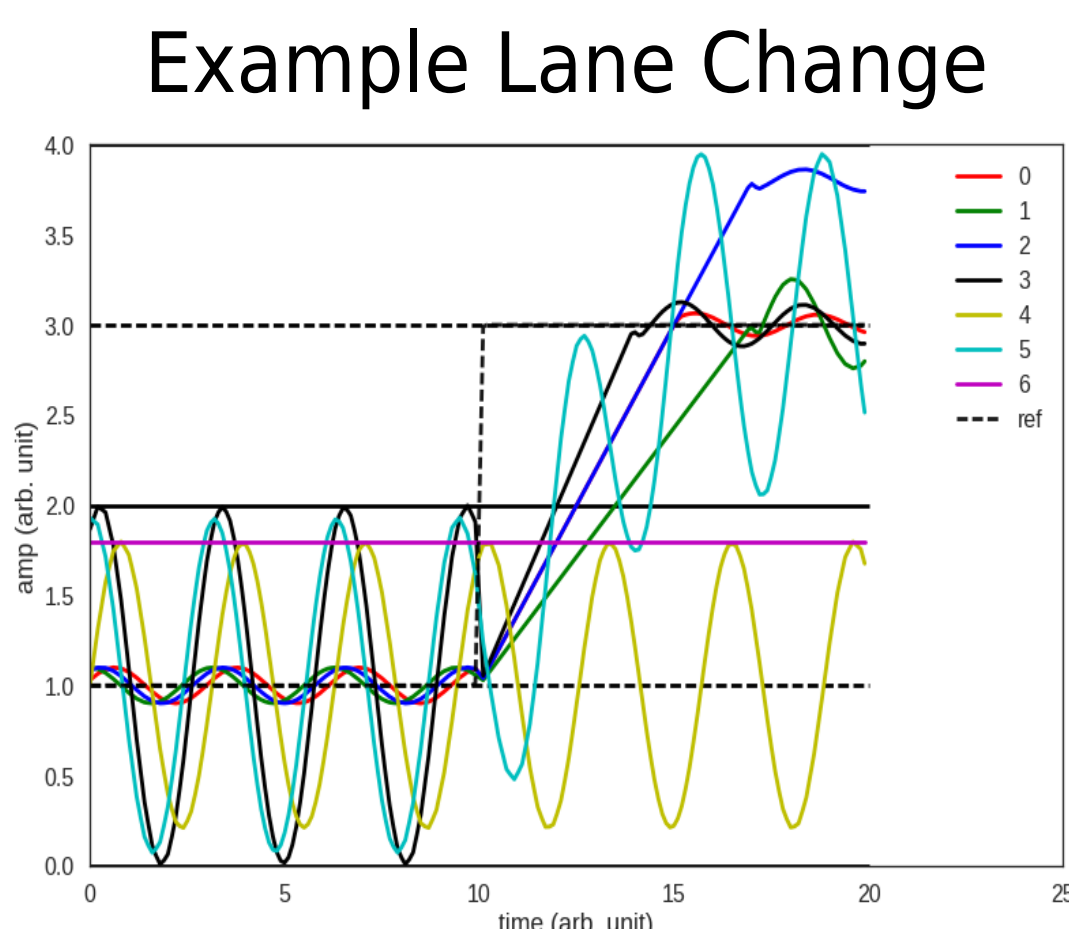
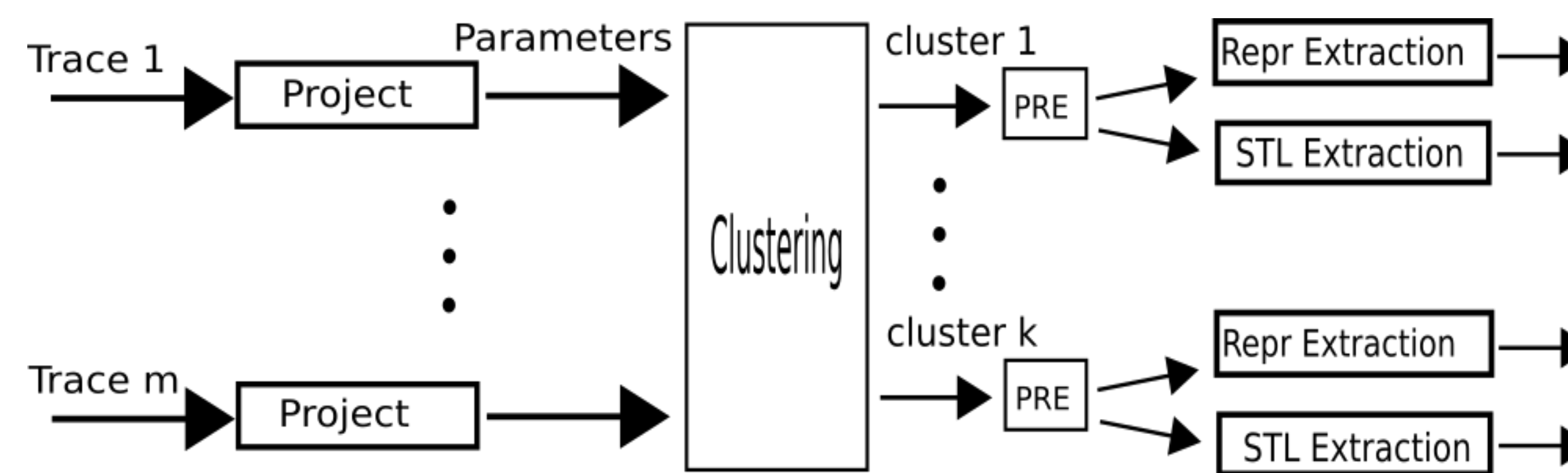The behaviors 3 behaviors exhibited after the lane change command.

1. Ignore Lane Change
2. Overshoot Kane
3. Correctly Lane Change

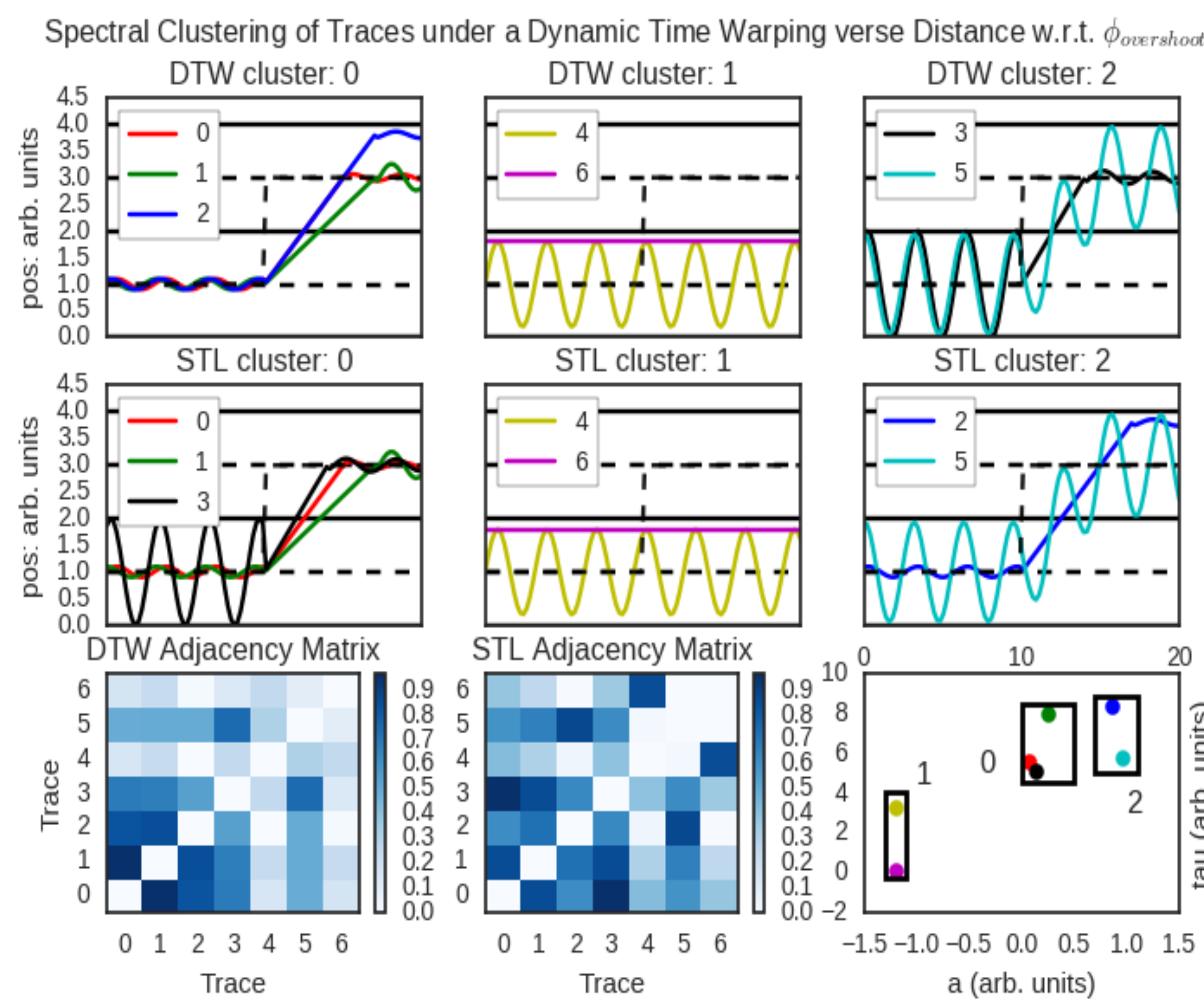Approximations of the boundary of satsifaction for each trace.

The X marks the lexicographic optima for each boundary.

We compare with the dynamic time warping distance of each trace (a common feature agnostic way to measure shape similarity of time series).

Overview

cluster 1 → Repr Extraction / STL Extraction

cluster k → Repr Extraction / STL Extraction

Results of Spectral Clustering

Spectral Clustering of Traces under a Dynamic Time Warping verse Distance w.r.t. $\phi_{overshoot}$

DTW cluster: 0   DTW cluster: 1   DTW cluster: 2

STL cluster: 0   STL cluster: 1   STL cluster: 2

DTW Adjacency Matrix   STL Adjacency Matrix

Cluster Bounding Box to STL

$$\psi_B \triangleq \varphi(\nu_w) \land \bigwedge_{\nu \in E_B} \neg\varphi(\nu)$$

$$\text{Red Black Green} \triangleq \varphi(8,0) \land \neg\varphi(4,0) \land \neg\varphi(8,0.5)$$
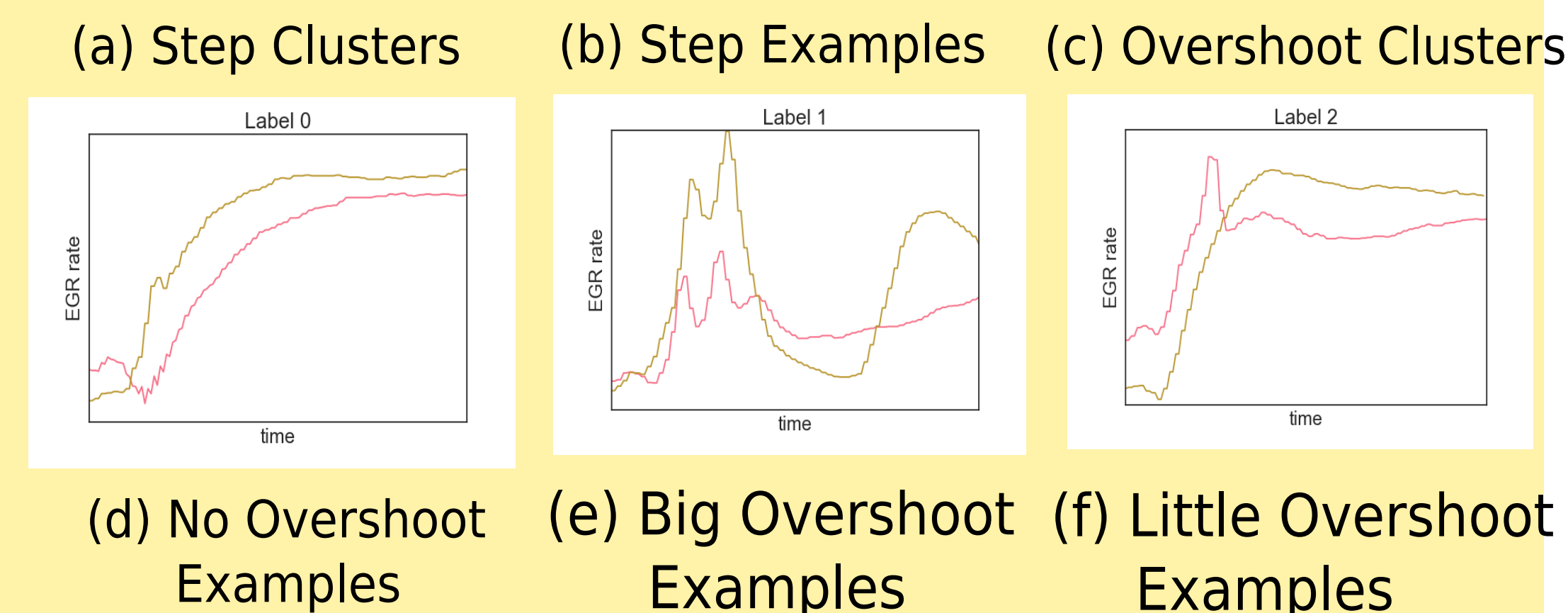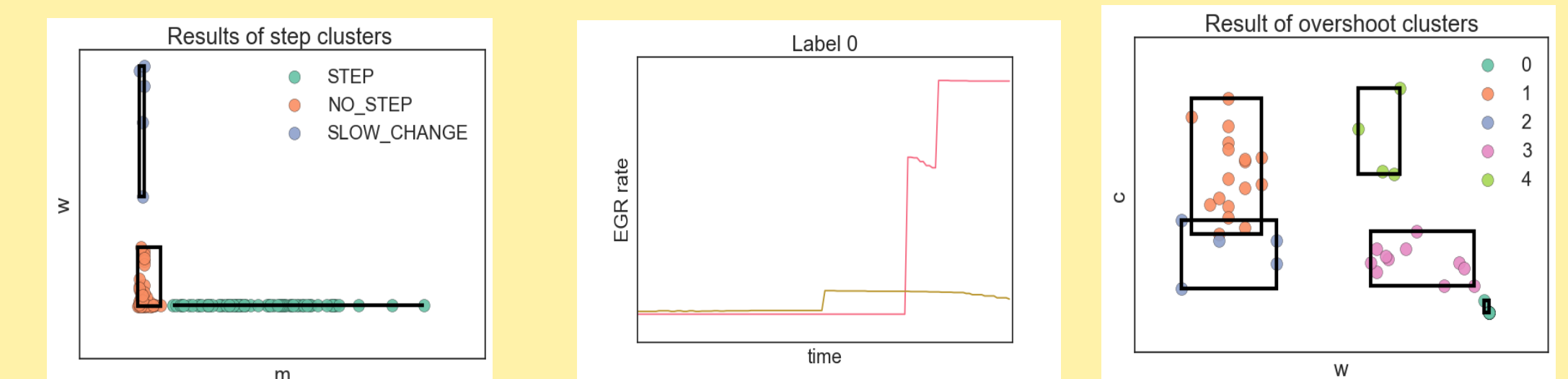
E_B is the set of "essential" corners

Case Studies

Diesel Engine

Task: Characterize overshoots due to step in reference flow rate

Overshoot Template same as Toy Example

Step Template: $F\left(\ddot{x} > m \land F_{[0,w]}(\ddot{x}) < -m\right)$

(a) Step Clusters   (b) Step Examples   (c) Overshoot Clusters

(d) No Overshoot Examples   (e) Big Overshoot Examples   (f) Little Overshoot Examples

Highway 101

Task: Characterize Lane Weaving on Highway 101

$$\bigvee_i (x \neq L_i) \land \left(F_{(0,\epsilon]}\left((x = L_i)U_{[\epsilon,\tau_i]}(x \neq L_i)\right)\right)$$

C	τ_2	τ_3	τ_4	T	|C|
0	T	T	T	T1	626
1	T	T	[⊥,78]	T2	115
2	[⊥,67]	T	T	T2	44
3	T	[.26, 30]	[⊥,65]	T3	52
4	T	[⊥,70]	T	T2	32
5	[.56, 54]	[⊥,40]	T	T3	14
6	[.56, 32]	[1.8, 24]	[⊥,31]	T4	12
7	T	76	T	T1	1

(a) Lane Clusters

(b) Examples from lane (c) Aggressive Behavior cluster C= 1