



# CPS: Medium: Collaborative Research: Multi-Objective Mitigation Strategies for Viability and Performance of Cyber-Physical Systems

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## Challenge:

This project aims to bring control theory, automotive & aerospace application domain-knowledge, and real-time resource allocation, to bear on the problem of run-time mitigation when complex CPSs operating in uncertain environments confront with unanticipated viability-compromising situations.

## Proposed solution:

Establish a systematic, general, and broadly applicable framework to approach failure mode effects management (FMEM) development for advanced autonomous systems:

- **Fault Detection:** Faults should be detected as soon as they happen (or are potentially likely to happen)
- **Multi-Mode Control:** Pointwise-in-time state and control constraints should be enforced at all times despite the presence of faults.
- **Effective Resource Reallocation:** Computing resources should be appropriately reallocated in a timely manner upon detection of faults to executing the corresponding control strategies
- **Mitigation Coordination:** Once a fault is detected, a set of strategies should be provided to the multi-mode control module and the resource allocator to assure system viability.

## Scientific impact:

- Providing a framework for formalizing the concept of “continuing operation” in the face of a variety of contingencies.
- Defining a set of mitigation strategies that addresses a broad range of contingencies that arise in several important applications.
- Obtaining a better understanding of emergent problems that arise from the interaction of resource-allocation and control strategies, and propose devising optimal (or near-optimal) strategies for dealing with such problems with guaranteed performance in the presence of uncertainty.

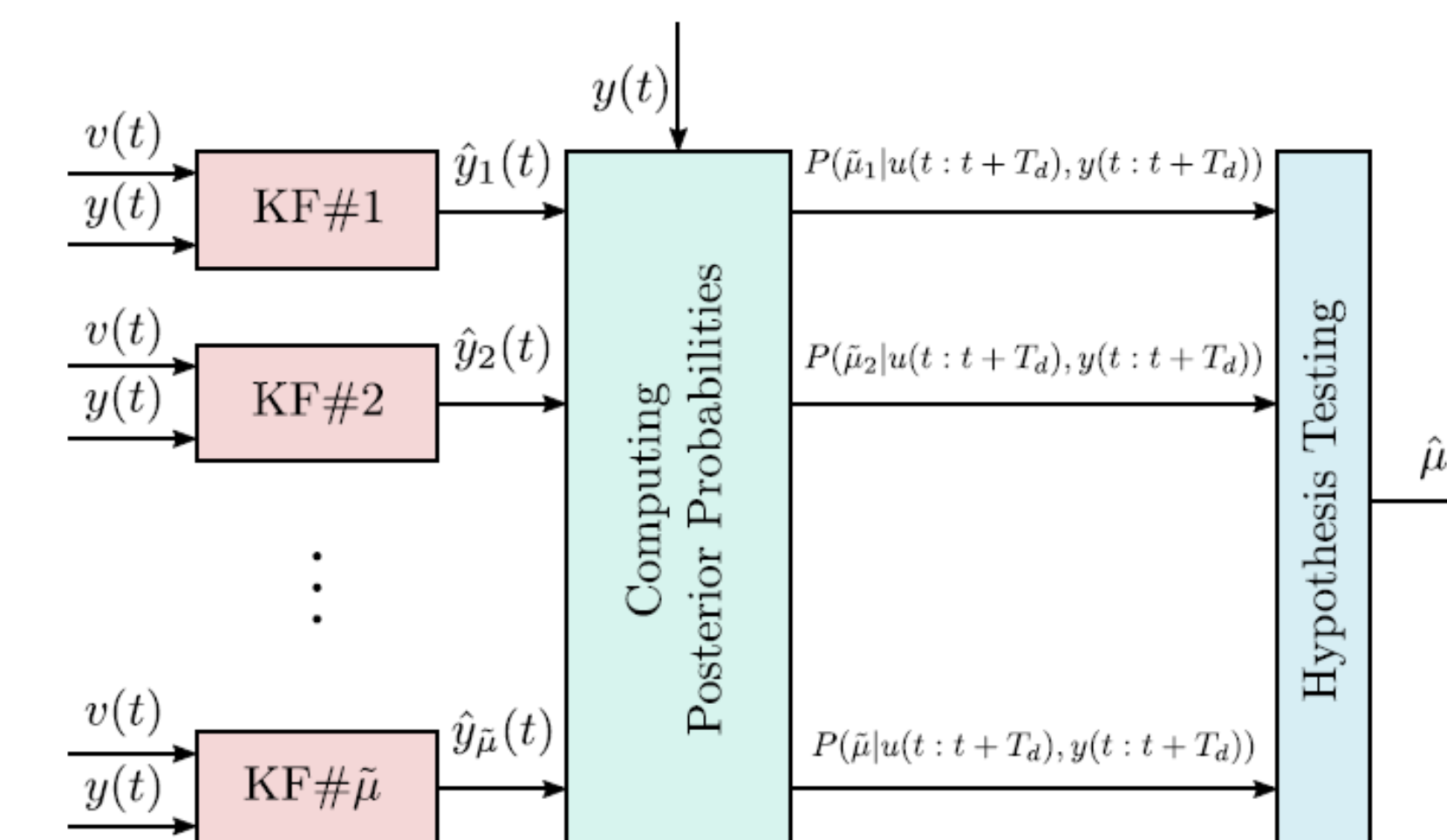
## Broader impact:

- Analysis techniques, algorithms and implementation methodologies developed in this project can support the automotive industry to develop safe advanced/autonomous vehicles.
- This project provides unique opportunities for students to develop expertise across the different areas.

## Proposed framework:

### Fault Detection

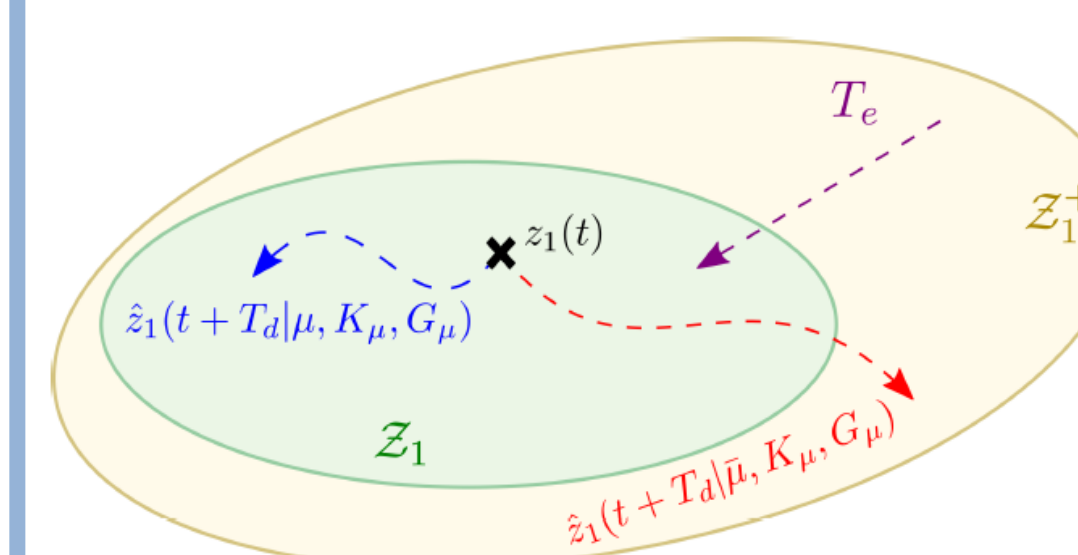
- LTI systems with  $F$  fault scenarios can be expressed as  $F$  distinct LTI systems with different system matrices.
 
$$\begin{cases} x(t+1|\mu) = A_\mu x(t|\mu) + B_\mu v(t) + \omega(t) \\ y(t|\mu) = C_\mu x(t|\mu) + \zeta(t) \end{cases}$$
- When the system is operating in mode  $\mu$ , its state and reference have to satisfy constraints
 
$$\begin{cases} \mathbb{E}[z_1(t|\mu)] \in Z_1 \\ \mathbb{P}(z_2(t|\mu) \in Z_2) \geq \beta \end{cases}$$
- The Multi-Model Adaptive Estimator (MMAE) uses residual signals to identify the actual mode of the system.
 
$$\hat{\mu} = \arg \max_{\hat{\mu}} \mathbb{P}(\hat{\mu} | y(t:t+T_d), v(t:t+T_d-1))$$
 with  $y(t:t+T_d) = [y(t)^T \dots y(t+T_d)^T]^T$  and  $v(t:t+T_d-1) = [v(t)^T \dots v(t+T_d-1)^T]^T$



- The control sequence  $v(t:t+T_d-1)$  can be determined via a suitably designed optimization problem so to improve detection performance without violating constraints

$$v^* = \arg \min_v \sum_{\hat{\mu}} \sum_{\tilde{\mu}} \sqrt{\mathbb{P}(\hat{\mu}) \mathbb{P}(\tilde{\mu})} e^{-\rho \hat{\mu} \tilde{\mu}}$$

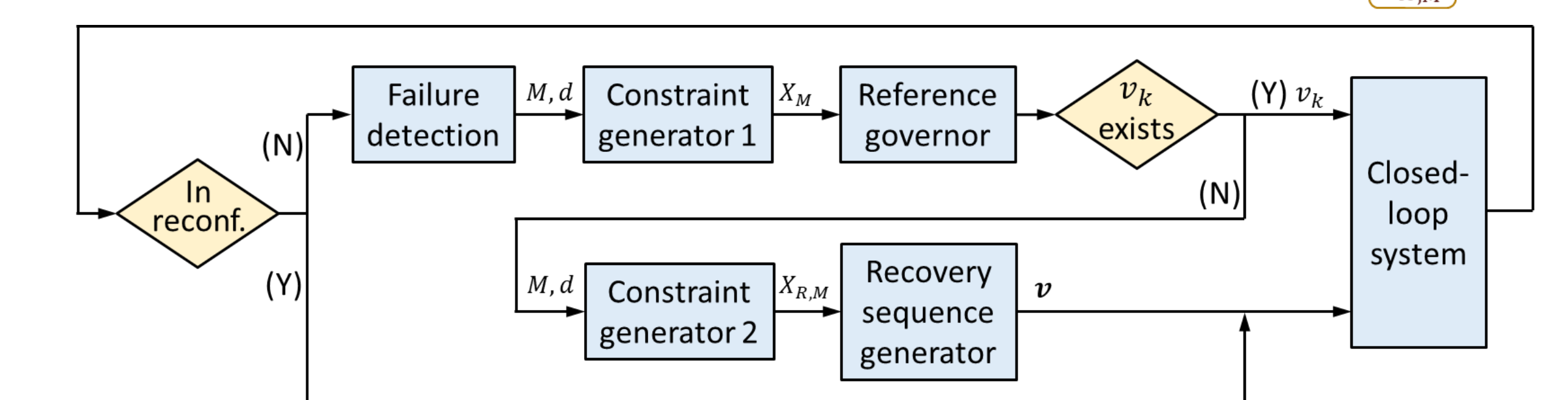
with  $\rho \hat{\mu} \tilde{\mu}$  as a quadratic function in the control sequence  $v(t:t+T_d-1)$ .



### Multi-Mode Control

#### Set-theoretic failure mode reconfiguration

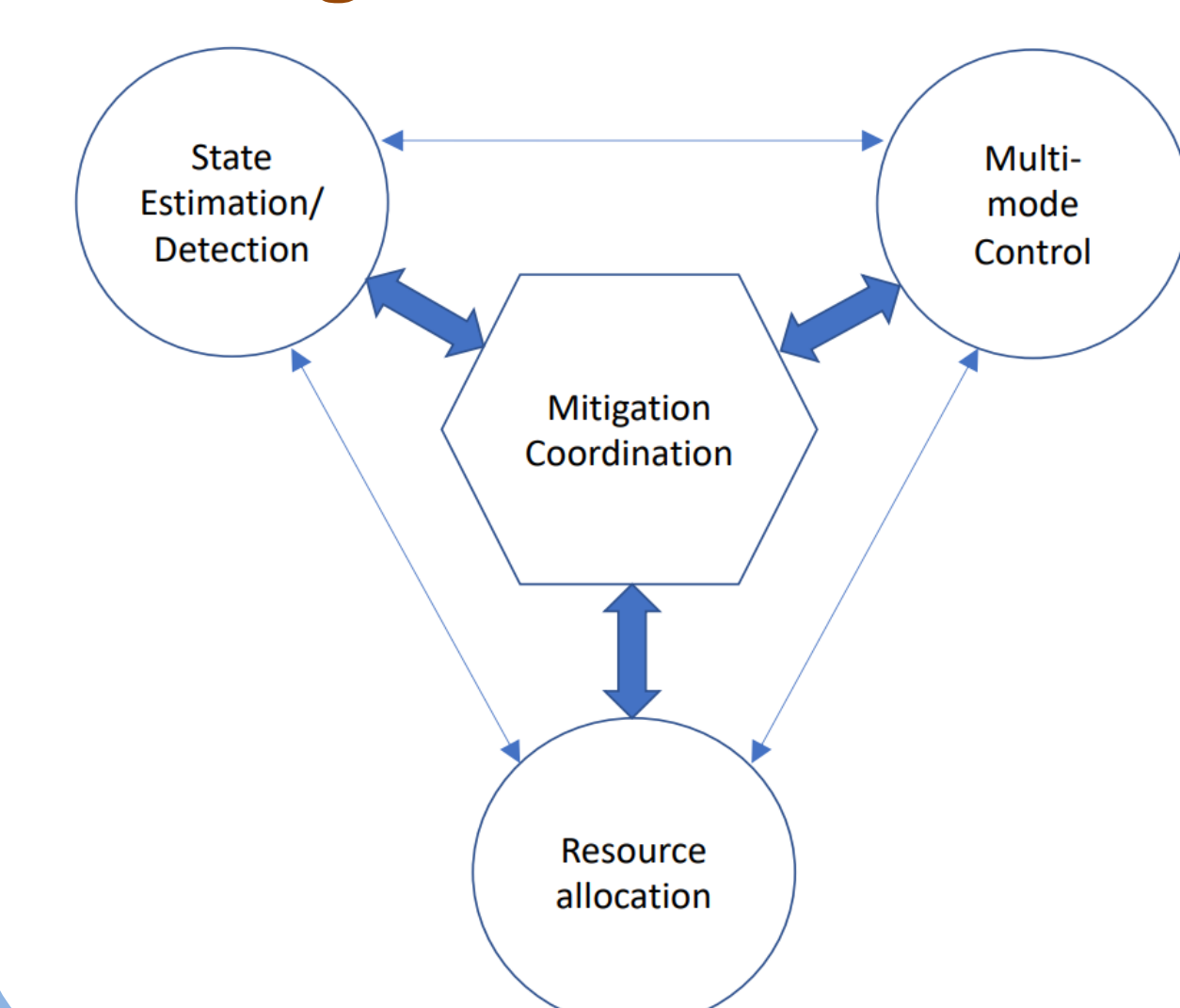
- Exploits nesting between constraint admissible sets  $O_{\infty, M}$  and recoverable sets  $R_{\infty, M}^{N_M}$  to ensure there exists a recovery sequence  $v = \{v_k, \dots, v_{k+N_M}\}$ .
- Reconfiguration condition for mode  $M$ :
 
$$\text{Proj}_X O_{\infty, M'} \subseteq R_{\infty, M}^{N_M} \quad \forall M' \in \text{pred}(M)$$
- Apply reference governor for reference tracking while imposing constraints.



#### Viability maximization and failure mode management

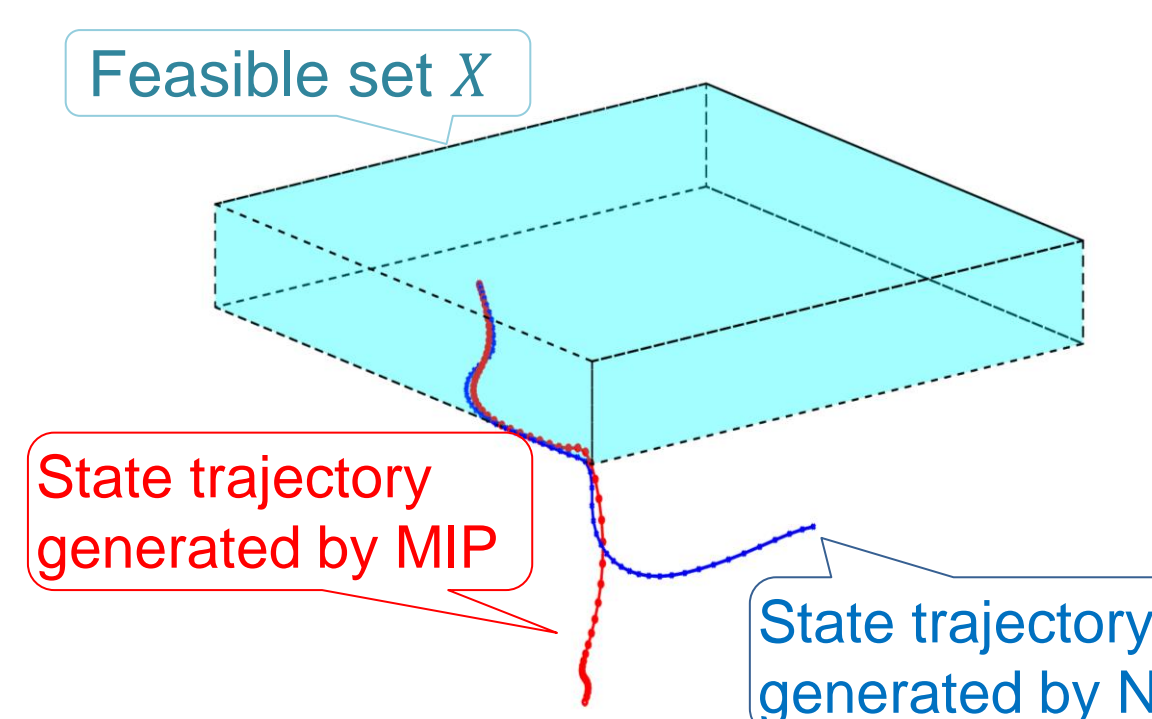
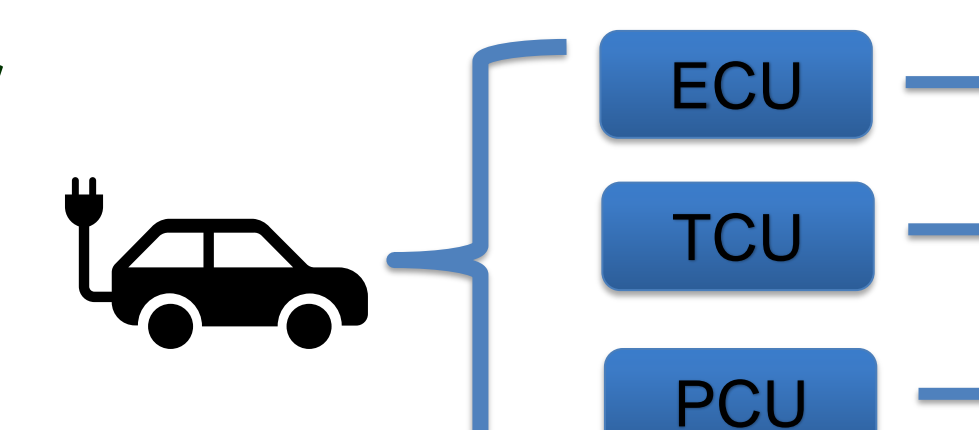
- State trajectory may eventually drift outside a desired operating region  $X$ .
- Maximize the time before trajectory exiting operating region  $X$  (violating constraint).

### Mitigation Coordination



### Resource Allocator

Responsible for marshalling and allocating the available platform computing resources



Continuous nonlinear programming (NLP) approach based on **exponential weighting**

$$\begin{aligned} \min_{\{u_k\}_{k=0}^{N-1}, \{\varepsilon_k\}_{k=0}^N} & \sum_{k=0}^N \theta^{N-k} \varepsilon_k \\ \text{Subject to } & x_{k+1} = f_d(x_k, u_k) \\ & u_k \in U \\ & 0 \leq \varepsilon_k \leq \varepsilon_{k+1} \\ & H(x_k) \leq h + 1M\varepsilon_k \end{aligned}$$

- Same “time-before-exit”
- Significantly improved computation efficiency over mixed-integer programming (MIP) approaches