Modeling and Maximizing Power for Wind Turbine Arrays

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Introduction

We consider the problem of maximizing power extraction from wind turbine arrays, and using partitioning and control-theoretic design methods. Wind turbine arrays can be viewed as large coupled networks, for which the application of traditional optimization techniques are impractical. In our work, we developed an extension to a dynamic programming solution previously developed under uniform wind and extend it to higher-fidelity wind models. We then update our solution for dynamically evolving wind conditions. Using a Markov model derived from real-world data, the underlying optimization problem is reformulated in a Model Predictive Control framework.

Figure 1: Image of the Rush Creek onshore wind farm in Colorado

High Fidelity Wind Farm Model

Given uniform wind with magnitude U_∞, the high-fidelity FLORIS wake model states that each turbine affects wind velocity downstream by a diminishing multiplicative factor

$$V_i = U_\infty(1 - \Delta V_i),$$

with

$$\Delta V_i = 2u_i \left(\frac{D}{D + 2K_e(u_i)m_{wake}\Delta d}\right)^2$$

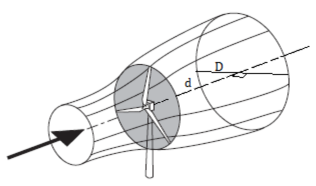

Figure 2: Wind propagation through a turbine

Coefficients K_e and m_{wake} are obtained by a wind farm simulator: SOWFA. At a given location in the intersection of wakes 1…k, the aggregate wind velocity is determined by

$$V_i = U_\infty \left(1 - 2\sqrt{\sum_{i=1}^{k}(p_i \Delta V_i)^2}\right)$$

The power P_i, extracted by the ith turbine is proportional to the cube of the wind velocity, correlation factor η, and corresponding axial induction factor u_i,

$$P_i(u_i) = C_i u_i (1 - u_i)^2 V_i^3 \eta$$

The axial induction factor setting serves as the exogenous control variable, and affects both rotational speed of the turbine and the resulting power extracted. The design goal is to optimize the total power extracted from the wind turbine array.

Varying Wind Dynamics and Control

Our prior work focused on static wind velocities. estimate varying wind dynamics as a Markov chain. Using real-world data, we built a stochastic transition matrix $\hat{P}$ to predict future wind velocities. We then are able to construct a finite-horizon Model Predictive Control optimization problem.

We seek to find the axial induction factors over a finite horizon that maximize the power extraction over an entire wind farm. This maps to the MPC problem

$$\max_{u(t)\ldots u(t+T)} \sum_{\tau=t}^{t+T} P_{total}(v(\tau), u(\tau))$$

$$u(\tau) \in \mathcal{U}$$

$$\vec{v}(\tau+1) = \vec{v}(\tau)\hat{P}$$

$$L(v(t+T), u(t+T)) \leq L(v(t), u(t)) \quad \forall[t, t+T]$$

We extended our Bellman-type approach derived for unchanging wind dynamics to solve for the axial induction factors of each turbine in the form of a planning problem:

- An initial set of axial induction factors are obtained, beginning with the farthest downstream turbine and are then calculated for each of the preceding upstream turbines under unchanging wind dynamics.
- Next, axial induction factors for each turbine are scheduled based on the maximum likelihood estimator applied to $\hat{P}$.
- This process is repeated under a finite horizon, updating each turbine in succession if the actual wind dynamics vary from our estimation outside of our control horizon.

Additionally, we are extending an LPV framework to use H∞ control theory to build a robust distributed controller.

$$G: \begin{cases} \dot{x} = A(\theta)x + B_v(\theta)\ddot{V} + B(\hat{\theta})\ddot{\Omega}_z, \\ \hat{T}_s = C_t x, \\ y = Cx + Du, \end{cases}$$

$$x = [\hat{\theta}_s \ \ \hat{\Omega}_r \ \ \hat{\Omega}_g]^T$$

$$y = [\hat{\Omega}_g \ \ \hat{T}_g]^T$$

$$\theta = [\bar{V} \ \ \bar{\Omega}]^T$$

$$A(\theta) = \begin{bmatrix} 0 & 1 & -1 \\ -\frac{K_s}{J_r} & -\frac{B_r(\bar{\Omega},\bar{V}) + B_s}{J_r} & \frac{B_s}{J_r} \\ \frac{K_s}{J_g} & \frac{B_s}{J_g} & -\frac{B_s + B_g}{J_g} \end{bmatrix},$$

$$B_v(\theta) = \begin{bmatrix} 0 & \frac{k_{r,V}(\bar{\Omega},\bar{V})}{J_r} & 0 \end{bmatrix}^T,$$

$$B(\theta) = \begin{bmatrix} 0 & 0 & \frac{B_g}{J_g} \end{bmatrix}^T,$$

$$C_t = [K_s \ \ B_s \ \ -B_s],$$

$$C = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & B_g \end{bmatrix},$$

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Simulations

We can demonstrate that our methods provide a higher power extraction when compared to simple control schemes.

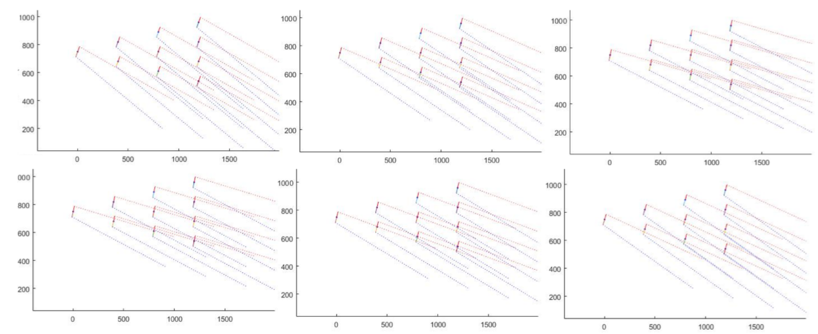

Figure 3: Simulated 10-turbine tree dynamics evolving over time using a Markov Chain obtained using real world data.

We demonstrated that our methodology improves on Persistence (the wind estimation model that wind velocities are identical across timesteps) and Greedy (local agent maximization) across the 10-turbine tree.

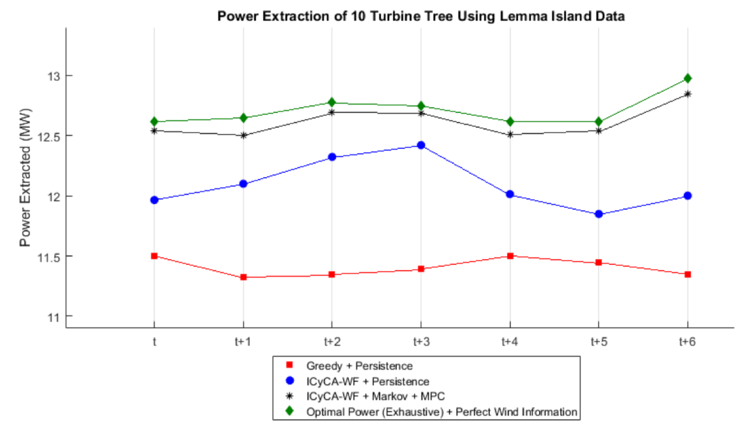

Figure 4: Power Extraction comparison for the 10-turbine tree using real world data. Optimal power comparison found via exhaustive search.

Conclusion

As we have shown, it is possible to determine an algorithm that terminates to a solution that can find control schemes to improve on current simple methods. Future work seeks to use distributed control designs to further improve validity of the results.

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