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Multi-Dimensional Forward Contracts under Uncertainty for Electricity Markets

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***Abstract*—We consider mechanism design problems for strategic agents with multi-dimensional private information and uncertainty in their utility/cost function. We show that the optimal mechanism with firm allocation can be implemented as a nonlinear pricing scheme, and the optimal mechanism with random allocation can be implemented as a menu of nonlinear pricing schemes. We provide two examples to demonstrate the results: an optimal energy procurement mechanism from a strategic seller with renewable (random) generation, and the design of an optimal demand response program for a network of heterogeneous loads.**

***Index Terms*—renewable energy, demand response, contract under uncertainty, electricity market**

I. Introduction

In recent years, electricity markets have undergone profound structural changes both in the generation and the demand side. The traditionally monopolistic government-regulated markets reformed toward liberalized electricity markets in order to introduce competition and increase efficiency in generation [19]. Privately-owned generators and utility companies possess private information about their cost/utility, behave strategically, and seek to maximize their profit. Moreover, the developing network of smart grids aims to utilize the available flexibility on the demand side to increase the efficiency of the grid. To involve the demand side actively into the operation of the grid, one needs to design appropriate mechanisms that incentivize the demand to exercise flexibility in its consumption behavior.

Long-term contracts, as an agreement between strategic parties with private information, is one of the main trading mechanisms used in electricity markets. Generators and utility companies sign long-term contracts to hedge themselves against the risk of pooling markets. In fact, it has been suggested that long-term contracts are necessary along with the existing pooling markets to ensure the stability and reliability of electricity markets [2].

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Contracts have been considered as one of the main mechanisms to induce a desired behavior on the demand side of smart grids. In comparison to real time pricing or direct market participation, contracts with incentive payments result in a direct control of resources, and thus, give reliability and stability guarantees [11]. Furthermore, contracts with incentive payments are simpler to implement and more appealing to smaller market participants (*e.g.* households) [16].

In this paper, we study a general contract design problem for electricity markets in a principal-agent (buyer-seller(s)) setup. We assume that both the buyer and the seller sides have multi-dimensional private information that and general utility/cost functions. Furthermore, we explicitly consider a general uncertainty in our problem formulation; uncertainty is becoming a critical issue in electricity markets.

As the share of intermittent generation from renewable generation increases, the uncertainty in the available generation will increase. Furthermore, the added flexibility on the demand side in smart grids also means a higher uncertainty on demand; such uncertainty should be properly managed through appropriately designed incentives. In general, both the buyer and the seller may have uncertainty, either in their cost/utility functions, or the availability of the resources being traded between them. By explicitly including uncertainty into our problem formulation we capture these facts and can address the problem of commitment (ex-post voluntary participation), risk sharing, and forward contracts with random allocation.

The problems formulated in this paper enable us to capture and analyze interaction between energy consumers and renewable energy generators, as well as interactions between an aggregator and a network of a demand population participating in the demand response program. We provide examples for each of these scenarios so as to illustrate our results.

A. Related Literature

There is a growing literature on contract design for electricity with information asymmetry and strategic behavior. A contract design problem for demand management with one-dimensional private information and linear utility has been studied in [6]. The work in [1] addresses the problem of contract design for deferrable demands with constant marginal utility for demand. The work in [4] considers a mechanism design problem for the forward reserved market assuming that the participants have constant marginal cost and no market power. Although the private information in [1], and [4] is multi-dimensional, the simplifying assumption of constant marginal cost/utility enables the authors to rank different types, and is critical to the solution approaches they provide. The specific structures of utility/cost functions assumed in [1], [4], and [6] enable the authors to provide solutions that are inspired by the solution methodology of the one-dimensional screening problem. Contract design problem for demand response with quadratic cost functions is investigated in [7] by numerical methods. The work in [14] considers a mechanism design problem for energy procurement with a general utility/cost function and uncertainty and applies a Vickery-Clacks-Gloves (VCG) based mechanism. However, the VCG mechanism is suboptimal for the problem formulated in [14] when the cost function cannot be parameterized by only a one-dimensional type (see [8], Ch. 14).

From the economics point of view, the problem we formulate in this paper belongs to the class of screening problems. In economics, the one-dimensional screening problem has been well-studied with both linear and nonlinear utility functions [3]. However, the extension to the multi-dimensional screening problem is not straightforward and no general solution is available. The authors in [9] study a general framework for a deterministic multi-dimensional screening problem with linear utilities. They discuss two general approaches, the parametric-utility approach and the demand-profile approach. The methodology we use to solve the problem formulated in this paper is similar to the demand-profile approach. We consider a multi-dimensional screening problem under uncertainty with nonlinear utilities. The presence of nonlinearities and uncertainty results in additional complications that are not present in [9] where the utilities are linear and there is uncertainty[2].

B. Contribution

The contribution of this paper is two-fold. First, we consider an optimal contract design problem for electricity markets with utility/cost functions that are more general than those considered in the literature ([1], [4], [6], [7]). The nature of utility/cost functions with multi-dimensional private information is such that the solution methodology presented in [1], [4], and [6] does not extend to our problem. The generality of our model enables us to capture many instances of problems arising in electricity markets. Two such instances are discussed in Section IV and VI.

Second, we explicitly incorporate a general uncertainty in the realized cost/utility of the buyer and the seller. The presence of uncertainty along with the nonlinearity of the utilities result in problems where the methodology used in previous works ([1], [4], [6]) cannot be applied, as in these works the utilities are linear and any uncertainty can be replaced by its expected value. The inclusion of uncertainty is crucial in the modeling and analysis of emerging electricity markets because: (1) the share of renewable generation increases; (2) the existing demand becomes less shielded from the market outcome and more elastic; and (3) new resources/loads (e.g. storage, plug-in electric vehicles) enter the market. Due to uncertainty, firm forward contracts (a priori fixed allocation and fixed payment) do not appear to be an appropriate form of contract for emerging electricity markets. Moreover, in the presence of uncertainty, interim voluntary participation (defined in Section III) of the seller does not necessarily imply ex-post voluntary participation of the seller (defined in Section V). Therefore, additional considerations are needed to ensure the commitment of the agents to the contract for every realization of the uncertainty. We show that, in general, the optimal mechanism for the problem formulated in this paper is a menu of nonlinear pricing schemes. We prove that by allowing the payment to depend on the uncertainty, we can achieve ex-post voluntary participation of the seller, and a desired risk-sharing (associated with the uncertainty) between the buyer and the seller. To the best of our knowledge, our results present the first optimal forward contract under uncertainty for electricity markets where the buyer and the seller have general utility/cost functions parameterized by multi-dimensional private information. We illustrate our results by providing two examples from electricity markets: an optimal demand response contract for ancillary service; and a bilateral trade between a buyer and a renewable energy generator.

C. Organization

The paper is organized as follows. We introduce the model in Section II. In Section III, we formulate and analyze an optimal forward contract with deterministic allocation, and address the problem of risk sharing between the buyer and the seller. We illustrate the result via an example for a contract design problem for demand response program in section IV. In Section V, we formulate and analyze an optimal forward contract with random allocation that depends on the uncertainty, and address the problem of the seller's imperfect commitment (ex-

[2]When a problem is linear, expectation of any random variable can be replaced by its expected value and reduce the problem to a deterministic one.

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post voluntary participation). We provide an example of a bilateral trade between a buyer and a renewable energy generator in Section VI. We discuss our results in Section VII. We conclude in Section VIII. In the Appendix we present the proofs of lemmas and corollaries appearing in the paper.

II. Model

A buyer wants to design a mechanism to procure energy/resource from a seller.[3] Let q be the amount of energy/resource the buyer procures, and t be his payment to the seller. The buyer's total profit is given by $\mathcal{V}(q)-t$, where $\mathcal{V}(q)$ is his utility by receiving q amount of energy/resource. The function $\mathcal{V}(\cdot)$ is the buyer's private information and $\mathcal{V}(0)=0$.

The seller's provision cost is given by $C(q,\mathbf{x},w)$, convex and increasing in q, $\mathbf{x}=(x_1,x_2,\cdots,x_n)\in\chi\subseteq\mathbf{R}^n$ is the seller's type, and w denotes the realization of a random variable W (uncertainty) with a probability distribution $F_W(w)$ that is common knowledge. We assume that $C(0,\mathbf{x},w)$ (zero-provision cost) does not depend on the realization of random variable w and is equal to x_1, *i.e.* $C(0,\mathbf{x},w)=C(0,\mathbf{x})=x_1$. The seller's utility is given by her total expected revenue $\mathbb{E}_W\left\{t-C(q,\mathbf{x},W)\right\}$. The seller's type $\mathbf{x}$ is her private information, the set χ is common knowledge, and there is a prior probability distribution $F_{\mathbf{x}}$ over χ which is common knowledge between the buyer and the seller.

Let $c(q,\mathbf{x}) := \frac{\partial\mathbb{E}_W\{C(q,\mathbf{x},W)\}}{\partial q}$ denote the expected marginal cost for the seller's type $\mathbf{x}$. We assume that $\exists m,\ 1<m\leq n$, such that $c(q,\mathbf{x})$ is increasing in x_i for $1\leq i\leq m$, and decreasing in x_i for $m<i\leq n$.[4] Moreover, there exists $\underline{\mathbf{x}}\in\chi$ (the seller's worst type) such that $\underline{x_i}\leq x_i$ and $\underline{x_j}\geq x_j$ for all $\mathbf{x}\in\chi$, $1\leq i\leq m$ and $m<j\leq n$.

Definition 1. *We say the seller's type $\mathbf{x}$ is better (resp. worse) than the seller's type $\hat{\mathbf{x}}$ if $c(q,\mathbf{x})\leq c(q,\hat{\mathbf{x}})$ for all $q\geq 0$ (resp. $c(q,\mathbf{x})\geq c(q,\hat{\mathbf{x}})$) with strict inequality for some q.*

Therefore, the seller's type $\mathbf{x}$ is better than the seller's type $\hat{\mathbf{x}}$ if and only if $x_i\leq\hat{x}_i$ for $1\leq i\leq m$, and $x_i\geq\hat{x}_i$ for $m<i\leq n$ with strict inequality for some i. The following example illustrates such ordering.

Example 1. *Consider an energy seller with a wind turbine and a gas generator. The generation from the wind turbine is free and given by γw^3, where γ is the turbine's technology and w is the realized weather. The gas generator has a fixed marginal cost θ_c. There is a fixed cost c_0 which includes the start-up cost for both plants and the capital cost for the seller. Therefore, the seller's type has $n=3$ dimensions. The generation cost for the seller is given by*

$$C(q,w,\mathbf{x})=c_0+\theta_c\max\left\{q-\gamma w^3,0\right\}. \tag{1}$$

The seller's type $\mathbf{x}=(c_0,\theta_c,\gamma)$ is better than the seller's type $\hat{\mathbf{x}}=(\hat{c}_0,\hat{\theta}_c,\hat{\gamma})$ if and only if $c_0\leq\hat{c}_0$, $\theta_c\leq\hat{\theta}_c$, and $\gamma\geq\hat{\gamma}$, with one of the above inequalities being strict.

Note that in the one-dimensional screening problem, the cost of production induces a complete order among the seller's types, which is crucial to the solution of the optimal mechanism design problem. However, in multi-dimensional screening problems, the expected cost of production induces, in general, only a partial order among the seller's types.

We assume that the buyer has all the bargaining power; thus, he can design the mechanism/set of rules that determines the agreement for the procurement quantity q, and the payment t. After the buyer announces the mechanism for procurement and the seller accepts it, both the buyer and the seller are fully committed to following the rules of the mechanism.

As a consequence of the assumption on the buyer's bargaining power and the fact that the seller's utility does not directly depend on the buyer's private information (private value), the solution of the problem formulated in this paper does not depend on whether the buyer's utility $\mathcal{V}(\cdot)$ is private information or common knowledge[5].

In the rest of the paper we formulate two contract design problems. In section III, we assume that the buyer can only accept an a priori fixed energy delivery and formulate a forward contract design problem with deterministic allocation. In Section V, we assume that the buyer can tolerate intermittency in the delivered energy by utilizing his existing storage/reserve resources, and formulate a forward contract design with random allocation.

III. Forward Contracts with Deterministic Allocation

In this section we consider a problem of forward contract design where the allocation q is deterministic and is decided in advance at the time of contract signing. Bilateral trades with conventional generators and demand response (DR) contracts for direct load control are forms of such a contract.

A. Problem Formulation

Let $(\mathcal{M},h)$ be the mechanism/game form (see [10], Ch. 23) for energy procurement designed by the buyer. In this

[3]From now on, we refer to the buyer as "he" and to the seller as "she".

[4]Note that for a general cost function $C(q,\mathbf{x},W)$ if the corresponding $c(q,\mathbf{x})$ changes sign for only finite number of times, one can expand the type space χ and reorder its dimensions so that it satisfies the assumption on the existence of m.

[5]This becomes more clear by looking at the result of Theorem 1.

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game form, $\mathcal{M}$ describes the message/strategy space for the buyer and the seller, respectively, and h determines the outcome function; $h : \mathcal{M} \rightarrow \mathbf{R}_+ \times \mathbf{R}$. For every message $m \in \mathcal{M}$ the outcome function h specifies the amount q of the procured energy/resource and the payment t made to the seller, i.e. $h(m) = (q(m), t(m))$.[6]

The objective is to determine a mechanism $(\mathcal{M}, h)$ so as to

$$\underset{(\mathcal{M},(q(\cdot),t(\cdot)))}{\text{maximize}} \quad \mathbb{E}_{\mathbf{x},W} \left\{\mathcal{V}(q(m^*)) - t(m^*)\right\}, \quad (2)$$

where $m^* \in \mathcal{M}$ is a Bayesian Nash equilibrium (BNE) of the game induced by the mechanism $(\mathcal{M}, h)$. We want the seller to voluntarily participate in the procurement process. The voluntary participation (VP) (or individual rationality) for each type of the seller can be written as

$$\text{interim VP: } \mathbb{E}_W\{t(m^*) - C(q(m^*), \mathbf{x}, W)\} \geq 0, \forall \mathbf{x} \in \chi \quad (3)$$

That is, at equilibrium m^* of the induced game the mechanism the seller must have an expected (with respect to the uncertainty W) non-negative payoff. We call the requirement expressed by (3) an *interim voluntary participation* constraint.

We call the above problem **(P1)**.

B. Analysis & Results

We prove that the optimal procurement mechanism is a pricing scheme that the buyer offers to the seller and the seller chooses a quantity according to her type. In such a pricing scheme we have $\mathcal{M} = \chi$, $q : \chi \rightarrow \mathbf{R}_+$, and the payment function $t(\cdot)$ can be defined indirectly as a function of the quantity $q(\mathbf{x})$, *i.e.* $t(q(\mathbf{x}))$. We characterize the optimal procurement mechanism by the following theorem, which reduces the original functional maximization problem (P1) to a set of equivalent pointwise maximization problems.

Theorem 1. *Under a certain concavity condition, stated in Lemma 3 below, the optimal mechanism $(q(\cdot), t(\cdot))$ for the buyer is a nonlinear pricing scheme given by*

$$p(q) = \arg\max_{\hat{p}} \{P[\mathbf{x} \in \chi | \hat{p} \geq c(q, \mathbf{x})] (\mathcal{V}'(q) - \hat{p})\}, \quad (4)$$

$$t(q) = \int_0^q p(l) dl + C(0, \underline{\mathbf{x}}), \quad (5)$$

$$q(\mathbf{x}) = \arg\max_{l \in \mathbf{R}_+} \{t(l) - \mathbb{E}_W \{C(l, \mathbf{x}, W)\}\} \quad (6)$$

where $\mathcal{V}'(q) := \frac{d\mathcal{V}(q)}{dq}$ and $\mathcal{M} = \chi$.

The assertion of Theorem 1 is established via several steps. Below we present these steps and the key ideas behind each step. The proofs of the lemmas and corollaries appearing in theses steps can be found in the appendix. In the sequel, we omit the argument of the functions $q(\cdot)$ and $t(\cdot)$ whenever such an omission causes no confusion.

[6] Note that we use q (resp. t) to denote both the quantity value (resp. payment value) and the quantity outcome function (resp. payment function) of mechanism $(\mathcal{M}, h)$.

Step 1. We set message space $\mathcal{M} = \chi$ and formulate the following problem **(P2)** that is equivalent to problem **(P1):**

$$\underset{(q(\cdot),t(\cdot))}{\text{maximize}} \quad \mathbb{E}_{\mathbf{x},W} \left\{\mathcal{V}(q(\mathbf{x})) - t(\mathbf{x})\right\} \quad (7)$$

subject to

$$IC : \mathbf{x} = \arg\max_{\mathbf{x}'} \mathbb{E}_W[t(\mathbf{x}') - C(q(\mathbf{x}'), \mathbf{x}, W)], \forall \mathbf{x} \in \chi \quad (8)$$

$$interim\ VP : \mathbb{E}_W[t(\mathbf{x}) - C(q(\mathbf{x}), \mathbf{x}, W)] \geq 0, \forall \mathbf{x} \in \chi, \quad (9)$$

where $q : \chi \rightarrow \mathbf{R}_+$ and $t : \chi \rightarrow \mathbf{R}$.

The equivalence follows from the revelation principle [5]. By invoking the revelation principle, without loss of optimality, we restrict attention to direct mechanisms (where $\mathcal{M} = \chi$) that are incentive compatible and individually rational. Incentive compatibility (IC) for a direct mechanism requires that truth-telling must be an optimal strategy for the seller.

Step 2. We show that for any incentive compatible mechanism (q, t) the seller's worst type $\underline{\mathbf{x}}$ gets the minimum utility among all of the seller's types. We utilize the partial order among the seller's different types to rank her utility for her different types (Lemma 1), and reduce the VP constraint (13) for all the seller's types to the VP constraint only for the seller's worst type (Corollary 1).

Lemma 1. *For a given incentive compatible mechanism (q, t), a better type of the seller gets a higher utility. That is, let $U(\mathbf{x}) := \mathbb{E}_W \{t(\mathbf{x}) - C(q(\mathbf{x}), \mathbf{x}, W)\}$ denote the expected profit of the seller with type $\mathbf{x}$. Then,*

1) $\frac{\partial U}{\partial x_i} \leq 0, 1 \leq i \leq m,$
2) $\frac{\partial U}{\partial x_i} \geq 0, m < i \leq n.$

A direct consequence of Lemma 1, is that the seller's worst type $\underline{\mathbf{x}}$ receives the minimum utility among all the seller's types.

Corollary 1. *The voluntary participation constraint is only binding for the worst type $\underline{\mathbf{x}}$. That is, the general VP constraint (13) can be reduced to*

$$U(\underline{\mathbf{x}}) := \mathbb{E}_W \{t(\underline{\mathbf{x}}) - C(q(\underline{\mathbf{x}}), \underline{\mathbf{x}}, W)\} \geq 0. \quad (10)$$

Step 3. We show, via Lemma 2 below, that the optimal mechanism (q, t) is a pricing scheme. That is the payment function $t(\mathbf{x})$ can be defined indirectly as a function of q as $t(q(\mathbf{x}))$.

Lemma 2. *For any pair of functions (q, t) that satisfies the IC constraint, we can rewrite $t(\mathbf{x}')$ as $t(q(\mathbf{x}'))$.*

With some abuse of notation we assume that the payment function $t : \mathbf{R} \rightarrow \mathbf{R}$ refers to the indirectly defined function $t(q(\mathbf{x}))$ (non-linear pricing scheme) and we denote $t(q(\mathbf{x}))$ by $t(q)$.

Lemma 2 implies that the VP constraint (10) can be written as

$$U(\underline{\mathbf{x}}) := \mathbb{E}_W \left\{ t(q(\underline{\mathbf{x}})) - C(q(\underline{\mathbf{x}}), \underline{\mathbf{x}}, W) \right\} \geq 0. \quad (11)$$

Step 4. We show that under a certain quasi-concavity condition, stated in Lemma 3 below, we can define indirectly the allocation function $q(\mathbf{x})$ as a function of the payment function $t(l)$ by utilizing the incentive compatibility constraint. We define the following problem **(P3)**, that is equivalent to problem **(P2)**, in terms of the marginal price $p(l) = \frac{dt(l)}{dl}$ and the minimum payment $t(0)$:

$$\max_{p(\cdot),t(0)} \int_0^\infty P[\mathbf{x}\in\chi|p(l)\geq c(l,\mathbf{x})](\mathcal{V}'(l)-p(l))\,dl - t(0) \quad (12)$$

subject to

$$\text{interim VP: } \mathbb{E}_W \left\{ t(0) + \int_0^{q(\underline{\mathbf{x}})} p(l)dl - C(q(\underline{\mathbf{x}}), \underline{\mathbf{x}}, W) \right\} \geq 0. \quad (13)$$

The equivalence is established in two steps. First, consider an arbitrary incentive compatible mechanism (q, t). The optimal quantity $q^*(\mathbf{x})$ for each type $\mathbf{x}$ of the seller is given by

$$q^*(\mathbf{x}) = \arg\max_l \mathbb{E}_W \left\{ t\left(l\right) - C(l, \mathbf{x}, W) \right\}. \quad (14)$$

Incentive compatibility then requires that the seller must tell the truth to achieve this optimal value, and cannot do better by lying, *i.e.* $q(\mathbf{x}) = q^*(\mathbf{x})$ for all $\mathbf{x} \in \chi$. For any function $t(\cdot)$, this last equality can be taken as the definition for the associated function $q(\cdot)$. Thus, the IC constraint can be eliminated by defining $q(\cdot) := q^*(\cdot)$ and the problem of designing the optimal direct revelation mechanism (q, t) can be reduced to an equivalent problem where we determine only the optimal payment function $t(\cdot)$ subject to the voluntary participation constraint for the worst type.

Next, using Lemma 3, stated below, we rewrite the buyer's expected utility in terms of the marginal price $p(q) := \frac{\partial t(q)}{\partial q}$ and the minimum payment $t(0)$ (which along with $p(\cdot)$ uniquely determines the payment function $t(\cdot)$).

Lemma 3. *Assume that the seller's problem defined by (14) is continuous and quasi-concave*[7]*. Then, the buyer's expected utility can be expressed in terms of* $p(.)$ *and* $t(0)$ *as*

$$\mathbb{E}_{\mathbf{x}}[\mathcal{V}(q^*(\mathbf{x}))] - \mathbb{E}_{\mathbf{x}}[t(q^*(\mathbf{x}))] = \int_0^\infty P\left(\mathbf{x} \in \chi | q^*(\mathbf{x}) \geq l\right) \mathcal{V}'(l)dl - t(0) - \int_0^\infty P\left(\mathbf{x} \in \chi | q^*(\mathbf{x}) \geq l\right) p(l)dl, \quad (15)$$

where

$$P\left(\mathbf{x} \in \chi | q^*(\mathbf{x}) \geq l\right) = P\left[\mathbf{x} \in \chi | p(l) \geq c(l, \mathbf{x})\right]. \quad (16)$$

[7]This is a standard assumption in economics literature, e.g. see [9] and [18]. Basically, it can be seen as a situation where the seller can decide for each marginal unit of production independently. Thus, in general, there is no guarantee that the seller's independent decisions about each marginal unit of production results in a continuous and plausible total production quantity q. Therefore, the continuity of the result must be checked a posteriori for each type of the seller.

Using (15) and (16), we can rewrite the objective of problem **(P2)** and obtain the equivalent problem **(P3)** given by (12) and (13)

Equation (16) states that the seller is willing to produce the marginal quantity at l if the resulting expected marginal profit is positive, *i.e.* the marginal price $p(l)$ exceeds the marginal expected cost of generation $c(l, \mathbf{x})$. Equation (15) expresses the buyer's total expected utility in term of an integral of his total marginal utility $\mathcal{V}'(l) - p(l)$ at quantity l, times the probability that the seller's production exceeds l, minus the minimum payment $t(0)$.

Step 5. We prove that the seller's worst type produces the minimum quantity among all the seller's types, *i.e.* $q(\underline{\mathbf{x}}) = \min_{x\in\chi} q(\mathbf{x})$. As a result, we show that problem **(P3)** is equivalent to the following problem **(P4):**

$$\max_{p(\cdot)} \int_0^\infty P\left[\mathbf{x} \in \chi | p(l) \geq c(l, \mathbf{x})\right] \left(\mathcal{V}'(l) - p(l)\right) dl \quad (17)$$

subject to

$$\text{iterim VP: } C(0, \underline{\mathbf{x}}) + \int_0^{q^*(\underline{\mathbf{x}})} p(l)dl \geq \mathbb{E}_W[C(q^*(\underline{\mathbf{x}}), \mathbf{x}, W)]. \quad (18)$$

We establish the equivalence by providing a ranking for the seller's optimal decision $q^*(\mathbf{x})$ based on the partial order among the seller's types.

Lemma 4. *For a given mechanism specified by* $(t(\cdot), q(\cdot))$*, a better type of the seller produces more. That is, the optimal quantity* $q^*(\mathbf{x})$ *that the seller with true type* $\mathbf{x}$ *wishes to produce satisfies the following properties:*

a) $\frac{\partial q^*(\mathbf{x})}{\partial x_i} \leq 0, 1 \leq i \leq m,$
b) $\frac{\partial q^*(\mathbf{x})}{\partial x_i} \geq 0, m < i \leq n.$

As a consequence of Corollary 1 and Lemma 4 we can then simplify the VP constraint (13) as follows.

Corollary 2. *The interim VP constraint is satisfied if* $t(0) = C(0, \underline{\mathbf{x}})$ *and the seller's worst type payment is equal to her expected production cost, i.e.* $t(q^*(\underline{\mathbf{x}})) = \mathbb{E}_W\{C(q^*(\underline{\mathbf{x}}), \underline{\mathbf{x}}, W)\}$.

The equivalence of problems **(P3)** and **(P4)** follows from Corollary 2 and by replacing the VP constraint (13) by (18). Note that we also dropped the constant term

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$t(0) = C(0,\underline{\mathbf{x}})$ (from Corollary 2) in the objective of problem **P4** given by (17).

Problem **(P4)** is in terms of the marginal price $p(l)$ and requires that the payment the seller's worst type receives is equal to her cost of production.

Step 6. We show that the solution of problem **(P4)** is given by

$$p(l) = \arg\max_{\hat{p}} \{P[\mathbf{x}\in\chi|\hat{p} \geq c(l,\mathbf{x})](\mathcal{V}'(l) - \hat{p})\}$$

To prove the claim of Step 6 we consider a relaxed version of (P4) without the VP constraint (18). The unconstrained problem can be solved pointwise at each quantity l to determine the optimal $p(l)$ as

$$p(l)=\arg\max_{\hat{p}} \{P[\mathbf{x}\in\chi|\hat{p} \geq c(l,\mathbf{x})](\mathcal{V}'(l) - \hat{p})\}, \quad (19)$$

which is the same as (4). From Corollary 2 and the fact that the worst type has the highest expected marginal cost, we can simplify (19), for $l \leq q^*(\underline{\mathbf{x}})$, as

$$p(l) = c(l,\underline{\mathbf{x}}), \quad \text{for } l \leq q^*(\underline{\mathbf{x}}). \quad (20)$$

Note that for $l \leq q^*(\underline{\mathbf{x}})$ we have $P[\mathbf{x}\in\chi|\hat{p} \geq c(l,\mathbf{x})] = 1$ from Lemma 4. Therefore, the minimum marginal price $p(l)$ that ensures all the seller's type are willing to produce more than $q^*(\underline{\mathbf{x}})$ is equal to the marginal expected cost for the seller's worst type $c(l,\underline{\mathbf{x}})$. Therefore, the solution to the unconstrained version of problem (P4) satisfies constraint (18) of problem (P4), and therefore, (19) is also the optimal solution of problem (P4).

We complete now the proof of Theorem 1. Using Claim 4 along with Corollary 2, the optimal payment function (nonlinear pricing) can be written as

$$t(q) = \int_0^q p(l)dl + C(0,\underline{\mathbf{x}})$$

which is the same as (5). From (14) we determine the optimal energy procurement function,

$$q(\mathbf{x}) = \arg\max_{l} \mathbb{E}_W \{t(l) - C(l,\mathbf{x},W)\}$$

which is the same as (6). The specification of $t(\cdot)$ and $q(\cdot)$ completes the proof of theorem 1 and the solution to problem (P1).

In essence, Theorem 1 states that at each quantity l, the optimal marginal price $p(l)$ is chosen so as to maximize the expected total marginal utility at l, which is given by the total marginal utility $(\mathcal{V}'(l) - p(l))$ times the probability that the seller generates at least l.

Remark 1. *In problem (P1), we assume that there exists a seller's worst type which has the highest cost at any quantity among all the seller's types, and we reduce the VP constraint for all of the seller's type to only the VP constraint for this worst type. As a result, we pin down the optimal payment function by setting $t(0) = C(0,\underline{\mathbf{x}})$ to ensure the voluntary participation of the worst type, which consequently implies the voluntary participation for all of the seller's types. In absence of the assumption on the existence of the seller's worst type, the argument used to reduce the VP constraint is not valid anymore and we cannot pin down the payment function and specify $t(0)$ a priori. Assuming that all types of the seller participate in the contract, their decision on the optimal quantity q^* only depends on the marginal price $p(q)$, and therefore, the optimal marginal price $p(q)$, given by (19), is still valid without the assumption on the existence of the worst type. To pin down the payment function $t(\cdot)$, we find the minimum payment $t(0)$ a posteriori so that all types of the seller voluntarily participate. That is,*

$$t(0)=\max_{\mathbf{x}\in\chi}\left[\mathbb{E}_W\{C(q(\mathbf{x}),\mathbf{x},W)\} - \int_0^{q^*(\mathbf{x})} p(\hat{l})d\hat{l}\right], \quad (21)$$

where the optimal decision of type $\mathbf{x}$ is given by

$$q^*(\mathbf{x}) = \arg\max_{l}\left[\int_0^l p(\hat{l})d\hat{l} - \mathbb{E}_W\{C(l,\mathbf{x},W)\}\right]. \quad (22)$$

Remark 2. *In a setup with a positive zero-provision cost for the seller, it might not be optimal for the buyer to require all the seller's types to voluntarily participate in the procurement process, since $t(0)$ depends on the zero-provision cost of the seller's worst type $C(0,\underline{\mathbf{x}})$. In such cases, it might be optimal for the buyer to exclude some "less efficient" types of the seller from the contract, select an admissible set of the seller's types, and then design the optimal contract for this admissible set of the seller's types*[8]*. Note that this is not the case for setups without a zero-provision cost. In such setups, if it is not optimal for some type $\mathbf{x}$ to be included in the optimal contract, it is equivalent to set $q(\mathbf{x}) = 0$ in a contract menu that considers all types of the seller.*

C. Risk Allocation

In the optimal mechanism/contract menu presented by Theorem 1, the buyer faces no uncertainty, and he is guaranteed to receive quantity $q(\mathbf{x})$, and all the risk associated with the realization of W is taken by the seller. We wish to modify the mechanism to reallocate the above-mentioned risk between the buyer and the seller. To do so, we modify the payment function so that the risk is reallocated between the buyer and the seller. Consider the following modified payment function with $\alpha\in[0,1]$,

[8]To find the optimal admissible set, the optimal contract can be computed for different potential admissible sets. Then, the resulting utilities can be compared to find the optimal admissible set.

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$$\hat{t}(\mathbf{x}, w) = \quad t(q(\mathbf{x})) + \alpha\left[C(q(\mathbf{x}), \mathbf{x}, w) - \mathbb{E}_W\left\{C(q(\mathbf{x}), \mathbf{x}, W)\right\}\right]. \qquad (23)$$

From (23) it follows that $\mathbb{E}_W\{\hat{t}(\mathbf{x}, W)\}=t(q(\mathbf{x}))$. Therefore, the strategic behavior of the seller does not change and the seller chooses the same quantity under the modified payment function $\hat{t}(\cdot)$ as under the original payment function $t(q)$ given by (5). Note that for $\alpha = 0$ we have the same payment as $t(q)$. For $\alpha = 1$, the seller is completely insured against any risk and all the risk is taken by the buyer. The parameter α determines the allocation of the risk between the buyer and the seller; the buyer undertakes α and the seller undertakes $(1 - \alpha)$ share of the risk.

The result of Theorem 1 is illustrated by Example 1 in section IV.

IV. Example - Demand Response (DR)

We consider a contract design for DR program. There is a load aggregator that offers contracts with incentive payments to a heterogeneous population of loads who are willing to yield the direct control of their load to the aggregator given that they are offered an appropriate incentive payment. The aggregator participates in an ancillary service market and sells the aggregated resources to the reserve market at exogenous marginal price p_r.[9] Formally, there are I types of loads with a population distribution f over different types. Each load of type i has a maximum controllable load L_i. Let $q_i \leq L_i$ denote the quantity that each load of type i yields its control to the aggregator to be dispatched. We assume that each load of type i has a quadratic cost (increasing marginal cost) given by

$$C_i = \alpha_i^0 + \alpha_i^1 q_i + \alpha_i^2 q_i^2. \qquad (24)$$

Therefore, the load's type is $\mathbf{x} = (L_i, \alpha_i^0, \alpha_i^1, \alpha_i^2)$. Let t_i denotes the incentive payment to each load of type i for yielding the control of load q_i. Then, the total utility of each load of type i is given by

$$U_i = t_i - \left(\alpha_i^0 + \alpha_i^1 q_i + \alpha_i^2 q_i^2\right). \qquad (25)$$

The aggregator participates in the ancillary service market and provides capacity $q = \sum f_i q_i$ at a given uniform price p_r. Therefore, the aggregator's revenue is given by

$$p_r \sum (f_i q_i) - \sum (f_i t_i). \qquad (26)$$

We consider $I = 5$ types of loads described in Table I along with a normalized population distribution f with $\sum f_i = 1$, and set $p_r = 2$ ¢$/kWh$. Note that no complete ordering can be defined based on their marginal cost and there exists no worst type; at lower quantities smaller loads (*e.g.* type (b)) have a lower marginal cost while at higher quantities larger loads (*e.g.* type (e)) have lower cost.

[9] If p_r is not exogenous, the aggregator's interactions with the reserve market on one hand and the demand population on the other hand become coupled. In this case these interactions must be studied simultaneously.

Via Theorem 1 we determine the optimal menu of contracts the aggregator offers to the heterogeneous population of loads (Table II). The optimal menu of contracts can be interpreted also as a nonlinear pricing that the aggregator offers to loads (Fig. 1).

The optimal choice and the resulting payoff for each type of load are summarized in Table III. We note that, unlike one-dimensional contracts, a type with a higher quantity does not necessarily get a higher payoff.

type	$L_{i(kWh)}$	α_i^0 (¢)	α_i^1 $(\frac{¢}{kWh})$	α_i^2 $(\frac{¢}{kWh^2})$	f_i
(a)	0.5	0.1	5	10	0.1
(b)	1	0.1	4	10	0.3
(c)	1.5	0.6	8	5	0.2
(d)	2	0.6	5	8	0.3
(e)	2.5	1.2	6	5	0.1

TABLE I: Different types of loads

Quantity $q(\cdot)$ $_{(kWh)}$	0.38	0.64	0.82	1.10	1.40
Payment $t(\cdot)$ (¢)	3847	7569	10498	15450	20991

TABLE II: Options menu offered by the aggregator

Type	Quantity	Payment	Cost	Profit
(a)	0.382	3847	3469	378
(b)	0.643	7569	6897	762
(c)	1.100	15450	15450	0
(d)	0.8185	10498	10052	446
(e)	1.400	20991	19400	1591

TABLE III: Optimal contract and the resulting outcome

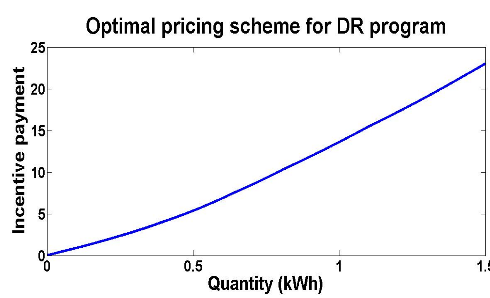

Fig. 1: The optimal pricing scheme for DR program

V. Forward Contracts with Random Allocation

In some instances of the problem considered in this paper, the buyer has a reserve resource [17] or wants to supply deferrable loads [1] that gives him the flexibility to accept a random allocation $q(\mathbf{x}, W)$ that depends on the uncertainty W, and compensate the randomness in the allocation utilizing the existing flexibility. In this section, we formulate and analyze a forward contract design problem with random allocation. We assume that the realization of the random variable W is common knowledge between the buyer and the seller.

A. Problem Formulation

Let $e(\mathbf{x})$ denote the forward scheduled quantity (deterministic) by the buyer and $q(\mathbf{x}, w)$ denote the random delivered quantity by the seller with type $\mathbf{x}$. Let $C_R(e(\mathbf{x}) - q(\mathbf{x}, w))$ denote the cost incurred by the buyer to compensate the real-time deviation $e(\mathbf{x}) - q(\mathbf{x}, w)$ from the forward schedule $e(\mathbf{x})$. Then, for a given set of contract menus $(q(\mathbf{x}, w), t(\mathbf{x}, w))$, the buyer's optimal schedule $e(\mathbf{x})$ for the seller's type $\mathbf{x}$ is defined by

$$e(\mathbf{x}) = arg \max_{\hat{e}} \mathbb{E}_W\{\mathcal{V}(\hat{e}) - t(\mathbf{x}, W) - C_R(\hat{e} - q(\mathbf{x}, W))\}, \quad (27)$$

and the buyer's expected utility is given by

$$\mathbb{E}_{W,\mathbf{x}}\{\mathcal{V}(e(\mathbf{x})) - t(\mathbf{x}, W) - C_R(e(\mathbf{x}) - q(\mathbf{x}, W))\}. \quad (28)$$

The buyer wants to design a mechanism $(q(\mathbf{x}, w), t(\mathbf{x}, w))$ so as to maximize his expected utility given by (28), subject to the voluntary participation of the seller. Formally, the contract design problem with random allocation for the buyer, called **(Q1)**, can be stated as follows:

$$\underset{\{q(\cdot,\cdot), t(\cdot,\cdot)\}}{\text{maximize}} \mathbb{E}_{W,\mathbf{x}}\{\mathcal{V}(e(\mathbf{x})) - t(\mathbf{x}, W) - C_R(e(\mathbf{x}) - q(\mathbf{x}, W))\} \quad (29)$$

subject to

$$\mathbb{E}_W\{t(\mathbf{x}, W) - C(q(\mathbf{x}, W), \mathbf{x}, W)\} \geq 0, \forall \mathbf{x} \in \mathcal{X}. \quad (30)$$

B. Analysis & Results

We show, via Theorem 2 below, that the optimal forward contract with random allocation is a menu of pricing schemes, one for each type of the seller.

Theorem 2. *The optimal forward contract with random allocation for problem* ***(Q1)*** *is a menu of pricing schemes given by*

$$e(\mathbf{x}) = \tilde{q}(\mathbf{x}) \quad (31)$$

$$q(\mathbf{x}, w) = \tilde{q}(\mathbf{x}) - q_R(\mathbf{x}, w), \quad (32)$$

$$t(\mathbf{x}, w) = \tilde{t}(\mathbf{x}) - C_R(q_R(\mathbf{x}, w)), \quad (33)$$

where $\{\tilde{q}(x), \tilde{t}(x)\}$ *denotes the optimal solution to the optimization problem*

$$\underset{\{\tilde{q}(\cdot), \tilde{t}(\cdot)\}}{\text{maximize}} \mathbb{E}_{W,\mathbf{x}}\{\mathcal{V}(\tilde{q}) - \tilde{t}\} \quad (34)$$

subject to

$$\mathbb{E}_{W,\mathbf{x}}\left\{\tilde{t}(\mathbf{x}) - \tilde{C}(\tilde{q}(\mathbf{x}), \mathbf{x}, W)\right\} \geq 0, \quad (35)$$

$$\tilde{C}(q, \mathbf{x}, w) := \min_l \{C(l, \mathbf{x}, w) + C_R(\tilde{q} - l)\}, \quad (36)$$

and

$$q_R(\mathbf{x}, w) := arg \min_l \{C(\tilde{q}(\mathbf{x}) - l, \mathbf{x}, w) + C_R(l)\}. \quad (37)$$

Proof. Consider the following contract design problem where the seller's cost function is defined as

$$\tilde{C}(\tilde{q}, \mathbf{x}, w) = \min_l \{C(l, \mathbf{x}, w) + C_R(\tilde{q} - l)\},$$

where $C(\cdot, \cdot, \cdot)$ is the seller's cost function in **(Q1)**, and the buyer's utility is defined as

$$\mathbb{E}_{W,\mathbf{x}}\{\mathcal{V}(\tilde{q}) - \tilde{t}(\tilde{q})\}. \quad (38)$$

The optimal contract design problem for the defined environment above, called **(Q2)**, can be stated as

$$\underset{\{\tilde{q}, \tilde{t}\}}{\text{maximize}} \mathbb{E}_{\mathbf{x},W}\{\mathcal{V}(\tilde{q}) - \tilde{t}\} \quad (39)$$

subject to

$$\text{IC: } \mathbf{x} = arg \max_{\mathbf{x}'} \mathbb{E}_W\left\{\tilde{t}(\mathbf{x}') - \tilde{C}(\tilde{q}(\mathbf{x}'), \mathbf{x}, W)\right\}, \forall \mathbf{x} \in \chi \quad (40)$$

$$\text{interim VP: } \mathbb{E}_W\left\{\tilde{t}(\mathbf{x}) - \tilde{C}(\tilde{q}(\mathbf{x}), \mathbf{x}, W)\right\} \geq 0, \forall \mathbf{x} \in \chi \quad (41)$$

where $\tilde{q}$ and $\tilde{t}$ denote the quantity and payment function for the defined problem above. By construction, problem **(Q2)** is the same as problem **(P2)**. Let $\{\tilde{q}(\mathbf{x}), \tilde{t}(\mathbf{x})\}$ denote the optimal contract for problem **(Q2)** obtained via Theorem 1. Note that through the cost function $\tilde{C}(\tilde{q}(\mathbf{x}), \mathbf{x}, w)$, defined by (36), we absorb the optimal schedule choice $e(\mathbf{x})$, given by (27), and internalize the compensation cost $C_R(e(\mathbf{x}) - q(\mathbf{x}, W))$ for the random deviation $e(\mathbf{x}) - q(\mathbf{x}, W)$ in problem **(Q1)** into the seller's cost function. Therefore, the optimal scheduled quantity $e(\mathbf{x})$ for problem **(Q1)** is equal to the optimal function $\tilde{q}(\mathbf{x})$ for problem **(Q2)**, *i.e.* $e(\mathbf{x}) = \tilde{q}(\mathbf{x})$. Consequently, one can reconstruct the optimal contract $\{q(\mathbf{x}, w), t(\mathbf{x}, w)\}$ for problem **(Q1)** using the optimal contract $\{\tilde{q}(\mathbf{x}), \tilde{t}(\mathbf{x})\}$ for the equivalent problem **(Q2)** as

$$q(\mathbf{x}, w) = \tilde{q}(\mathbf{x}) - q_R(\mathbf{x}, w),$$
$$t(\mathbf{x}, w) = \tilde{t}(\mathbf{x}) - C_R(q_R(\mathbf{x}, w)),$$

where

$$q_R(\mathbf{x}, w) := arg \min_l \{C(\tilde{q}(\mathbf{x}) - l, \mathbf{x}, w) + C_R(l)\},$$

denotes the random reserve quantity required to compensate the random allocation $q(\mathbf{x}, w)$. □

Theorem 2 has the following interpretation. The

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buyer offers different pricing schemes (quantity-payment curves), and each type of the seller chooses one based on her private information and expectation about W. Then, in real time as W is realized, based on the realization w, one point from the chosen pricing scheme is selected and the payment t and the energy delivery q are determined.

C. Imperfect Commitment and Ex-post Voluntary Participation

The voluntary participation constraint imposed in problem (Q1) is interim. That is, the expected profit with respect to W must be non-negative for each type of the seller. Up until now (problem **(P1)** and **(Q1)**) we have assumed that once the seller agrees to sign the contract (such an agreement takes place before the realization of random variable W) she is fully committed to following the agreement, even if the realized profit is negative (due to some realization w)[10]. Therefore, it would be desirable to modify the contract in order to ensure a positive payoff for the seller for every realization of W and full commitment without any outside enforcement. To ensure that the seller's realized profit is non-negative for every realization w, we impose an *ex-post voluntary participation* constraint and replace the interim VP constraint (30) by

$$\text{Ex-post VP: } t(\mathbf{x}) - C(q, \mathbf{x}, w) \geq 0, \forall w, \ \forall \mathbf{x} \in \chi. \quad (42)$$

To satisfy the ex-post voluntary participation constraint, we modify the payment function of the mechanism given by Theorem 2 as follows:

$$\check{t}(\mathbf{x}, w) = \mathbb{E}_W \{t(\mathbf{x}, W)\} - \mathbb{E}_W \{C(q(\mathbf{x}, W), \mathbf{x}, W)\} + C(q(\mathbf{x}, w), \mathbf{x}, w). \quad (43)$$

We have $\mathbb{E}_W\{\check{t}(\mathbf{x}, W)\} = \mathbb{E}_W \{t(\mathbf{x}, W)\}$, and therefore, the seller always chooses the same quantity q under the modified payment function $\check{t}$ as under the original payment function t given by (33). Furthermore, we have $\check{t}(\mathbf{x}, w) - C(q(\mathbf{x}, w), \mathbf{x}, w) = \mathbb{E}_W \{t(\mathbf{x}, W)\} - \mathbb{E}_W \{C(q(\mathbf{x}, W), \mathbf{x}, W)\} \geq 0$ for all $w, \mathbf{x}$, where the last inequality is true since $\{q, t\}$ satisfies the interim VP constraint (30). Therefore, under the modified payment function $\check{t}$, $\{q, \check{t}\}$ satisfies the expost VP constraint (42).

VI. Example - Forward Bilateral Trade

Consider a forward bilateral trade between a buyer and a seller with wind generation. The buyer has an (almost inelastic) energy demand curve given by Fig. 2.

The seller has a wind farm and (possibly) a reserve generator/storage that can be used to compensate for wind generation intermittency. The seller's wind generation is given by $g(w, v_{ci}, v_r, v_{co}, \gamma)$ as in Fig. 3, where w denotes the wind speed and $(v_{ci}, v_r, v_{co}, \gamma)$ denotes the specification of the wind turbine. The wind speed is random and the wind forecast f_W is given by Fig. 4, which is a Weibull distribution with shape parameter $k = 3$ and average wind speed of $5m/s$. We assume that the wind forecast f_W as well as the wind realization w are common knowledge between the buyer and the seller. The wind generation has a marginal operational cost θ_w. The seller (possibly) has a reserve generator/storage with capacity r and a marginal cost θ_r that can be utilized if needed. The seller has a zero-production cost c_0 which accounts for her capital cost and the start-up cost of her facilities. Therefore, the seller's private information is as $\mathbf{x} = (c_0, \theta_w, \theta_r, v_{ci}, v_r, v_{co}, \gamma, r)$.

We assume that the buyer has a reserve generator/deferrable load that can be utilized to compensate the real-time random energy delivery by the seller. We assume that deviation from the scheduled energy has an increasing marginal cost for the buyer given by $b_0 + b_1 q$.

We consider 4 types for the seller as in Table IV, and set $b_0 = 1.4 \frac{\$}{kwh}$ and $b_1 = 0.05 \frac{\$}{kWh^2}$.

The optimal forward contract menu for the buyer is given by Fig. 5. Since the energy demand considered in this example is almost inelastic, the scheduled quantity $e(\mathbf{x})$, and therefore, the quantity-demand curves are also

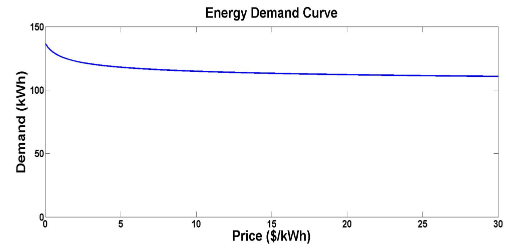

Fig. 2: The buyer's demand curve

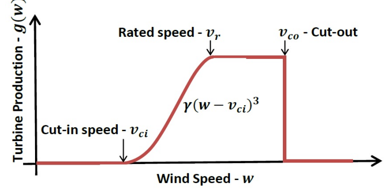

Fig. 3: The wind turbine generation curve $g(w, v_{ci}, v_r, v_{co}, \gamma)$

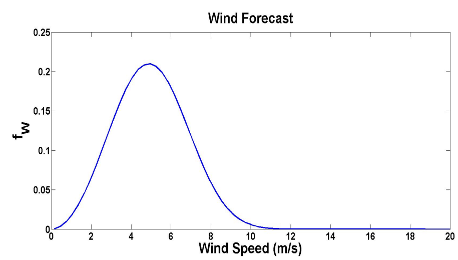

Fig. 4: The wind forecast F_W

[10]Since the seller's reserved utility is zero by not participating (outside option), we can always think of the seller walking away from the agreement for these negative profit realizations and not following the mechanism rules.

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type	c_0	θ_w	θ_c	v_{ci}	v_r	v_{co}	γ	r	f_i
a	90	0.20	1.2	0.4	13	23	1	60	0.1
b	60	0.25	1.4	0.8	17	25	1.25	30	0.3
c	40	0.10	1.0	0.1	15	20	1.5	10	0.2
d	20	0.15	-	1.0	17	28	1.7	0	0.3

TABLE IV: Different types of the seller

close to each others.[11] Table V summarizes the optimal energy schedule $e(\mathbf{x})$, and the expected utility $U(\mathbf{x})$ for different types of the seller.

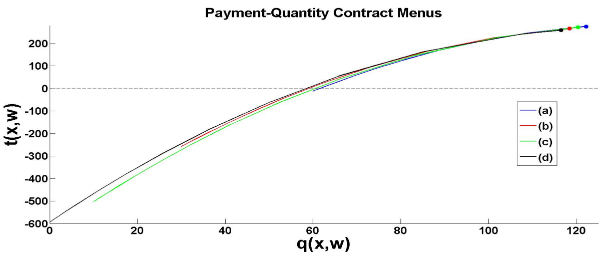

Fig. 5: The optimal forward contract menus

type	a	b	c	d
$e(\mathbf{x})$	122.3	118.5	120.4	116.5
$U(\mathbf{x})$	84.47	35.10	101.82	0

TABLE V: The outcomes of the optimal contract menus

The energy $q(\mathbf{x}, w)$ delivered to the buyer, the payment $t(\mathbf{x}, w)$ made to the seller, and the seller's utility $u(\mathbf{x}, w)$ in terms of wind w are given by Figures 6, 7, and 8, respectively.

For low realizations of wind speed, the delivered energy is low and the seller may even incur some penalty for very low energy delivery (Fig. 5). For higher realization of wind speed, the energy delivery increases, and therefore, the payment and the realized utility increase. However, for very high realization of wind speed that surpasses the cut-off speed v_{co} (see Figure 3), the energy delivery, and consequently the payment and the realized utility, drop.

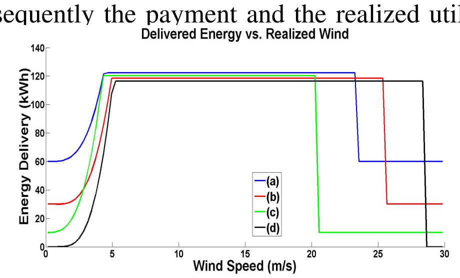

Fig. 6: Energy deliver $q(\mathbf{x}, w)$ in terms of wind w

VII. Discussion

For the problem on energy/service procurement formulated in this paper the optimal mechanism is a menu of

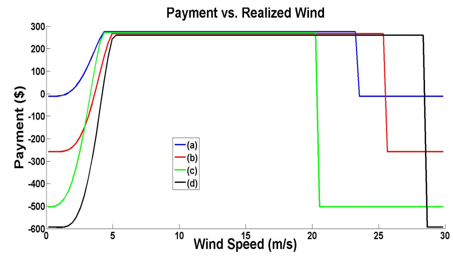

Fig. 7: Payment $t(\mathbf{x}, w)$ in terms of wind w

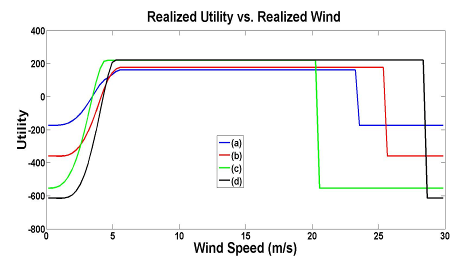

Fig. 8: The seller's utility $u(\mathbf{x}, w)$ in terms of wind w

contracts/nonlinear pricing schemes. The nonlinearity is due to three factors. First, the buyer's utility function $\mathcal{V}(q)$ is not linear in the quantity q. Second, for each type of the seller, the cost function is a nonlinear function of the quantity . Third, the seller has private information about her technology and cost (seller's type).

The buyer has to pay information rent (monetary incentive) to the seller to incentivize her to reveal her true type. Therefore, the payment the buyer makes to the seller includes the cost of provision the seller incurs plus the information rent, which varies with the seller's type; the better the seller's type, the higher is the information rent.

The optimal forward contracts discovered in this paper can be implemented as follows: the buyer offers the seller a menu of contracts (nonlinear pricing schemes); the seller chooses one of these contracts based on her type.

The optimal forward contracts induce some incentives for investment in infrastructure and technology development. From Lemma 1, the seller with the higher type has a higher utility. Therefore, there is an incentive for the seller to improve her technology and decrease her cost of generation.

It is well-known that in the presence of private information and strategic behavior, in general, there exists no mechanism/contract that is (1) individually rational, (2) incentive compatible, and (3) efficient (Pareto-optimal) [13]. In the optimal forward contract given by Theorems 1, and 2 the allocation for the seller's different types is not ex-post efficient (Pareto-optimal) except for the seller's worst type who gets zero utility.

In this paper, we formulated the contract design problem in a principal-agent setup. Therefore, the result can be applied to the contract design problem for a setup with

[11] For a completely inelastic energy demand, we have $e(\mathbf{x})$ fixed and independent of the seller's type $\mathbf{x}$. Therefore, all the quantity-demand curves coincide and are equal to the quantity-cost curve for the worst type.

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one buyer (principal) and a heterogeneous population of sellers (agents), if the buyer has a linear utility function, as in Example 1, or if the share of each individual agent is small and their effect on the market is negligible. However, if one considers a setup with nonlinear utility for the buyer or market power for each individual agent, the associated problem for such setup with multiple agents becomes equivalent to the design of optimal multi-unit auctions in economics. It is known that there exist no closed form solution to the general problem of optimal multi-unit auctions, and their solutions can only be computed numerically or with approximation [8].

VIII. Conclusion

We investigated the problem of optimal forward contract design under uncertainty and multi-dimensional private information. The consideration of multi-dimensional private information and general utility/cost functions enables us to capture many applications in electricity markets as well as other disciplines. We assume that the buyer and/or the seller has uncertainty in their utility/cost function which is realized after the time of contract signing. We considered froward contracts with random allocation that depends on the real-time realization of the uncertainty. We characterized the optimal forward contract under uncertainty as a menu of contracts. We addressed the problem of commitment (ex-post voluntary participation), and risk sharing in the presence of uncertainty. We demonstrated our results by two examples; an optimal contract design for a demand response program, and an optimal forward bilateral trade between a buyer and a seller with wind energy generation.

Appendix - Proofs

Proof of lemma 1. The given mechanism (q,t) is incentive compatible, so we can rewrite $U(\mathbf{x})$ as

$$U(\mathbf{x}) = \max_{\mathbf{x}'} \mathbb{E}_W \left\{ t(\mathbf{x}') - C(q(\mathbf{x}'), \mathbf{x}, W) \right\} \tag{44}$$

By applying the envelope theorem [12] on (44), we get

$$\frac{\partial U}{\partial x_i} = - \left. \frac{\partial \mathbb{E}_W \left\{ C(q(\mathbf{x}'), \mathbf{x}, W) \right\}}{x_i} \right|_{\mathbf{x}'=\mathbf{x}}. \tag{45}$$

The above equation along with the assumption on the monotonicity of the marginal expected cost $c(q,\mathbf{x})$ with respect to $\mathbf{x}_i$ (Section II.A) gives

$$\frac{\partial U}{\partial x_i} \leq 0, 1 \leq i \leq m \tag{46}$$

$$\frac{\partial U}{\partial x_i} \geq 0, m < i \leq n. \tag{47}$$

□

Proof of lemma 2. The proof is by contradiction. Assume that there exist $\mathbf{x}, \mathbf{x}' \in \chi$ such that $q(\mathbf{x}) = q(\mathbf{x}')$ but $t(\mathbf{x}') > t(\mathbf{x})$. Then a seller with type $\mathbf{x}$ is always better off by reporting $\mathbf{x}'$ instead of her true type $\mathbf{x}$, which contradicts the IC constraint. □

Proof of Lemma 3. Consider the buyer's objective (7). For any function $t(\cdot)$, we can determine from (14) the cumulative distribution function for q^*, called F_{q^*}. Consequently, we can rewrite the buyer's objective as

$$\begin{aligned} &\mathbb{E}_{q^*}\left[\mathcal{V}(q^*) - t(q^*)\right] = \int_0^\infty \left(\mathcal{V}(l) - t(l)\right) dF_{q^*}(l) \\ &= \left(F_{q^*}(l) - 1\right)\left(\mathcal{V}(l) - t(l)\right)\Big|_0^\infty \\ &+ \int_0^\infty \left(1 - F_{q^*}(l)\right) \frac{d\left(\mathcal{V}(l) - t(l)\right)}{dl} dl. \end{aligned} \tag{48}$$

We have

$$\left(F_{q^*}(l) - 1\right)\left(\mathcal{V}(l) - t(l)\right)\Big|_0^\infty = -t(0) \tag{49}$$

because $\mathcal{V}(0) = 0$ by assumption, and $(F_{q^*}(\infty) - 1) = 0$.

Because of (49), we can rewrite (48) as

$$\begin{aligned} \mathbb{E}_{q^*}\left[\mathcal{V}(q^*) - t(q^*)\right] &= \int_0^\infty P\left(q^* \geq l\right)\left(\mathcal{V}'(l) - p(l)\right) dl \\ &-t(0) \end{aligned} \tag{50}$$

where $\mathcal{V}'(l) = \frac{d\mathcal{V}(l)}{dl}$.

We can rewrite $P\left(q^* \geq l\right)$ as

$$P(q^* \geq l) = P\left[\mathbf{x} \in \chi | \arg\max_{\hat{l}} \mathbb{E}_W \left\{ t(\hat{l}) - C(\hat{l}, \mathbf{x}, W) \right\} \geq l\right]. \tag{51}$$

We implicitly assume that the seller's problem given by (14) is continuous and quasi-concave, so that from the first order optimality condition for (14) we obtain

$$p(q^*(\mathbf{x})) = \left. \frac{\partial \mathbb{E}_W \left\{ C(l, \mathbf{x}, W) \right\}}{\partial l} \right|_{q^*(\mathbf{x})} = c(q^*(\mathbf{x}), \mathbf{x}). \tag{52}$$

Therefore, from the optimality of $q^*(\mathbf{x})$ and the quasi-concavity of (14), we must have $p(l) > c(l;\mathbf{x})$ and $p(l) < c(l;\mathbf{x})$ for $l < q*(\mathbf{x})$ and $l > q^*(\mathbf{x})$, respectively. That is, each type of the seller wishes to produce more than quantity l if and only if the marginal price $p(q)$ that she is paid at l is higher than the expected marginal cost of production $c(l,\mathbf{x})$ that she incurs at l. Consequently, combining (51) and (52) we obtain

$$P\left(q^* \geq l\right) = P\left[\mathbf{x} \in \chi | p(l) \geq c(l, \mathbf{x})\right]. \tag{53}$$

Substituting (53) in (50), we obtain the following alternative expression for the buyer's objective

$$\begin{aligned} \mathbb{E}_{q^*}\left[\mathcal{V}(q^*) - t(q^*)\right] = &\int_0^\infty P[\mathbf{x} \in \chi | p(l) \geq c(l, \mathbf{x})] \\ &\left(\mathcal{V}'(l) - p(l)\right) dl - t(0). \end{aligned} \tag{54}$$

□

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Proof of lemma 4. Let $\mathbf{x}, \mathbf{x}' \in \chi$, where $\mathbf{x}$ is a better type than $\mathbf{x}'$. From IC for seller's type $\mathbf{x}$ we have

$$t(q(\mathbf{x})) - \mathbb{E}_W\{C(q(\mathbf{x}), \mathbf{x}, W)\} \geq t(q(\mathbf{x}')) - \mathbb{E}_W\{C(q(\mathbf{x}'), \mathbf{x}, W)\} \quad (55)$$

Similarly from IC for seller's type $\mathbf{x}'$ we have

$$t(q(\mathbf{x}')) - \mathbb{E}_W\left\{C(q(\mathbf{x}'), \mathbf{x}', W)\right\} \geq t(q(\mathbf{x})) - \mathbb{E}_W\left\{C(q(\mathbf{x}), \mathbf{x}', W)\right\} (56)$$

Subtracting (56) from (55), we get

$$\mathbb{E}_W\left\{C(q(\mathbf{x}), \mathbf{x}', W)\right\} - \mathbb{E}_W\left\{C(q(\mathbf{x}'), \mathbf{x}', W)\right\} \geq \mathbb{E}_W\left\{C(q(\mathbf{x}), \mathbf{x}, W)\right\} - \mathbb{E}_W\left\{C(q(\mathbf{x}'), \mathbf{x}, W)\right\} (57)$$

By assumption, $\frac{d\mathbb{E}_W\{C(l,\mathbf{x},W)\}}{dl} \leq \frac{d\mathbb{E}_W\left\{C(l,\mathbf{x}',W)\right\}}{dl}$ if $\mathbf{x}$ is a better type than $\mathbf{x}'$. Therefore, (57) holds if and only if

$$q(\mathbf{x}) \geq q(\mathbf{x}'). \quad (58)$$

□

Proof of corollary 2. Because of corollary 1, the VP constraint implies

$$U(\underline{\mathbf{x}}) = t(q(\underline{\mathbf{x}})) - \mathbb{E}_W\left[C(q^*(\underline{\mathbf{x}}), \underline{\mathbf{x}}, W)\right] = 0, \quad (59)$$

which is equivalent to

$$t(0) + \int_0^{q^*(\underline{\mathbf{x}})} p(l)dl = \mathbb{E}_W\left[C(q^*(\underline{\mathbf{x}}), \mathbf{x}, W)\right]. \quad (60)$$

Furthermore, from Lemma 4 it follows that if the worst type wishes to produce more than $q^*(\underline{\mathbf{x}})$, then all types produce more than $q^*(\underline{\mathbf{x}})$. Therefore,

$$P\left[\mathbf{x} \in \chi | p(l) \geq c(l, \mathbf{x})\right] = 1, \quad \text{for } l \leq q^*(\underline{\mathbf{x}}). \quad (61)$$

Using (61), we can rewrite the objective function of problem (P3) as,

$$- \left(t(0) + \int_0^{q^*(\underline{\mathbf{x}})} p(l)dl\right) + \int_0^{q^*(\underline{\mathbf{x}})} \mathcal{V}'(l)dl + \int_{q^*(\underline{\mathbf{x}})}^{\infty} P\left[\mathbf{x} \in \chi | p(l) \geq c(l, \mathbf{x})\right] (\mathcal{V}'(l) - p(l))\, dl. (62)$$

The term $t(0) + \int_0^{q^*(\underline{\mathbf{x}})} p(l)dl$ appears in both the objective (62) and the VP constraint (60). Therefore, without loss of optimality, we can assume $t(0) = C(0, \underline{\mathbf{x}})$, and set $t(q(\underline{\mathbf{x}})) = \mathbb{E}_W\left\{C(q(\underline{\mathbf{x}}), \underline{\mathbf{x}}, W)\right\}$. □

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