UNIVERSITY OF
MICHIGAN

Optimal Design of Robust Compliant Actuators for Ubiquitous Co-Robots

NSF Awards
1830360 → 1953908
(Gregg/Rezazadeh)
1830338 (Rouse)

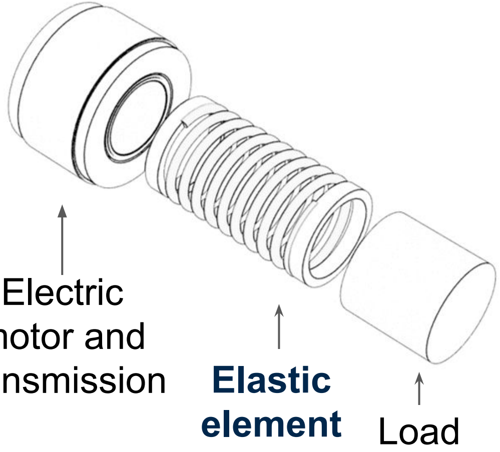
Robert D. Gregg
March 12, 2021

Poster #40

actuation and **energy** remain major bottlenecks in robotics

G. Yang et al., “*The grand challenges of Science Robotics,*” Sci. Robot., **Jan. 2018.**
A. M. Dollar and H. Herr, “*Lower Extremity Exoskeletons and Active Orthoses: Challenges and State-of-the-Art,*” IEEE Trans. Robot., **Feb. 2008**.

Elasticity!



Series elastic actuator

How to increase battery life and quality of force control?

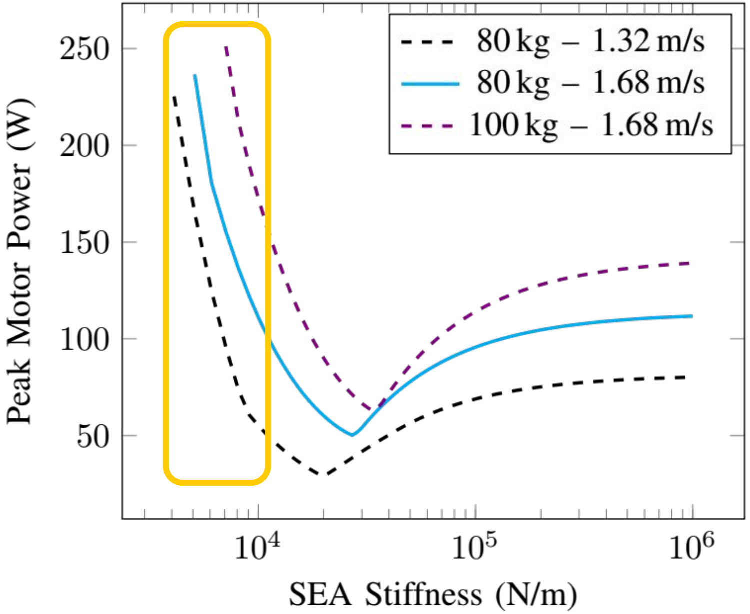
Possible side-effects of SEAs

Poor selection of stiffness for given task may:

- Increase peak power
- Increase energy consumption

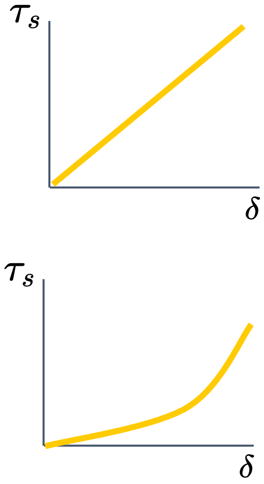
We lack fundamental understanding for robust design!

Example case for a **powered prosthetic ankle**



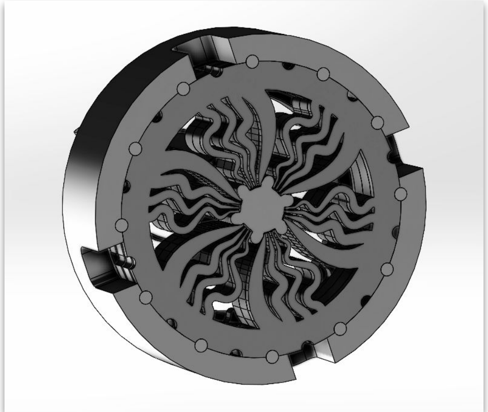
M. Grimmer and A. Seyfarth, "*Stiffness adjustment of a Series Elastic Actuator in an ankle-foot prosthesis for walking and running: The trade-off between energy and peak power optimization*," 2011 IEEE International Conference on Robotics and Automation, Shanghai, 2011

1. Design of **energy efficient series springs** as a **convex** optimization program

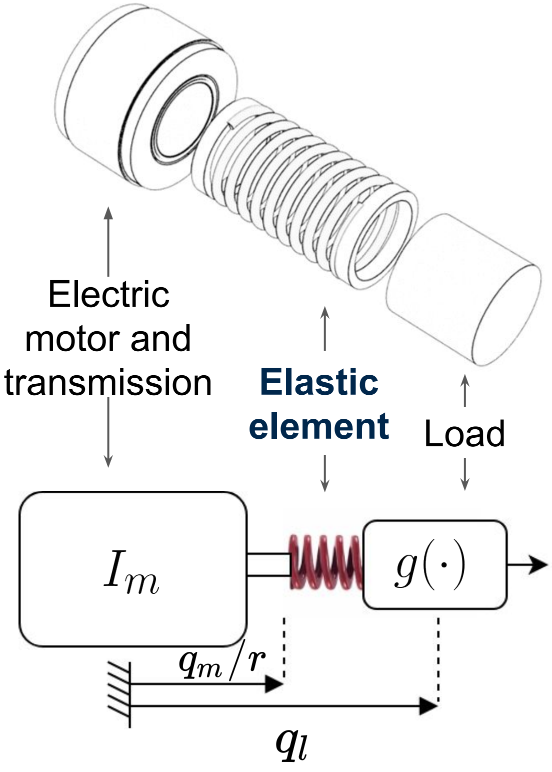


2. Robust feasible design of SEAs

3. Mechanical design of **optimal springs**



Modeling of SEAs



Equations of motion for motor shaft:

$$I_{\mathrm{m}}\ddot{q}_{\mathrm{m}} = \tau_{\mathrm{m}} - b_{\mathrm{m}}\dot{q}_{\mathrm{m}} - \frac{\tau_{\mathrm{s}}}{r}$$

Inertia motor & transmission

Motor torque

Viscous friction

Spring torque divided by the ratio of transmission

Spring torque $\tau_s = f(\delta)$

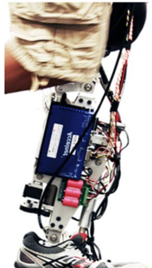
$$\delta = q_l - \frac{q_m}{r}$$

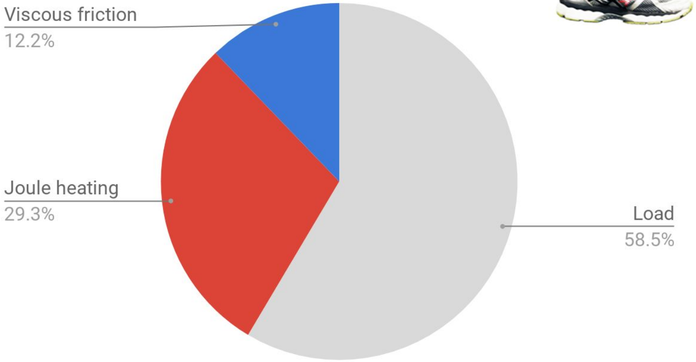
Elongation

Spring torque is equal to load torque:

$$\tau_s = g(\underbrace{q_l, \dot{q}_l, \ddot{q}_l, \tau_e}_{\text{Known load kinematics and kinetics}})$$

Energy consumption





UTD prosthetic leg V1 during walking

Siavash Rezazadeh
U. Denver
(Co-PI)

Edgar Bolívar-Nieto
U. Michigan

$$E_{\mathrm{m}} = \int_{t_0}^{t_f} i_{\mathrm{m}} v_{\mathrm{m}} dt = \int_{t_0}^{t_f} \left(\frac{\tau_{\mathrm{m}}^2}{k_{\mathrm{m}}^2} + \tau_{\mathrm{m}} \dot{q}_{\mathrm{m}} \right) dt$$

$$= \int_{t_0}^{t_f} \left(\frac{\tau_{\mathrm{m}}^2}{k_{\mathrm{m}}^2} + \left(I_{\mathrm{m}} \ddot{q}_{\mathrm{m}} + b_{\mathrm{m}} \dot{q}_{\mathrm{m}} + \frac{\tau_{\mathrm{s}}}{r} \right) \dot{q}_{\mathrm{m}} \right) dt$$

$$= \int_{t_0}^{t_f} \left(\underbrace{\frac{\tau_{\mathrm{m}}^2}{k_{\mathrm{m}}^2}}_{\text{Winding Joule Heating}} + \underbrace{b_{\mathrm{m}} \dot{q}_{\mathrm{m}}^2}_{\text{Viscous friction}} - \underbrace{\tau_{\mathrm{l}} \dot{q}_{\mathrm{l}}}_{\text{Energy load}} \right) dt,$$

where

$$k_{\mathrm{m}} = \frac{k_{\mathrm{t}}}{\sqrt{R}}$$

E. A. Bolivar Nieto, S. Rezazadeh, and R. D. Gregg, "**Minimizing Energy Consumption and Peak Power of Series Elastic Actuators: A Convex Optimization Framework for Elastic Element Design,**" IEEE/ASME Trans. Mechatronics, vol. 24, no. 3, pp. 1334–1345, Jun. 2019.

Energy as a quadratic function of compliance, α

$$E_m = \int_{t_0}^{t_f} \left(\frac{\tau_{\mathrm{m}}^2}{k_{\mathrm{m}}^2} + b_{\mathrm{m}} \dot{q}_{\mathrm{m}}^2 - \tau_{\mathrm{l}} \dot{q}_{\mathrm{l}} \right) dt$$

$$\boxed{E_m = a\alpha^2 + b\alpha + c}$$

Spring Compliance — $\alpha = 1/k$

where

$$a = \int_{t_0}^{t_f} \left(\frac{\gamma_1^2}{k_m^2} + b_m r^2 \dot{\tau}_s^2 \right) dt$$

$$b = \int_{t_0}^{t_f} \left(\frac{2\gamma_1 \gamma_2}{k_m^2} - 2 b_m r^2 \dot{q}_l \dot{\tau}_s \right) dt$$

$$c = \int_{t_0}^{t_f} \left(\frac{\gamma_2^2}{k_m^2} + b_m \dot{q}_l^2 r^2 - \dot{q}_l \tau_s \right) dt$$

E. Bolivar, S. Rezazadeh, T. Summers, and R. D. Gregg, "**Robust Optimal Design of Energy Efficient Series Elastic Actuators: Application to a Powered Prosthetic Ankle**," in 2019 IEEE 16th International Conference on Rehabilitation Robotics (ICORR), 2019, pp. 740–747.

Properties of the quadratic expression

$$E_m = a\alpha^2 + b\alpha + c$$

The quadratic expression is *convex*:

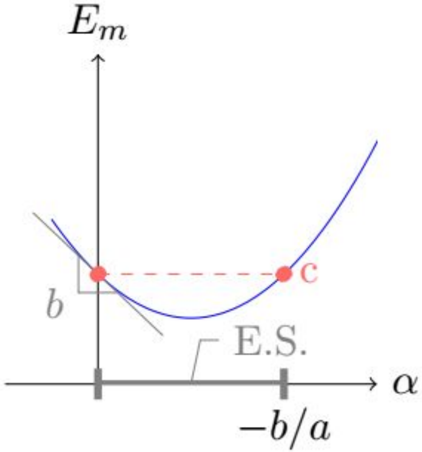
$$\frac{d^2 E_m}{d\alpha^2} = 2a \geq 0$$

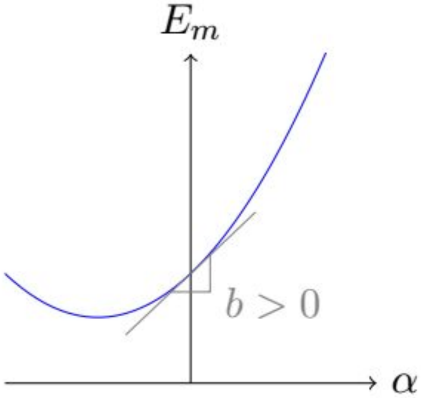
***C* is the energy consumption of a *rigid* actuator:**

$$\lim_{k \to \infty} E_m = c$$

The **necessary** condition for a linear series elastic element to **save energy**: $b < 0$

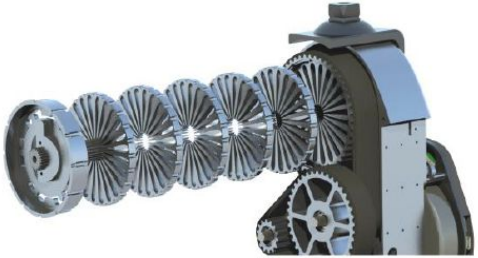
$$\int_{t_0}^{t_f} \left(\frac{2\gamma_1\gamma_2}{k_m^2} - 2b_m r^2 \dot{q}_l \dot{\tau}_s \right) dt < 0$$



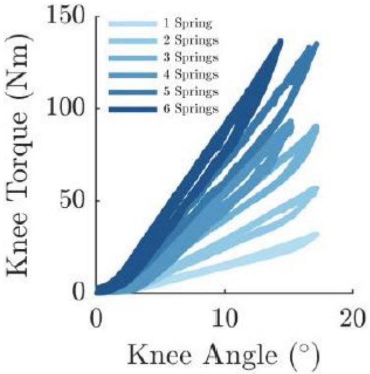


Experimental validation

Open Source Leg
U. Michigan

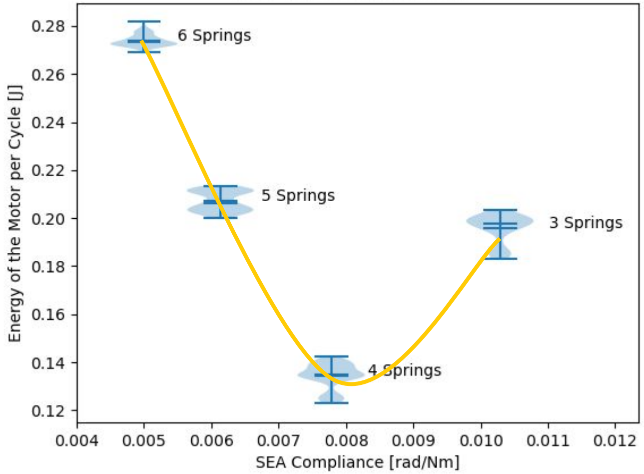


Elliott Rouse
(PI)



Gray Thomas

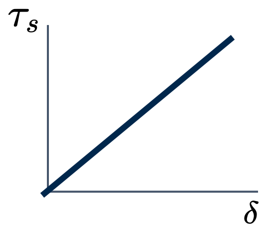
*IROS 2021

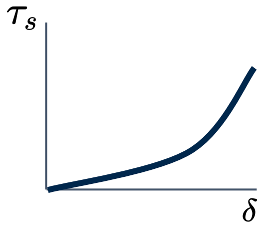


$$\begin{aligned} \underset{\boldsymbol{\alpha}}{\text{minimize}} \quad & \boldsymbol{\alpha}^T \boldsymbol{G} \boldsymbol{\alpha} + \boldsymbol{h} \boldsymbol{\alpha} + w, \\ \text{subject to} \quad & \boldsymbol{M} \boldsymbol{\alpha} \leq \boldsymbol{p} \end{aligned}$$

Design of nonlinear and linear springs that globally minimize energy consumption

E. A. Bolivar Nieto, S. Rezazadeh, and R. D. Gregg, "**Minimizing Energy Consumption and Peak Power of Series Elastic Actuators: A Convex Optimization Framework for Elastic Element Design,**" IEEE/ASME Trans. Mechatronics, vol. 24, no. 3, pp. 1334–1345, Jun. 2019.





1. Design of **energy efficient series springs** as a **convex** optimization program

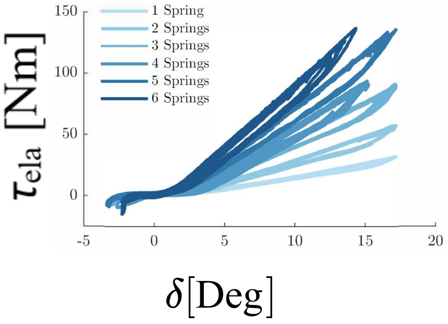
2. Robust feasible design of SEAs

3. Mechanical design of **optimal springs**

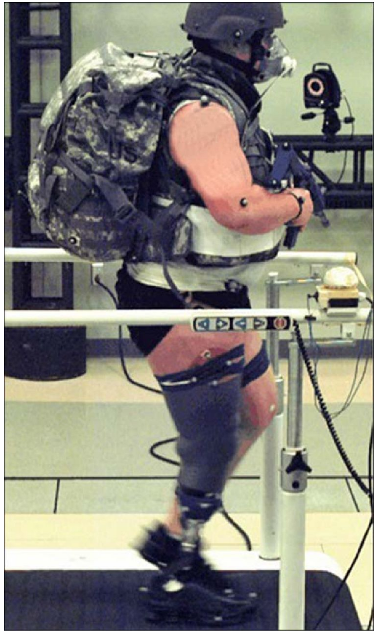
What is robust feasible design?

Uncertainty from:

Manufacturing



Task



Leads to uncertainty in the optimization parameters:

$$\begin{aligned} \underset{\boldsymbol{x}}{\text{minimize}} \quad & f(\boldsymbol{x}, \boldsymbol{\beta}) \\ \text{subject to} \quad & \boldsymbol{x} \in \mathcal{X}(\boldsymbol{\beta}) \\ & \boldsymbol{\beta} \in \mathcal{U}_{\beta} \end{aligned}$$

Uncertainty set

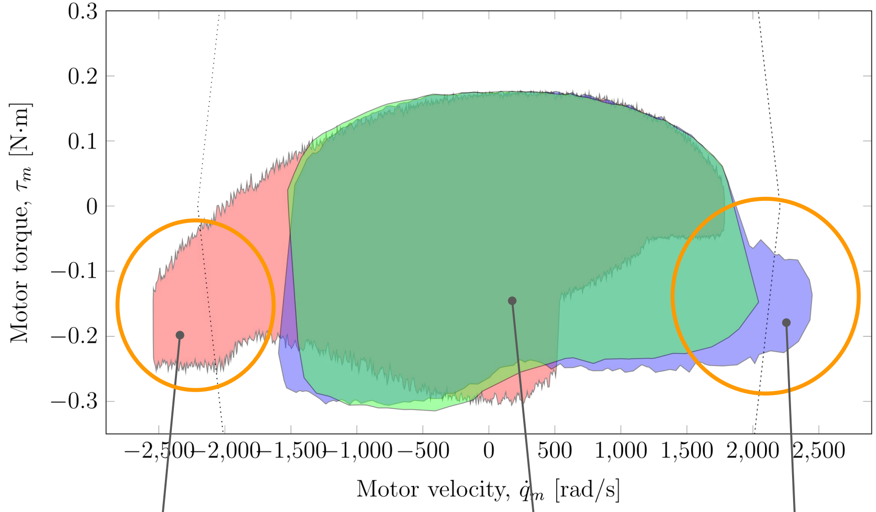
Robust feasible: optimal point is feasible

$$\boldsymbol{x} \in \mathcal{X}(\boldsymbol{\beta}) \quad \forall \boldsymbol{\beta} \in \mathcal{U}_{\beta}$$

Robust optimal: solution is optimal for the worst case condition

$$\underset{\boldsymbol{x}}{\text{minimize}} \quad \left[\sup_{\boldsymbol{\beta}} f(\boldsymbol{x}, \boldsymbol{\beta}) \right]$$

Motor torque-speed regions for prosthetic ankle



Motor without spring

Robust series spring

Series spring designed with nominal requirements

Uncertain **affine and quadratic** constraints have a computationally tractable counterpart*

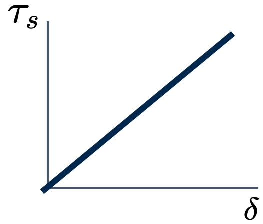
Tyler Summers
UT Dallas

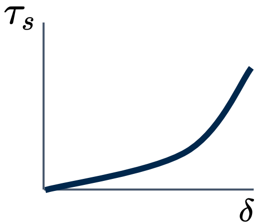
Siavash Rezazadeh
U. Denver

Edgar Bolívar-Nieto
U. Michigan

E. Bolívar-Nieto, T. Summers, R. D. Gregg and S. Rezazadeh, "**Series Spring Design for Robust-Feasible Quasi-Direct Drives,**" *Mechatronics*, Under review

1. Design of **energy efficient series springs** as a **convex** optimization program

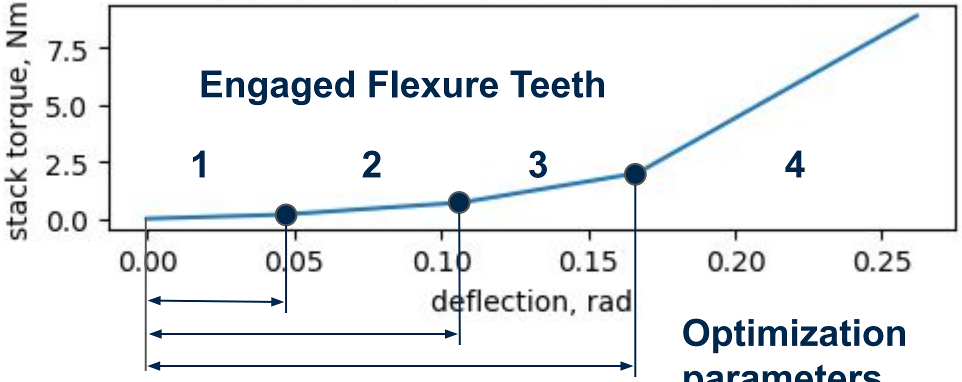




2. Robust feasible design of SEAs

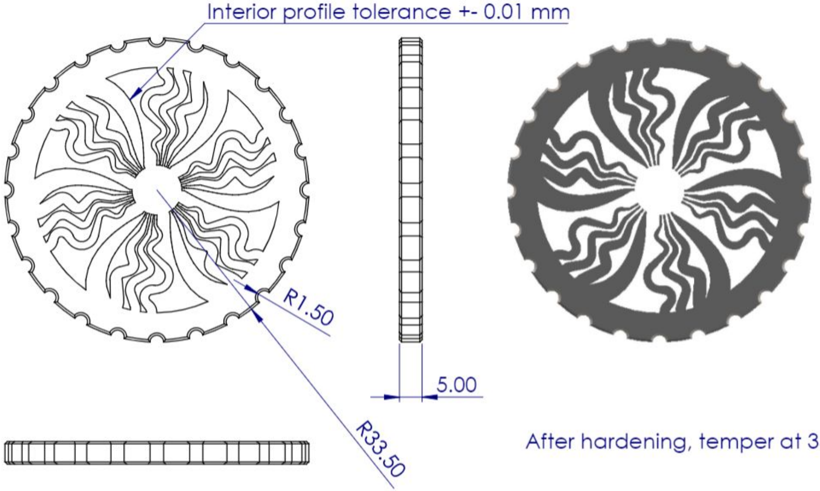
3. Mechanical design of **optimal springs**

Non-Linear Disk Spring Design



Engaged Flexure Teeth

Optimization parameters



After hardening, temper at 343 C for 2 hours

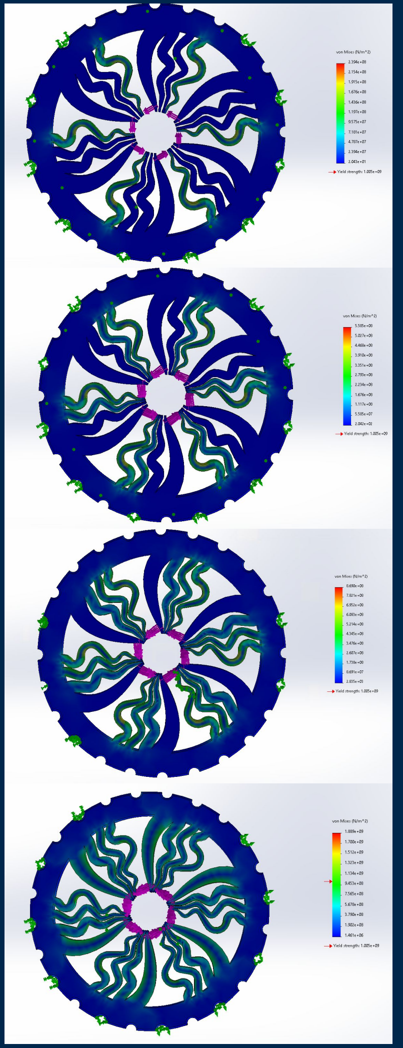
Gray Thomas
U. Michigan

1st flexure, until 0.05 rad

Collision with 2nd flexure

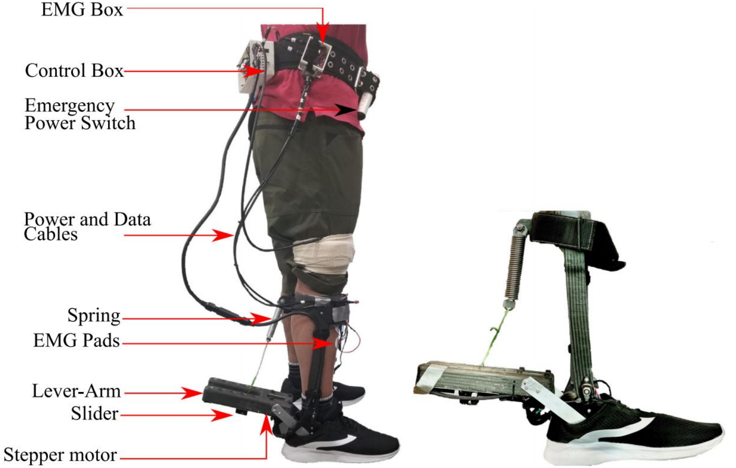
...

At peak loads, all flexures are engaged



Broader Impacts

Senior design project led to *RAL* paper!



(a) Experimental setup (b) Quasi-passive exoskeleton

Universidad de los Andes

M | ROBOTICS

Control and Optimal Design of Wearable Robots

Summer school 2020

Universidad de los Andes

For Women. For Success.

WEST Inc.

WOMEN'S ENTERPRISE SKILLS TRAINING OF WINDSOR INC.

WESTOFWINDSOR.COM

PhD student graduated

Edgar Bolívar-Nieto, PhD
UT Dallas → U. Michigan

Postdoc supported

Gray Thomas, PhD
U. Michigan

Thank you!

LocoLab and Neurobionics Lab - University of Michigan

NSF Awards
1830360 / 1953908
1830338

ROBOTICS

UNIVERSITY of DENVER

THE UNIVERSITY OF TEXAS AT DALLAS

See you at Poster #40!