

Optimal Placement of Energy Storage in Distribution Networks

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Motivation

Energy storage devices

- shift generation and consumption across time
- help integrate renewable energy resources



Challenges

- Joint optimization over both *time* and *space*
- Numerical algorithms available but few *structural results*

Formulation

Total storage budget: $\sum_x B(x) \leq B_{\text{tot}}$

At location x :

storage state of charge

$$b(x, t) \in [0, B(x)]$$

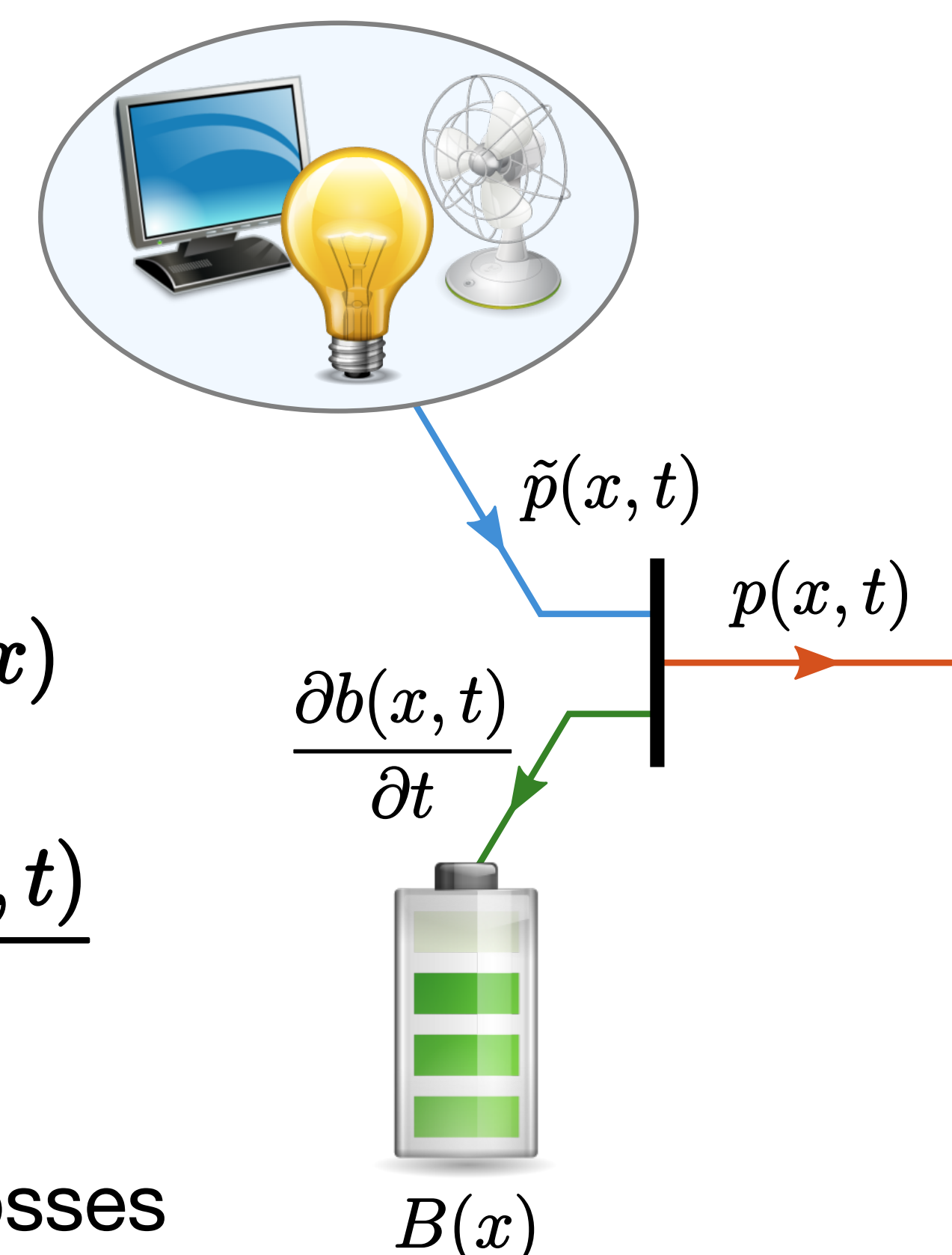
background injection

$$\tilde{p}(x, t) \approx \alpha(x)p(t) + \beta(x)$$

net power injection

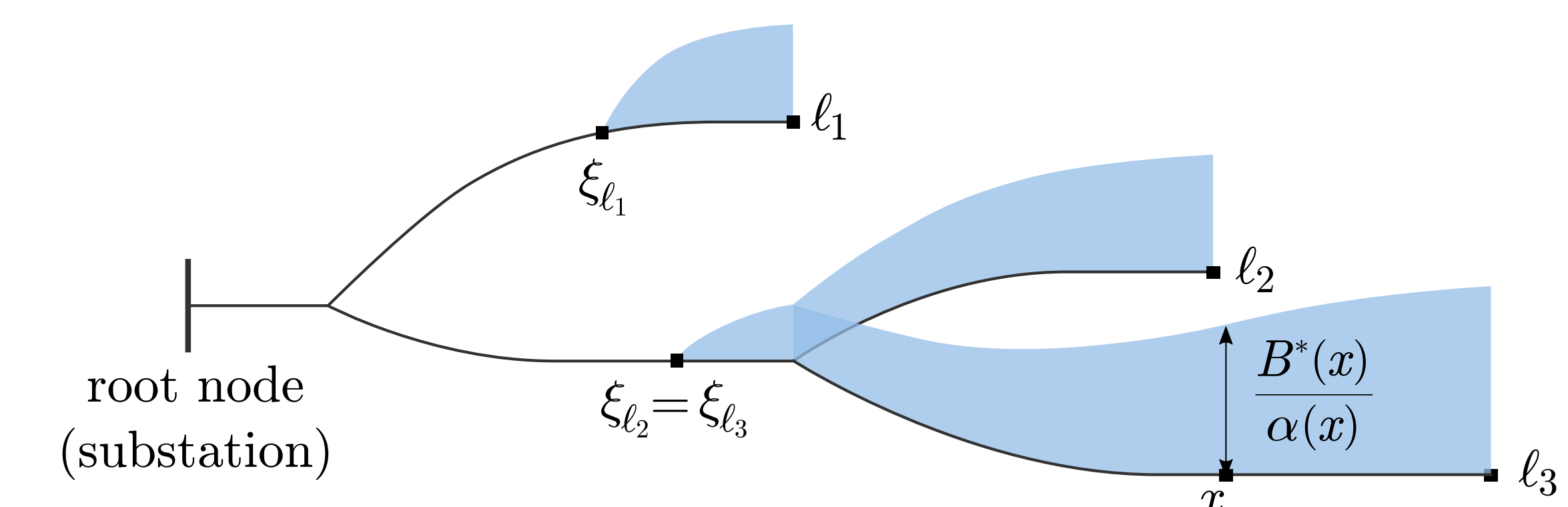
$$p(x, t) = \tilde{p}(x, t) - \frac{\partial b(x, t)}{\partial t}$$

Goal: minimize network losses

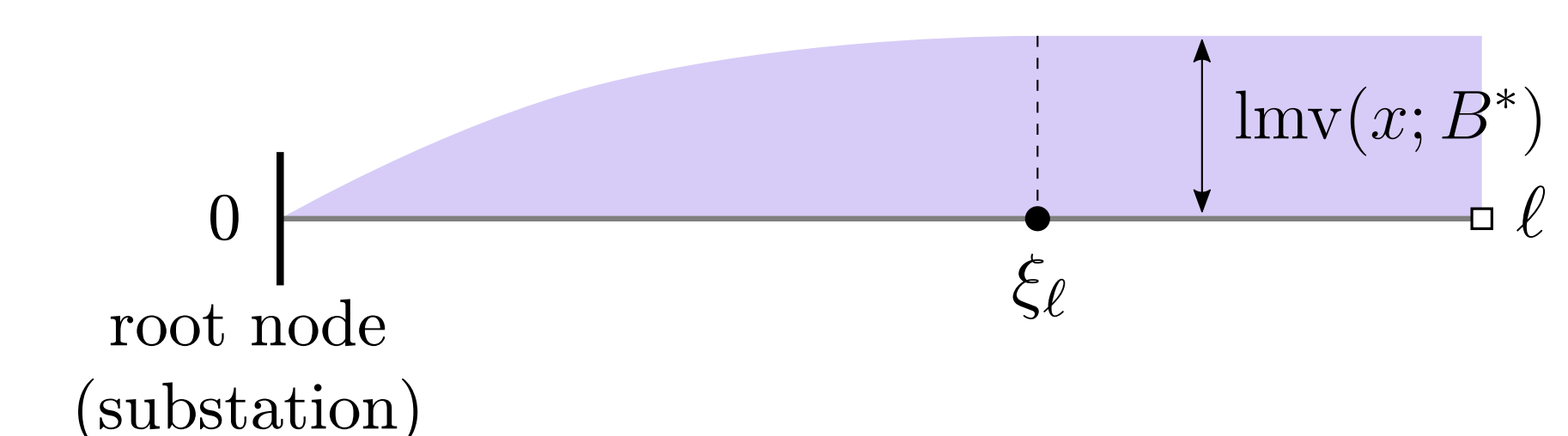


Structure of Optimal Placement

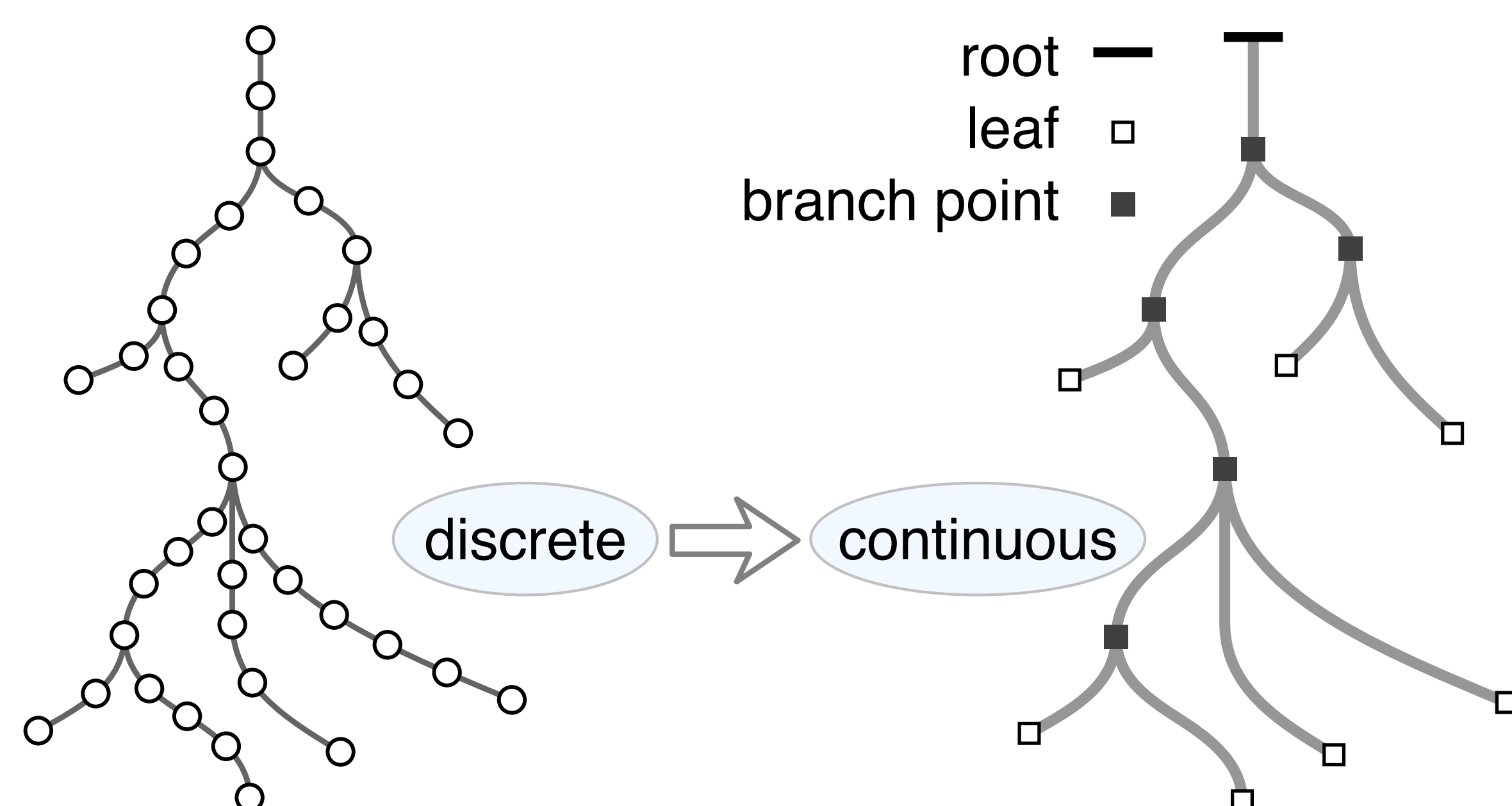
1. On each path connecting the root and a leaf, a point separates the path into two segments:
 - no storage on the segment containing the root
 - storage everywhere on the segment containing the leaf
2. The scaled optimal capacity $B^*(x)/\alpha(x)$ is *increasing* from the root to any leaf.



3. The locational marginal value of storage under optimal placement
 - is *increasing* from the root to any leaf,
 - is *equalized* over places where nonzero storage is allocated.



Solution



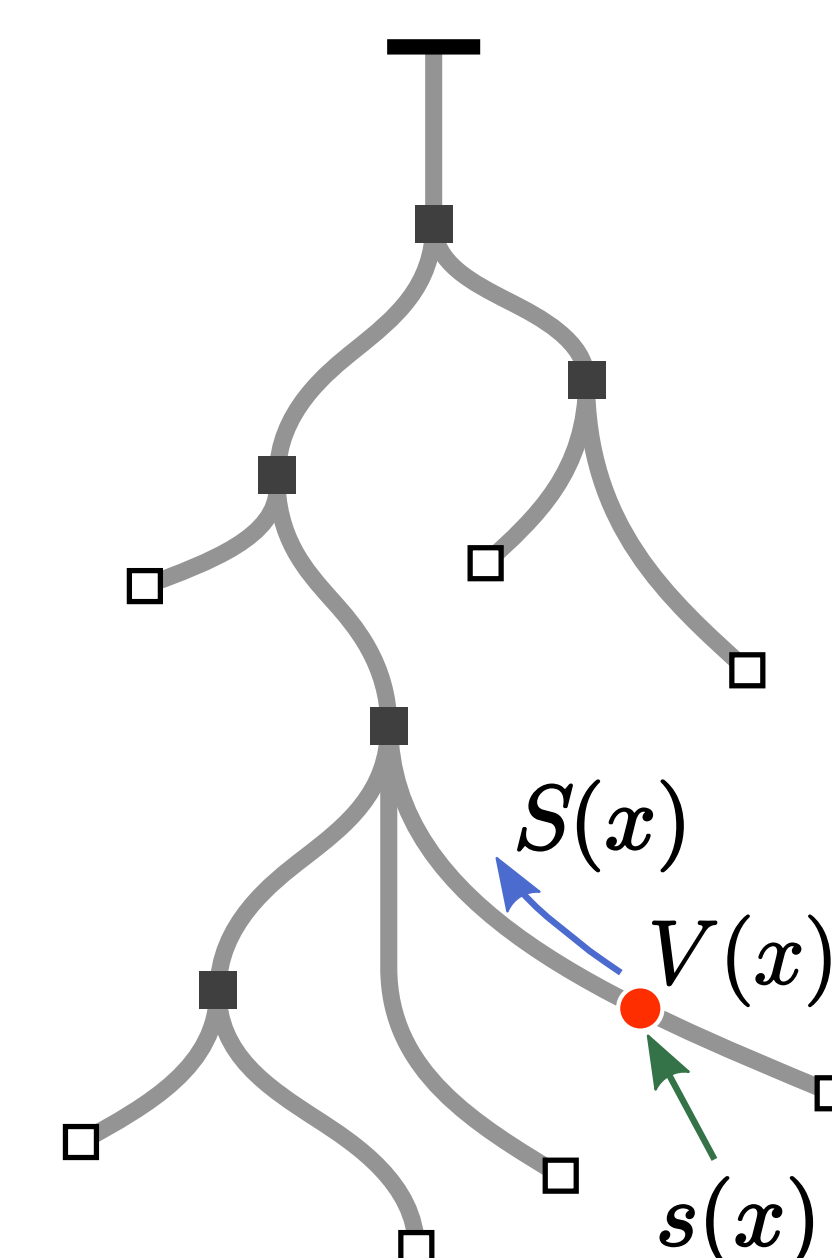
Power Flows on *Continuous Trees*

$$S(x) = \int_{y \geq x} \left(s(y) - z(y) \frac{|S(y)|^2}{|V(y)|^2} \right) dy$$

$$|V(x)|^2 = |V_{\text{root}}|^2 + 2 \int_{\text{root}}^x \text{Re} [z^*(y)S(y)] dy$$

✓ Continuous version of *DistFlow Equations*

✓ Allow doing *calculus* on the network



Reference

- [1] Y. Tang and S. H. Low. "Optimal placement of energy storage in distribution networks," IEEE Transactions on Smart Grid, 10.1109/TSG.2017.2711921