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Passivity and Symmetry for Control in CPS

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UND

Science of Integration for Cyber Physical Systems

Vanderbilt University

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Fundamentals of Composition in Heterogeneous Systems

- Decoupling cross layer interaction.

  Achieving orthogonality of design concerns

  Develop passivity and symmetry based tools

Role of Passivity and Symmetry

•How do we guarantee desirable properties in a network of approximately known heterogeneous systems, which may change dynamically, and expand or contract?

•Need results that offer insight on how to do synthesis – e.g. how do we grow the system in particular ways to preserve its properties?

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Team

- Mike McCourt, Cherry Yu, Feng Zhu, Po Wu, Eloy Garcia, Yingbo Zhao, Meng Xia, Vahideh Ghanbari, Jason Nightingale, Ashley Kulczycki (Graduate Students)
- Yue Wang, Surya Shravan, (Post-docs)
- Getachew Befekadu (Post-doc ,Research Faculty)
- Vijay Gupta, Bill Goodwine, Panos Antsaklis (PIs)

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Progress and Accomplishments

- Dissipativity and Passivity in Approximate Models.

•Dissipativity and Passivity in Switched, Hybrid and Discrete Event Systems.

•Experimental Determination of Passivity Indices.

•Symmetry and Approximately Symmetric Systems.

•Networking, Asynchrony, Uncertainty, Performance Scalability.

Some Publications (Journals)

- H. Yu and P.J. Antsaklis, “Event-Triggered Output Feedback Control for Networked Control Systems Using Passivity: Achieving L2 Stability in the Presence of Communication Delays and Signal Quantization,” Automatica, Vol.49, No.1, pp. 30-38, January 2013.
- Eloy Garcia and Panos J. Antsaklis, “Model-Based Event-Triggered Control for Systems with Quantization and Time-Varying Network Delays.” IEEE Trans Auto. Control, Vol.58,No.2, pp. 422-434, February 2013.
- Eloy Garcia and Panos J. Antsaklis, “Parameter estimation and adaptive stabilization in time-triggered and event-triggered model-based control of uncertain systems.” Int. Journal Control, Vol.85, No.9, pp. 1327-1342, 2012.
- B. Goodwine and P.J. Antsaklis, “Multiagent Compositional Stability Exploiting System Symmetries,” Automatica. Accepted.
- G. Befekadu, V. Gupta, P.J. Antsaklis, "Characterization of feedback Nash equilibria for multi-channel systems via a set of non-fragile stabilizing state-feedback solutions and dissipativity inequalities," Journal of Math. Control Signals Syst., January 2013.
- G. K. Befekadu, V. Gupta and P. J. Antsaklis, "Reliable decentralized stabilization via extended LMIs and constrained dissipativity," International Journal of Robust and Nonlinear Control. To appear.
- Yue Wang, Vijay Gupta and Panos J. Antsaklis, “Generalized Passivity in Discrete-Time Switched Nonlinear Systems,” TAC, accepted.

Some Publications (Conferences)

***Proceedings of the 2012 American Control Conference*, Montreal, June 2012.**

•Han Yu and P.J. Antsaklis, "Quantized Output Synchronization of Networked Passive Systems with Event-driven Communication"

•Eloy Garcia and P.J. Antsaklis, "Decentralized Model-Based Event-Triggered Control for Networked Systems"

•Michael J. McCourt and Panos J. Antsaklis, "Stability of Interconnected Switched Systems using QSR Dissipativity with Multiple Supply Rates"

•Feng Zhu, Han Yu, Michael J. McCourt and Panos J. Antsaklis, "Passivity and Stability of Switched Systems under Quantization," *Proceedings of the 2012 HSCC*, Beijing, China.

•Panos J. Antsaklis, Michael J. McCourt, Han Yu, Feng Zhu, "Cyber-Physical System Design Using Dissipativity," (Plenary), *Proc. of the 31st Chinese Control Conference (CCC'2012)*, Hefei, China, July 2012.

•Eloy Garcia and P.J. Antsaklis, "Model-Based Control of Continuous-Time and Discrete-Time Systems with Large Network Induced Delays," *Proceedings of the 20th Mediterranean Conference on Control and Automation (MED'12)*, Barcelona, Spain, July 2012.

Some Publications (Conferences)

***Proceedings of the 51st Conference on Decision and Control*, Maui, Dec 2012.**

•Eloy Garcia and P.J. Antsaklis, “Output Feedback Model-Based Control of Uncertain Discrete-Time Systems with Network Induced Delays”

•Yue Wang, Vijay Gupta and Panos J. Antsaklis, “Generalized Passivity in Discrete-Time Switched Nonlinear Systems”

•Han Yu and Panos J. Antsaklis, “Distributed Formation Control of Networked Passive Systems with Event-driven Communication”

•Han Yu and Panos J. Antsaklis, “Formation Control of Multi-agent Systems with Connectivity Preservation by Using both Event-driven and Time-driven Communication”

***Proceedings of the 2013 American Control Conference*, Washington, June 2013.**

•Yue Wang, Vijay Gupta and Panos J. Antsaklis, “Stochastic Passivity of Discrete-Time Markovian Jump Nonlinear Systems”

•Meng Xia, Panos J. Antsaklis and Vijay Gupta, “Passivity Analysis of a System and its Approximation”

•Han Yu, Eloy Garcia and P.J. Antsaklis, “Model-Based Scheduling for Networked Control Systems”

•Meng Xia, Vijay Gupta, Panos J. Antsaklis, “Networked State Estimation over a Shared Communication Medium”

•Han Yu, Feng Zhu, Meng Xia and Panos J. Antsaklis, “Robust Stabilizing Output Feedback Nonlinear Model Predictive Control by Using Passivity and Dissipativity,” 2013 European Control Conference (ECC13), Zurich, Switzerland, July 2013.

PhD Dissertations,
Dept. Elect. Eng., University of Notre Dame

•M.J. McCourt, “Dissipativity Theory for Hybrid Systems with Applications to Networked Control Systems,” May 2013.

•Po Wu, “Symmetry and Dissipativity in the Design of Large Scale Complex Control,” June 2013.

•Han (Cherry) Yu, "Passivity and Dissipativity as Design and Analysis Tools for Networked Control Systems," January 2013.

•Eloy Garcia, "Model-Based Control over Networks: Architecture and Performance," June 2012.

Some Activities

Control of Cyber-Physical Systems Workshop

at the University of Notre Dame London Centre, October 20-21, 2012.

http://controls.ame.nd.edu/mediawiki/index.php/London_CPS_Workshop

IEEE Transactions on Automatic Control, Special Issue on the

Control of Cyber-Physical Systems. In progress.

Talks

Universite de Lorraine – Nancy, France. December 4, 2012.

"On the Control of Cyber-Physical Systems"

Imperial College of Science, Technology and Medicine –London, UK October 16, 2012.
"Control of Cyber-Physical Systems Using Passivity and Symmetry"

Technische Universitaet Berlin (TU Berlin) – Electrotechnik und Informatik Berlin, Germany March 13, 2012.

"Control of Cyber-Physical Systems"

Presentations at Conferences and Workshops

Some Activities

Keynote Addresses and Plenaries

2012 NSF Cyber-Physical Systems (CPS) PI Meeting – Science Keynote Address

Washington DC, October 3-5, 2012.

“Cyber-Physical Systems Design Using Dissipativity and Symmetry”

The 31st Chinese Control Conference (CCC’12), Hefei, China, July 25 -27, 2012.

“Cyber-Physical Systems Design Using Dissipativity”

International Workshop on Emerging Frontiers in Systems and Control, Tsinghua University, Beijing, China, May 18, 2012.

“Cyber-Physical Systems, Symmetry and Passivity”

Invited Panel Panelist

Panel Discussion on “Control Engineering Impact on the Society of the Future: Challenges of Cyber-Physical Systems” at the 2012 Mediterranean Conference on Control and Automation (MED12), Barcelona, Spain, July 4, 2012.

Panel Discussion on “Cyber-Physical Systems” at the NIST Workshop on Performance Metrics for Intelligent Systems (PerMIS 2012), University of Maryland, March 22, 2012.

Outline

- **Passivity & Dissipativity in Dynamical Systems**.
- **Experimental Determination of Passivity Indices and Vanderbilt's Testbed.**
- **Dissipativity and Passivity in Approximate Models.**
- **Dissipativity and Passivity in Hybrid and Discrete Event Systems (Surya Shravan)**
- **Symmetry and Stability. (Bill Goodwine)**
- **Networking, Asynchrony, Uncertainty. (Vijay Gupta)**

Dissipativity in Hybrid and Discrete Event Systems (Surya Shravan)

1. Dissipativity for Hybrid Input / Output Automata and its compositional nature

This work is based on a modified version of the framework introduced by Nancy Lynch's work. It also includes some important assumptions from Zhao and Hill's work on passivity of switched systems.

2. Dissipativity for Finite State Automata and Dissipativity for Discrete Event Systems based on Kevin Passino's work

3. Preserving passivity using finite state approximations based on Paulo Tabuada's work on approximate simulations and bisimulations.

Tabuada's symbolic subsystems framework allows us to define passivity for symbolic systems, which is consistent with a notion of passivity of conventional (infinite state) discrete time systems.

We show that under some assumptions input output passivity can be preserved, however the passivity condition we obtain is practical passivity. These assumptions are based on Oishi's work on Discretization.

Symmetry Topics (Bill Goodwine)

Extensions to approximately symmetric systems

a. Stability of approximately symmetric systems (stability to the origin)
b. Stability of approximately symmetric systems to sets (likeLaSalle's invariance principle)

Boundedness of nonautonomous symmetric systems

a. Boundedness of solutions about a point
b. Boundedness of solutions about a set (in progress)

Network Topics (Vijay Gupta)

Passivity in the presence of communication networks

Fundamental performance limitations in distributed control

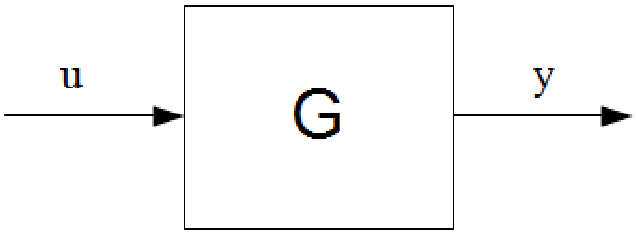
Processor cooperation in CPS systems

Passivity & Dissipativity

Definition of Passivity in Continuous-time

- Consider a continuous-time nonlinear dynamical system

$$\dot{x} = f(x,u)$$
$$y = h(x,u).$$



- This system is *passive* if there exists a continuous storage function *V(x) ≥ 0 (for all x)* such that

$$\int_{t_1}^{t_2} u^T(t)y(t)dt + V(x(t_1)) \geq V(x(t_2))$$

for all $t_2 \geq t_1$ and input $u(t) \epsilon U$.

- When V(x) is continuously differentiable, it can be written as:

$$u^T(t)y(t) \geq \dot{V}(x(t))$$

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Extended Definitions of Passivity

Passive	$u^T y \geq \dot{V}(x)$
Lossless	$u^T y = \dot{V}(x)$
Strictly Passive	$u^T y \geq \dot{V}(x) + \psi(x)$
Strictly Output Passive	$u^T y \geq \dot{V}(x) + \varepsilon y^T y$
Strictly Input Passive	$u^T y \geq \dot{V}(x) + \delta u^T u$

- Note that *V(x)* and *Ψ(x)* are positive definite and continuously differentiable. The constants ε and δ are positive. These equations hold for all times, inputs, and states.

Stability of Passive Systems

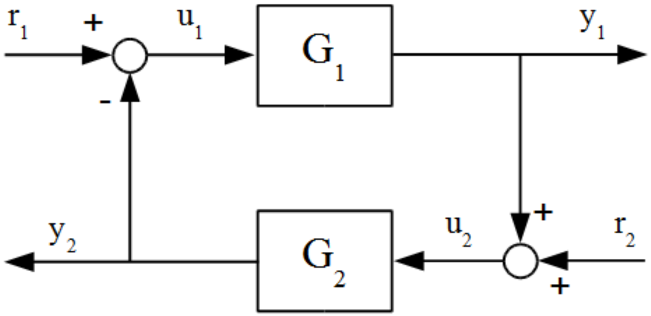
- Strictly passive systems ($\psi(x)>0$) are asymptotically stable
- Output strictly passive systems ($\delta>0$) are L_2 stable
- The following results hold in feedback
  - Two passive systems → passive and stable loop
  - Passive system and a strictly passive system → asymptotically stable loop
  - Two output strictly passive systems → L_2 stable loop
  - Two input strictly passive systems ($\varepsilon>0$) → L_2 stable loop

$$u^T y \geq \dot{V}(x) + \varepsilon u^T u$$

Interconnections of Passive Systems

- One of the strengths of passivity is when systems are interconnected. Passive systems are stable and passivity is preserved in many practical interconnections.

- For example, the negative feedback interconnection of two passive systems is passive.

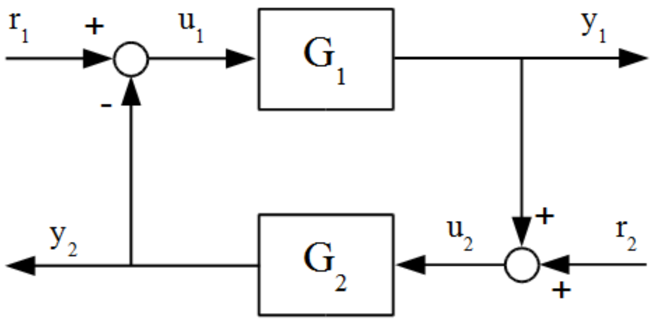


- If $u_1 \rightarrow y_1$ and $u_2 \rightarrow y_2$ are passive then the mapping $r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \rightarrow y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ is passive

- Note: the other internal mappings ($u_1 \rightarrow y_2$ and $u_2 \rightarrow y_1$) will be stable but may not be passive

Interconnections of Stable Systems

- Compared with passive systems, the feedback interconnection of two stable systems is not always stable



- One notable special case is the small gain theorem where if G_1 and G_2 are finite-gain L_2 stable with gains γ_1 and γ_2 then the interconnection is stable if $\gamma_1\gamma_2 < 1$.

- Both Passivity theory and the small gain theorem are special cases of larger frameworks including the conic systems theory and the passivity index theory.

Definition of Dissipativity in Continuous-time

- This concept generalizes passivity to allow for an arbitrary energy supply rate *ω(u,y).*
- A system is *dissipative* with respect to supply rate *ω(u,y)* if there exists a continuous storage function $V(x) \geq 0$ such that

$$\int_{t_1}^{t_2} \omega(u,y)dt \geq V(x(t_2)) - V(x(t_1))$$

  for all t_1, t_2 and the input $u(t) \in U$.

- A special case of dissipativity is the QSR definition where the energy supply rate takes the following form:

$$\omega(u,y) = y^T Q y + 2y^T S u + u^T R u.$$

- *QSR dissipative systems are* L_2 *stable when* $Q<0$

QSR Dissipativity (CT)

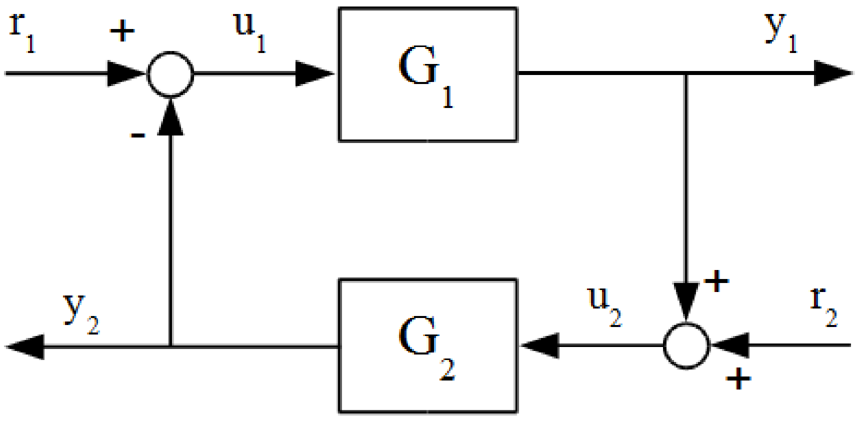
- Consider the feedback interconnection of G_1 and G_2
  - G_1 is QSR dissipative with Q_1, S_1, R_1
  - G_2 is QSR dissipative with Q_2, S_2, R_2

- The feedback interconnection

$$r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \to y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

is stable if

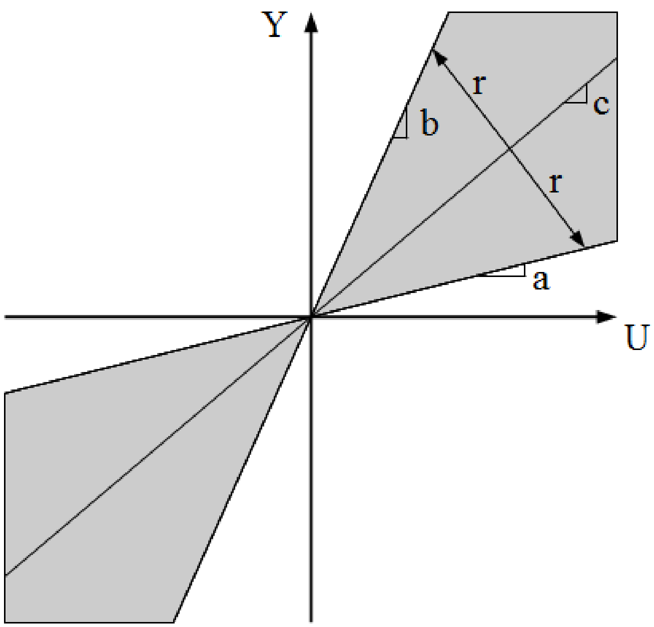
$$\tilde{Q} = \begin{bmatrix} Q_1 + R_2 & S_1 - S_2{}^T \\ S_1{}^T - S_2 & Q_2 + R_1 \end{bmatrix} < 0$$



- Other mappings ($r_1 \to y_2$ and $r_2 \to y_1$) are stable but may not be passive
- Large scale sytems (with multiple feedback connections) can be analyzed using QSR dissipativity to show stability of the entire system

Conic Systems

- A conic system is one whose input-output behavior is constrained to lie in a cone of the *U x Y* inner product space



- A system is conic if the following dissipative inequality holds for all $t_2 \geq t_1$

$$\int_{t_1}^{t_2} \left[(1+\frac{a}{b})u^T y - a u^T u - \frac{1}{b} y^T y \right] dt \geq V(x(t_2)) - V(x(t_1))$$

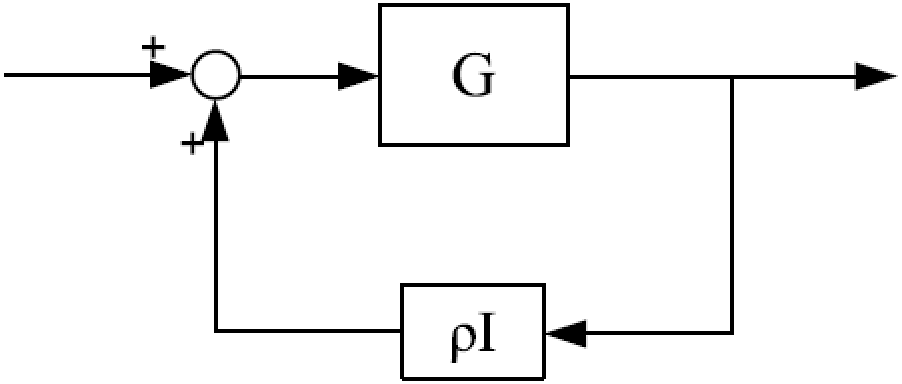
Passivity Indices

- Conic systems and passivity indices capture similar information about a system

- A passivity index measures the level of passivity in a system
- Two indices are required to characterize the level of passivity in a system
    1. The first measures the level of stability of a system
    2. The second measures the extent of the minimum phase property in a system
- They are independent in the sense that knowing one provides no information about the other
- Each has a simple physical interpretation

Output Feedback Passivity Index

The output feedback passivity index (OFP) is the largest gain that can be put in positive feedback with a system such that the interconnected system is passive.

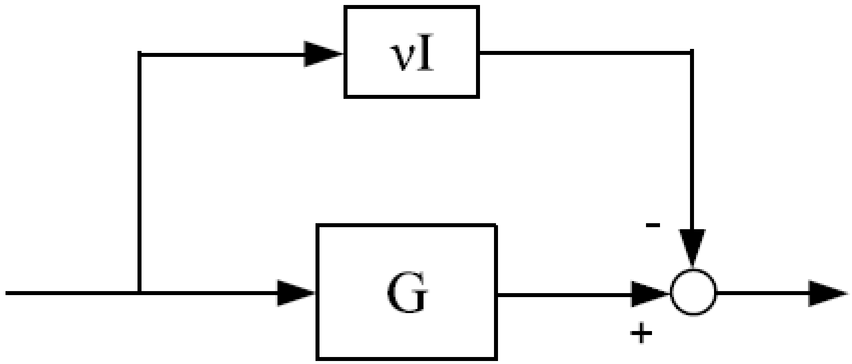


Equivalent to the following dissipative inequality holding for G

$$\int_{t_1}^{t_2} u^T y\, dt \geq V(x(t_2)) - V(x(t_1)) + \rho \int_{t_1}^{t_2} y^T y\, dt$$

Input Feed-Forward Passivity Index

The input feed-forward passivity index (IFP) is the largest gain that can be put in a negative parallel interconnection with a system such that the interconnected system is passive.

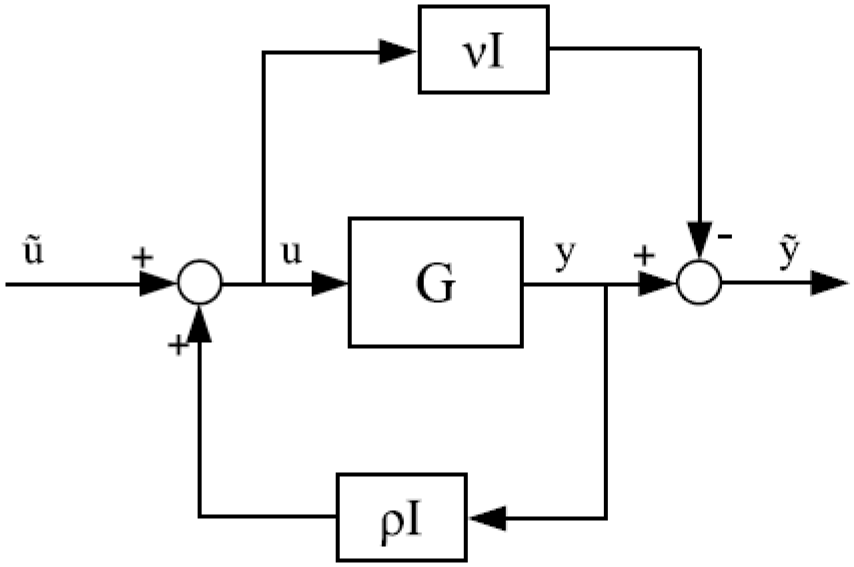


Equivalent to the following dissipative inequality holding for G

$$\int_{t_1}^{t_2} u^T y\,dt \geq V(x(t_2)) - V(x(t_1)) + \nu \int_{t_1}^{t_2} u^T u\,dt$$

Simultaneous Indices

When applying both indices the physical interpretation as in the block diagram

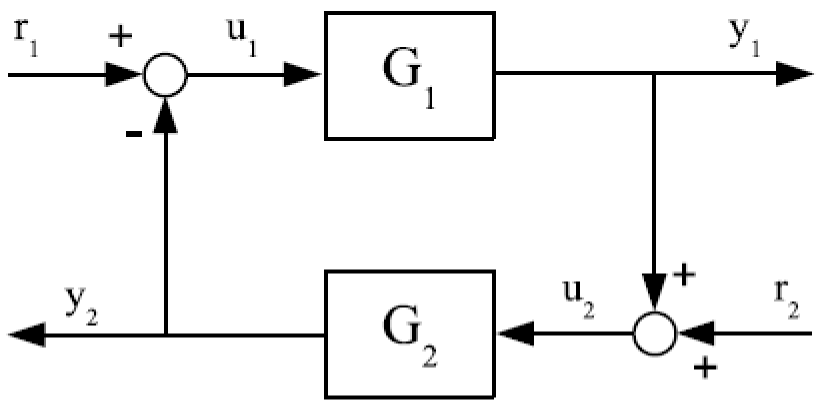


Equivalent to the following dissipative inequality holding for G

$$(1+\rho\nu)\int_{t_1}^{t_2} u^T y\,dt \geq V(x(t_2)) - V(x(t_1)) + \rho\int_{t_1}^{t_2} y^T y\,dt + \nu\int_{t_1}^{t_2} u^T u\,dt$$

Stability

We can assess the stability of an interconnection using the indices for G_1 and G_2



G_1 has indices ρ_1 and ν_1

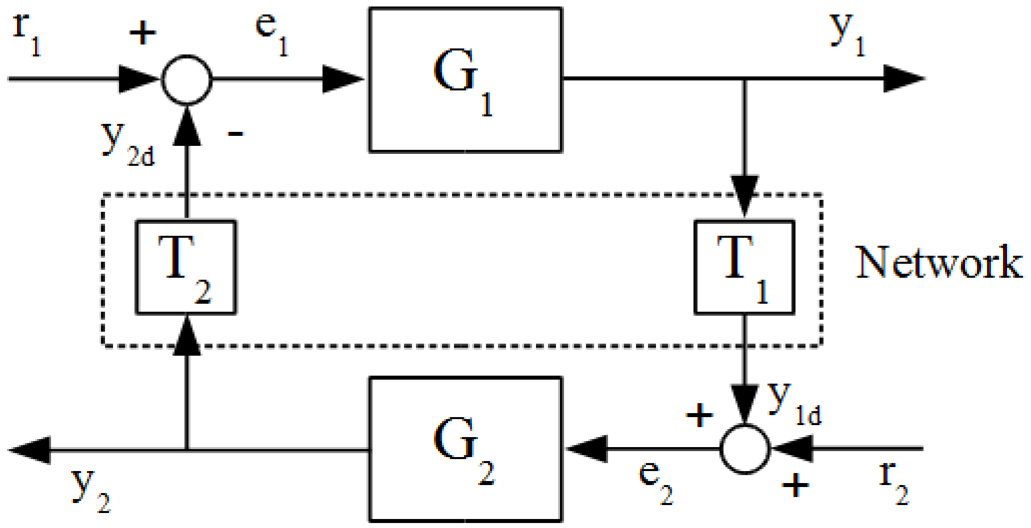
G_2 has indices ρ_2 and ν_2

The interconnection is $\mathcal{L}_2$ stable if the following matrix is positive definite

$$\begin{bmatrix} (\rho_1 + \nu_2)I & \frac{1}{2}(\rho_2\nu_2 - \rho_1\nu_1)I \\ \frac{1}{2}(\rho_2\nu_2 - \rho_1\nu_1)I & (\rho_2 + \nu_1)I \end{bmatrix} > 0$$

Networked Systems

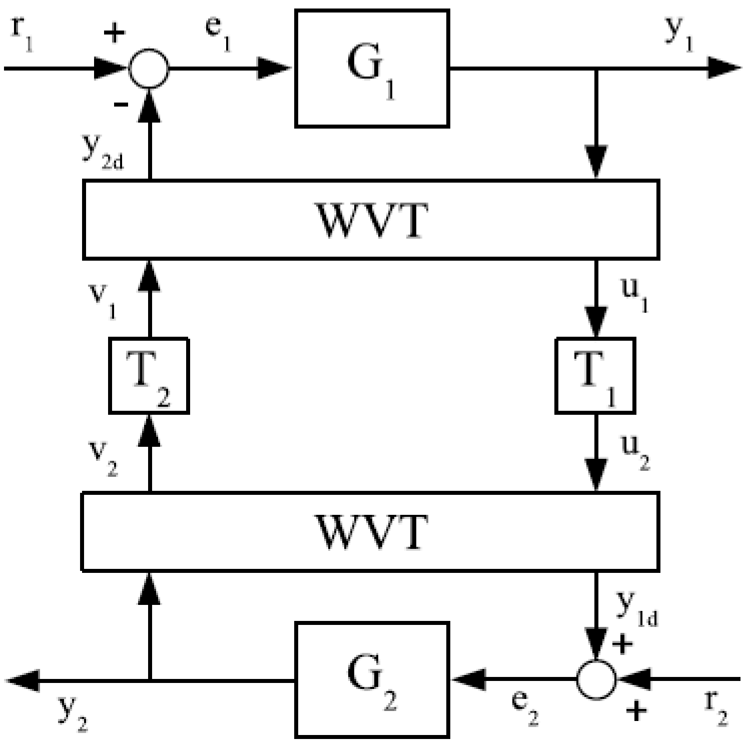
- Motivating Problem: The feedback interconnection of two passive systems is passive and stable. However, when the two are interconnected over a delayed network the result is not passive so stability is no longer guaranteed. How do we recover stability?



The systems G_1 and G_2 are interconnected over a network with time delays T_1 and T_2

Stability of Networked Passive Systems

- One solution to interconnecting passive systems over a delayed network is to add an interface between the systems and the network
- The wave variable transformation forces the interconnection to meet the small gain theorem. Stability is guaranteed for arbitrarily large time delays
- The WVT is defined below



$$\begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \frac{1}{\sqrt{2b}} \begin{bmatrix} bI & I \\ bI & -I \end{bmatrix} \begin{bmatrix} y_1 \\ y_{2d} \end{bmatrix}$$

$$\begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \frac{1}{\sqrt{2b}} \begin{bmatrix} bI & I \\ bI & -I \end{bmatrix} \begin{bmatrix} y_{1d} \\ y_2 \end{bmatrix}$$

Experimentally Determining Passivity Indices

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Determining Passivity Experimentally

- The properties of passivity and dissipativity are traditionally demonstrated by finding an energy storage function that satisfies an inequality

- Passivity is independent of internal realization and can be shown by testing a sufficient class of inputs with respect to the following

> ***Definition:*** *A system is passive if there exists a constant β such that the following inequality holds for all inputs u(t) in a set U and all times T*
>
> $$\int_0^T y^T(t)u(t)dt \geq -\beta.$$

- In practice, this test can be performed with respect to an input set U and zero initial conditions

$$\int_0^T y^T(t)u(t)dt \geq 0$$

Experimental Passivity Indices

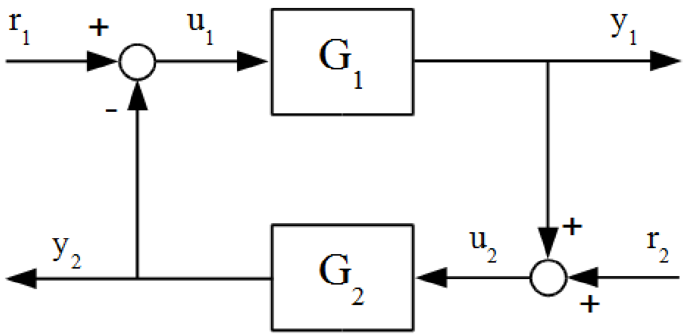
- Passivity indices can be determined similarly from input-output data

$$\int_0^T \left[(1+\rho\nu)y^T u - \rho y^T y - \nu u^T u\right] dt \geq 0$$

- For each input *u(t)* in *U* and all times *T*, constraints are placed on the indices

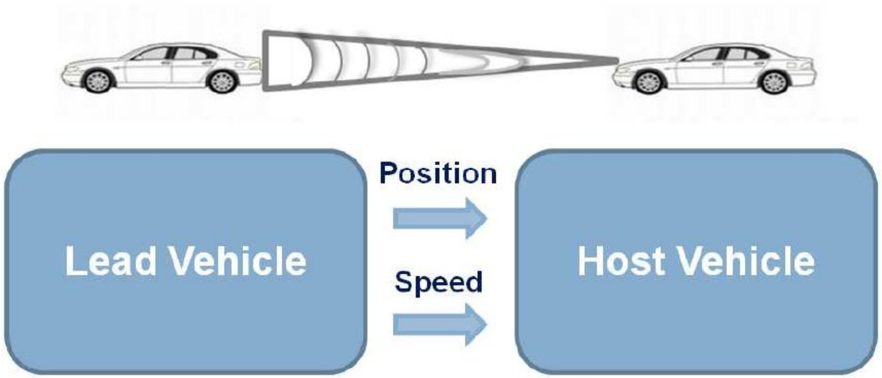
- This sort of testing requires extensive data collection but it is similar to data collection for modeling purposes

- After data collection, a controller can be designed for closed loop stability and/or closed loop indices

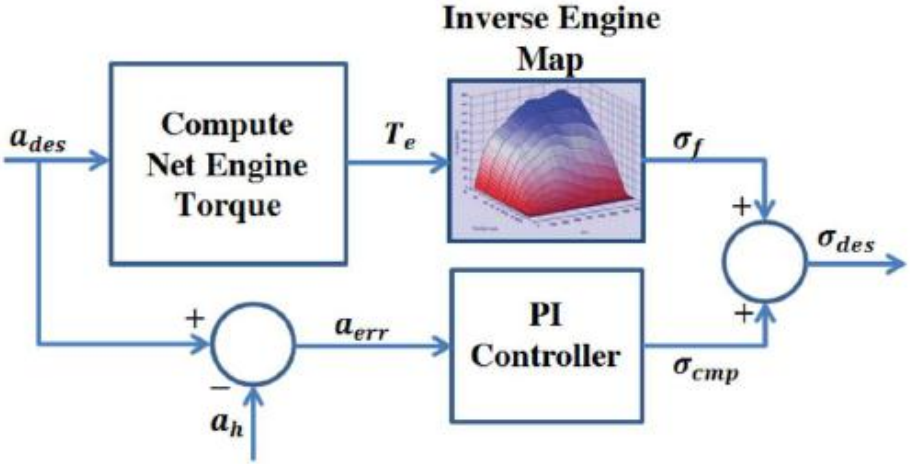


Adaptive Cruise Control

- The ACC algorithm has two modes: traditional cruise control and maintaining a following time behind a lead vehicle



- The ACC algorithm can be adjusted by tuning the gains of a PI controller
- These gains can adjusted for closed loop passivity or passivity indices

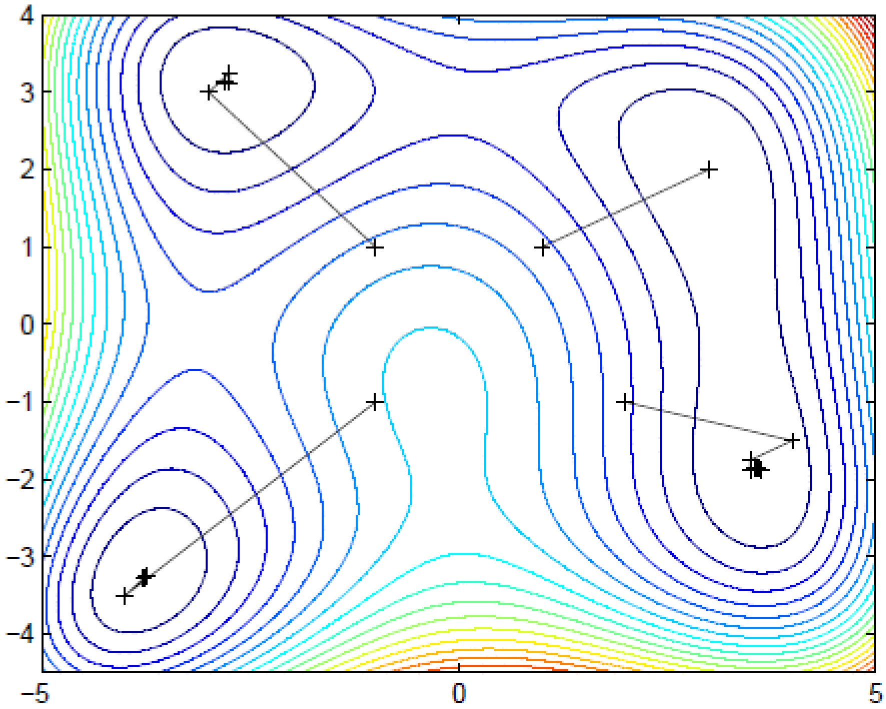


Optimization of Passivity Indices

- The algorithm gives a range of passivity indices that depend on the PI gains chosen
- Finding the maximum indices may allow for more flexibility for future design

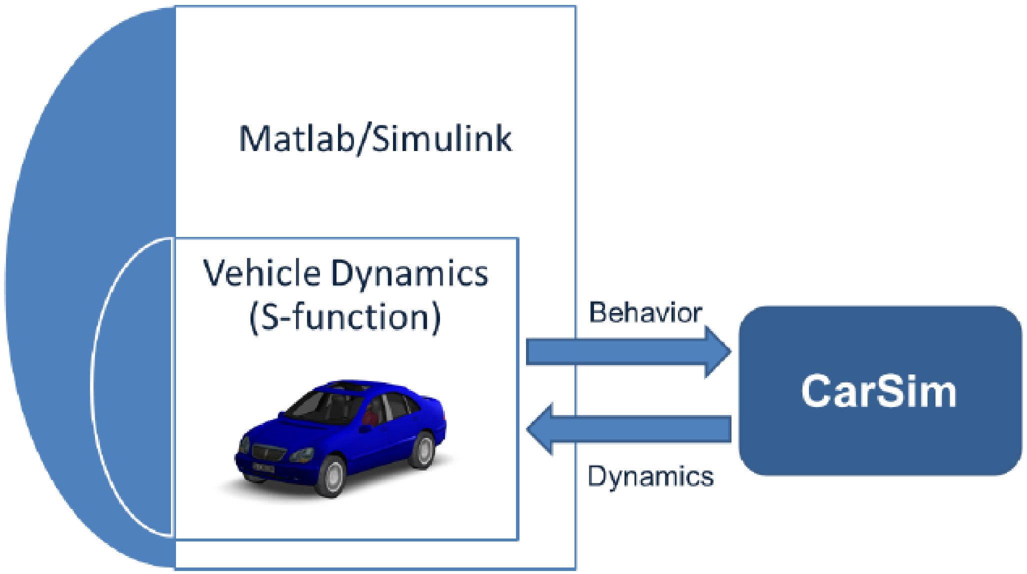
- Can use a numerical optimization method to optimize indices
- Since models are not specified, must use an optimization method that is not based on gradient methods
- The method used in this work is from Hooke and Jeeves

[Hooke and Jeeves 1969]



The method works by evaluating the cost function in several directions and then updates by moving in a direction to reduce the cost function

Simulation Platform



- The approach was verified by simulation using CarSim and MATLAB
- The results gave a maximal set of passivity indices for the closed loop system consisting of vehicle and ACC
- The work is currently being implemented on a virtual platform (Vanderbilt)

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On Passivity Bounds using Approximate Models

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Approximate Models

In a large scale CPS, precise knowledge of the mathematical model will be difficult to obtain.

Trade off between model accuracy and tractability may lead to the use of a simple model.

e.g. linear models, lower-order models…

➢ *Question: Can passivity of a system be guaranteed if a model 'close' to it is passive*

Definition 1: Consider a system Σ with input u and output y where $u(t), y(t) \in \mathbb{R}^m$. It is said to be

- **passive**, if there exists a constant $\beta \leq 0$ such that

$$\langle u, y \rangle_T \geq \beta.$$

- **input strictly passive (ISP)**, if there exist constants $\nu > 0$ and $\beta \leq 0$ such that

$$\langle u, y \rangle_T \geq \beta + \nu \langle u, u \rangle_T. \tag{1}$$

- **output strictly passive (OSP)**, if there exist constants $\rho > 0$ and $\beta \leq 0$ such that

$$\langle u, y \rangle_T \geq \beta + \rho \langle y, y \rangle_T. \tag{2}$$

- **very strictly passive (VSP)** , if there exist constants $\rho > 0$, $\nu > 0$ and $\beta \leq 0$ such that

$$\langle u, y \rangle_T \geq \beta + \rho \langle y, y \rangle_T + \nu \langle u, u \rangle_T. \tag{3}$$

For the system Σ,

- the **input feedforward passivity index** (IFP) is the largest $\nu > 0$ such that (1) holds $\forall u$ and $\forall T \geq 0$,
- the **output feedback passivity index** (OFP) is the largest $\rho > 0$ such that (2) holds $\forall u$ and $\forall T \geq 0$.

Denoted by **IFP(ν)** and **OFP(ρ)**, respectively.

Definition 2: Consider a system Σ with input u and output y where $u(t), y(t) \in \mathbb{R}^m$,

- any $\tilde{\nu} \in (0, \nu]$ is an **ISP level** of Σ if Σ has IFP(ν);
- any $\tilde{\rho} \in (0, \rho]$ is an **OSP level** of Σ if Σ has OFP(ρ);
- any $(\tilde{\rho}, \tilde{\nu})$ are **VSP levels** of Σ if Σ is VSP for (ρ, ν) such that $0 < \tilde{\rho} \leq \rho, 0 < \tilde{\nu} \leq \nu$.

- **finite-gain $\mathcal{L}_2$ stable** , if there exist $\kappa > 0$ and $\beta \leq 0$ such that

$$\langle y, y \rangle_T \leq -\beta + \kappa^2 \langle u, u \rangle_T. \quad (4)$$

- **QSR-dissipative**, if there exist $Q = Q^T$, $R = R^T$ and S and a constant $\beta \leq 0$, such that

$$r(u, y) \triangleq \langle y, Qy \rangle_T + 2\langle y, Su \rangle_T + \langle u, Ru \rangle_T \geq \beta. \quad (5)$$

  The function $r(u, y)$ is called the **supply rate** for Σ.

In all cases, we require that the inequality holds $\forall u(t), \forall T \geq 0$ and the corresponding $y(t)$.

✓For linear systems, LMI to test dissipativity:

$$\begin{bmatrix} A^T P + PA - C^T QC & PB - C^T QD - C^T S \\ B^T P - D^T QC - S^T C & -D^T QD - S^T D - D^T S - R \end{bmatrix} \leq 0$$

Problem Statement

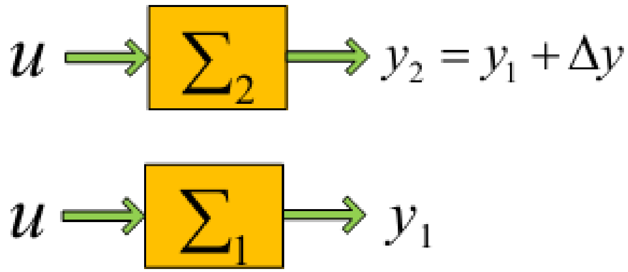


Fig. 1. Illustration of two systems: system Σ_1 with input u and output y_1, system Σ_2 with input u and output $y_2 = y_1 + \Delta y$, where u, y_1, y_2 and Δy are of the same dimensions.

- ✓ One could represent *a physical system* and the other *an approximation* of the system.
- ✓ Two different approximations of a physical system.

➢ 'Error' Constraint: $\langle \Delta y, \Delta y \rangle_T \leq \gamma^2 \langle u, u \rangle_T + \epsilon$

for stable linear systems γ is an upper bound on the H_∞ norm of the difference between the transfer functions G_1 and G_2.

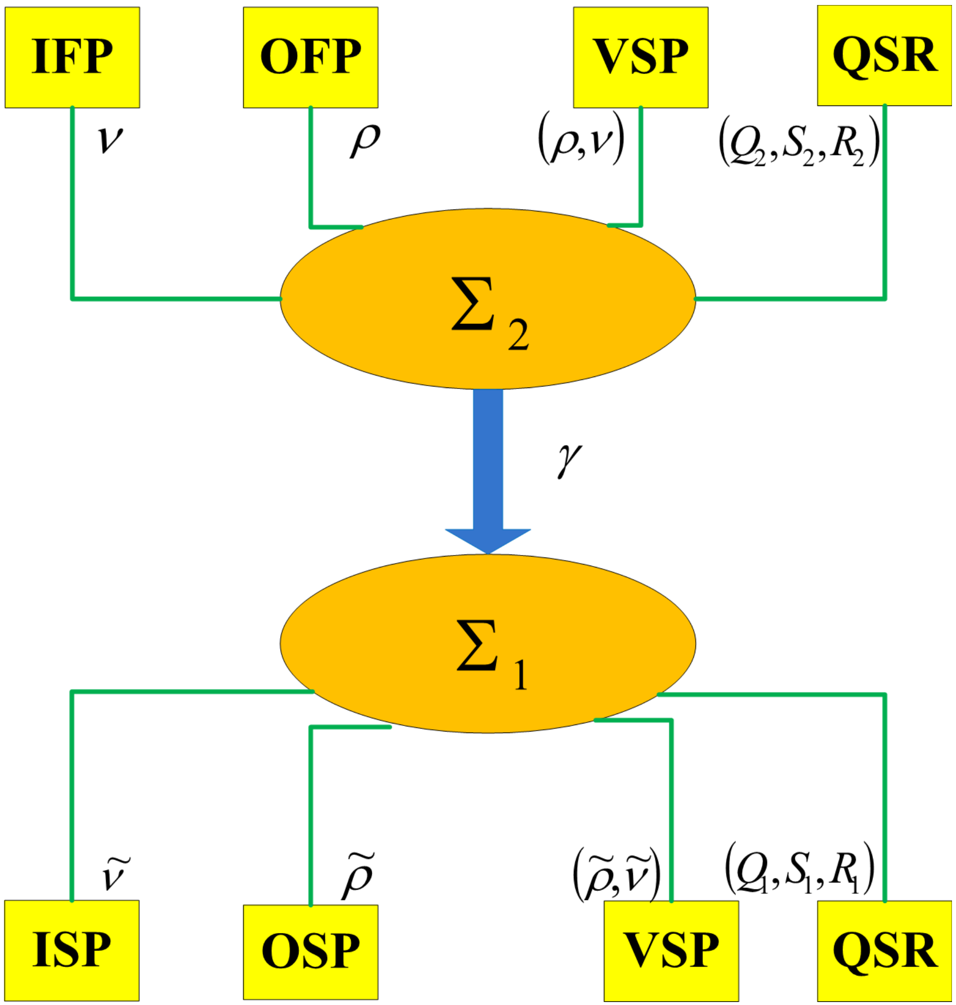
Problem 1: Suppose that an approximate model Σ_2

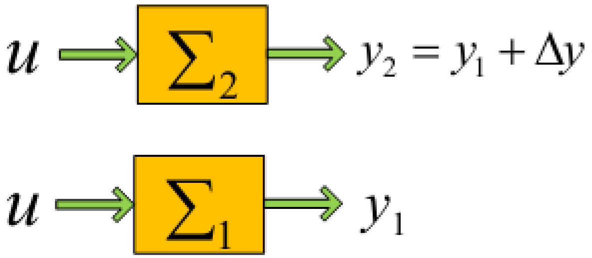
- has IFP(ν); or
- has OFP(ρ); or
- is VSP for (ρ, ν); or
- is (Q_2, S_2, R_2)-dissipative.

What corresponding passivity or QSR-dissipativity properties can be derived for the system Σ_1 based on (10)?

- General result: if the error between the system and its approximation is small,
  - ✓ when the approximate model has an excess of passivity, the system has a guaranteed passivity level as well.
  - ✓ QSR dissipativity has a similar robustness property.

Main Results





- *The following results:*
  - ✓ *hold when* Σ_1 and Σ_2 *exchange places;*
  - ✓ *can be developed in discrete-time domain.*

- *When an approximation is ISP:*

Theorem 1. *Consider* Σ_1 *and* Σ_2 *in Fig. 1. Suppose (8) is satisfied for some* $\gamma > 0, \epsilon \geq 0$. *Suppose* Σ_2 *has IFP*(ν), *the following results hold:*

1. *If* $\gamma < \nu$, *then* Σ_1 *is ISP for* $\tilde{\nu} = \nu - \gamma$;
2. *If* $\gamma \leq \nu$, *then* Σ_1 *is passive.*

Particular Approximation Methods

- *Model reduction* of linear systems.
  - ✓ algorithm: balanced truncated realization
  - ✓ 'error' bounds: $$\|G_1 - G_2\|_{H_\infty} \le 2 \sum_{i=r+1}^{n} \sigma_i$$

Corollary 5: Consider a stable LTI system G_1 with order n. Let G_2 be a reduced order model of G_1 with order r $(0 \le r < n)$ obtained using the TBR procedure. Define $\gamma \triangleq 2\sum_{i=r+1}^{n} \sigma_i$, where σ_i is the ith Hankel singular value of G_1 and $\sigma_1 \ge \sigma_2 \ge ...\sigma_n \ge 0$. Then,

1) If G_2 has IFP(ν) and $\gamma < \nu$, then G_1 is ISP for $\nu - \gamma$;
2) If G_2 is VSP for (ρ, ν) and $\gamma < \min\{\rho, \nu\}$, then G_1 is VSP for $(\rho - \gamma, \nu - \gamma)$ if $\gamma^2 - (\rho - \frac{2}{\rho})\gamma + \nu^2 - 2 \ge 0$.

➢ *Sampled-data systems*:

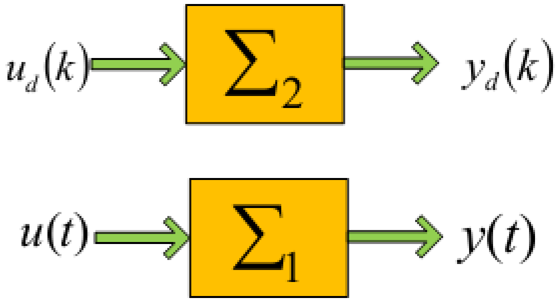
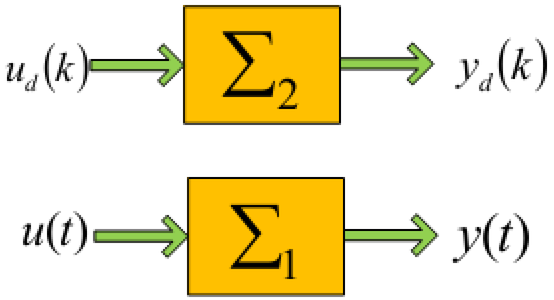


Fig. 2. Sampled-data System with an ideal sampler and a ZOH device, for which $u(t) = u_d(k)$ for $kh \leq t < (k+1)h$, $y_d(k) = y(kh)$ for all $k \geq 0$, where h represents the sampling period.

✓ assumption: $$\int_0^T \|\dot{y}(t)\|^2 \mathrm{d}t \leq \alpha^2 \int_0^T \|u(t)\|^2 \mathrm{d}t.$$

✓ 'error' constraint: $\gamma = \alpha h$

$$\langle \Delta y, \Delta y \rangle_T \leq \gamma^2 \langle u, u \rangle_T + \epsilon$$



Corollary 6: Consider a continuous-time system Σ_1 and its sampled-data system Σ_2 obtained from standard discretization, as shown in Fig. 2. Suppose that Assumption 2 is satisfied.

1) If Σ_2 has IFP(ν) and $\alpha h < \nu$, then Σ_1 has IFP no less than $\nu - \alpha h$;
2) If Σ_2 has IFP(ν) and $\alpha h \leq \nu$, then Σ_1 is passive;
3) If Σ_2 is VSP for (ρ, ν) and $\rho\nu^2 + \nu - \alpha h \geq 0$, then Σ_1 is passive.

Remarks

- Passivity properties of a system can be obtained by analyzing its approximation.
- Robustness properties of passivity and dissipativity with respect to model uncertainties.
- May be conservative without considering specific characterization.
  - Model reduction, sampled data systems, quantization, linearization
- Extension to hybrid switched dynamical systems.

M. Xia, P. J. Antsaklis, V. Gupta, "Passivity Analysis of a system and its approximation", *American Control Conference*, June 2013.

- Other Approximation methods (such as linearization and quantization) and hybrid switched systems

M. Xia, P. J. Antsaklis, V. Gupta, "Passivity and Dissipativity Analysis of a system and its approximation", *submitted to Special Issue on Cyber-Physical Systems in the Transactions on Automatic Control*, Feb 2013.

Summary

- Dissipativity / Passivity and Approximate Models.
- Dissipativity / Passivity in Switched, Hybrid and Discrete Event Systems.
- Symmetry, Approximate Symmetry, Dissipativity / Passivity and Stability.
- Experimental Determination of Passivity Indices and Vanderbilt's Testbed.
- Dissipativity / Passivity in Networked Dynamical Systems. Networking, Model-Based Control, Event-Triggered Control, Multi-Agent Systems.