# CNS-1329619: Robust team-triggered coordination for real-time control of networked cyber-physical systems


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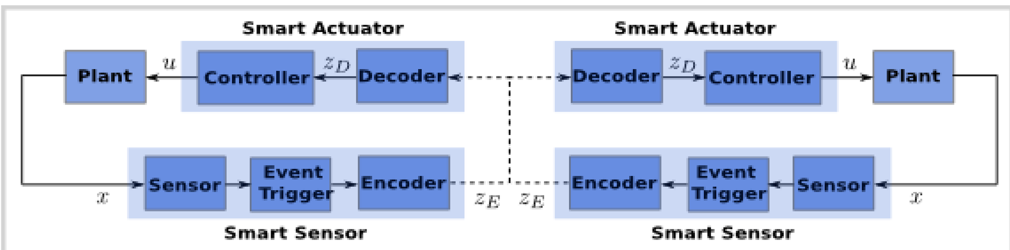
## Event-triggered stabilization of linear systems under channel blackouts

### Networked cyber-physical systems




- Shared communication channels with possibly low, time-varying, unreliable capacity
- Induced communication delays and finite precision (quantized) feedback
- Time intervals during which the channel is not available (channel blackouts)

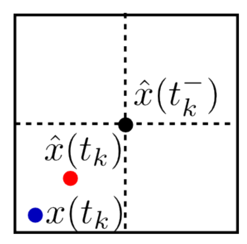
Time-triggered strategies are conservative in such scenarios as opposed to event-triggered ones.

### System description

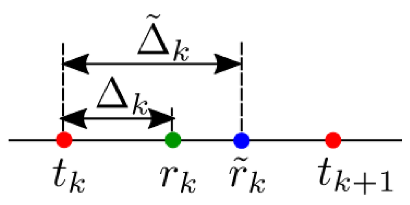
**Plant dynamics:** $\dot{x}(t) = Ax(t) + Bu(t), \quad u(t) = K\hat{x}(t), \quad x(t) \in \mathbb{R}^n$

**Dynamic controller flow:** $\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) = \bar{A}\hat{x}(t), \quad t \in [\tilde{r}_k, \tilde{r}_{k+1})$

**Dynamic controller jump:** $\hat{x}(\tilde{r}_k) \triangleq q_k(x(t_k), \hat{x}(t_k^-)),$ (quantization)



**Communication model:**

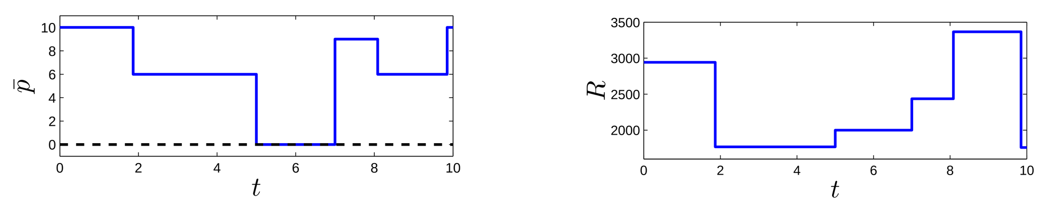


$$\Delta_k \leq \Delta(t_k, p_k) \triangleq \frac{b_k}{R_a(t_k)} = \frac{p_k}{R(t_k)}$$

$b_k = np_k$ is the # of bits transmitted at $t_k$

Can choose $\{t_k\}$, $\{p_k\}$, $\{\tilde{r}_k\}$

**Time-slotted channel model:**



$R(t) = R_j, \quad \forall t \in (\theta_j, \theta_{j+1}]$, **min comm. rate:** $\frac{p_k}{\Delta(t_k, p_k)} \geq R(t_k)$

$\bar{p}(t) = \bar{\pi}_j, \quad \forall t \in (\theta_j, \theta_{j+1}]$, **max packet size:** $p_k \leq \bar{p}(t_k)$

- Channel is not available when $\bar{p} = 0$ (*channel blackout*)
- Channel evolution is known a priori

### Objective

Lyapunov function: $x \mapsto V(x) = x^T P x$

Desired performance function: $V_d(t) = V_d(t_0) e^{-\beta(t-t_0)}$

Performance objective: ensure $h_{\mathrm{pf}}(t) \triangleq \frac{V(x(t))}{V_d(t)} \leq 1$, for all $t \geq t_0$

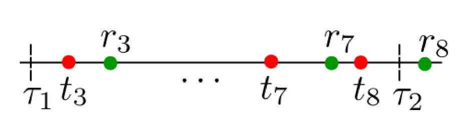
**Design objective:**

- Quantify data capacity in real time to overcome blackouts
- Design event-triggered communication policy
- Recursively determine $\{t_k\}$, $\{p_k\}$ and $\{\tilde{r}_k\}$
- Ensure a uniform positive lower bound for $\{t_k - t_{k-1}\}_{k \in \mathbb{Z}_{>0}}$

### Data capacity

max # of bits that can be *communicated* during the time interval $[\tau_1, \tau_2]$, overall all possible $\{t_k\}$ and $\{p_k\}$

$$\mathcal{D}(\tau_1, \tau_2) \triangleq \max_{\substack{\{t_k\}, \{p_k\} \\ \text{s.t. } \dots}} n \sum_{k=\underline{k}_{\tau_1}}^{\bar{k}_{\tau_2}} p_k$$



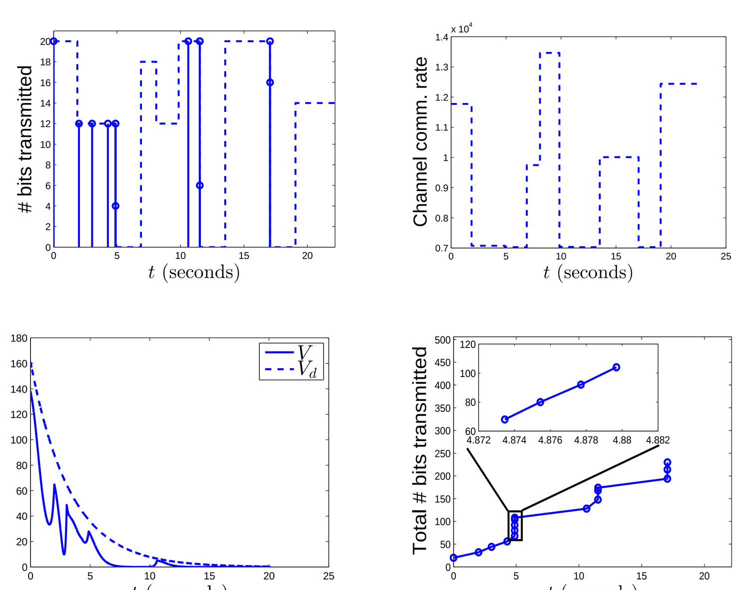
$\underline{k}_{\tau_1} = 3, \bar{k}_{\tau_2} = 7$

- Equivalent to optimal allocation of *discrete* # bits to be transmitted in each time slot
- Proposed a real-time algorithm to compute a suboptimal solution to the data capacity problem, with quantified bound on the suboptimality

### Results

**Main ideas of the control policy:**

- Impose artificial bound on the max packet size so as not to affect future data capacity
- Transmit whenever either:
  - performance objective might be violated in a look-ahead time interval if no action is taken
  - current 'artificial max packet size' is about to be not "sufficient"
  - data capacity from current time to the next blackout is about to be not "sufficient"
- Rules to determine sufficient packet size and update times




### References


- Event-triggered stabilization of linear systems under channel blackouts, P. Tallapragada, M. Franceschetti, J. Cortés, Allerton Conference on Communications, Control, and Computing, Monticello, Illinois, USA, 2015, to appear
- Event-triggered control under time-varying rates and channel blackouts, P. Tallapragada, M. Franceschetti, J. Cortés, Automatica, submitted


## Team-triggered coordination for real-time networked control

### Gist of team-triggered coordination

Novel approach for implementation of distributed controllers on networked cyberphysical systems

- Combine best properties of event- and self-triggered strategies into unified approach
- Agents make promises to neighbors about their future states – this information allows agents to autonomously schedule information requests in the future
- Agents are responsible for warning each other when promises need to be broken – reminiscent of event-triggered implementations

### Time-, Event-, and Self-triggered control

*Coordination task:* Drive $N$ agents with linear dynamics

$$\dot{x}_i = A_i x_i + B_i u_i, \qquad x_i \in \mathcal{X}_i, \quad u_i \in \mathcal{U}_i,$$

to a set $D$. Agents can *communicate* with other agents through graph with a *fixed topology*

Given *Lyapunov function* $V$ with distributed gradient

$$\frac{d}{dt} V(x) = \sum_{i=1}^{N} \nabla_i V(x^i_{\mathcal{N}}) \dot{x}_i,$$

distributed *continuous control* law $u^*$ monotonically optimizing $V$, design real-time implementation

Time-triggered implementation: a priori computation of period, conservative, assumes all agents synchronized

Event-triggered implementation: requires global information. Local versions require continuous information from neighbors to check triggers

Self-triggered implementation: takes into account reachability sets to identify local triggers, but generally conservative updates

### Promises

Agent $j$ makes either *control promise* to agent $i$ at time $t_{\text{last}}$

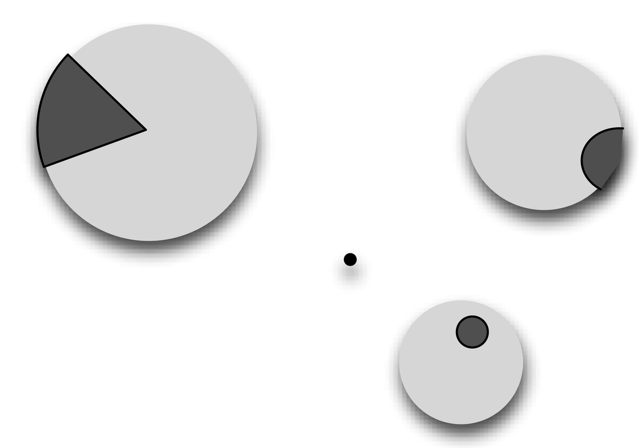
$$U^i_j[t_{\text{last}}] \in C^0([t_{\text{last}}, \infty); 2^{\mathcal{U}_j}),$$

or *state promise*

$$X^i_j[t_{\text{last}}] \in C^0([t_{\text{last}}, \infty); 2^{\mathcal{X}_j}),$$

*Promises* contain *more information* than reachability sets

$$x_j(t) \in X^i_j[t_{\text{last}}](t) \subset \mathbf{X}^i_j(t)$$



Promises that agent sends depends on its information. Promises can be adaptively tuned to the degree of execution of the task

### Opportunistic state-triggered updates

*Self-trigger information update*

At any time $t$ agent $i \in \{1, \dots, N\}$ receives new promise(s) $X^i_j[t]$ from neighbor(s) $j \in \mathcal{N}(i)$:

1: compute time $t_{\text{next}} \geq t$
2: request information from neighbors at time $t_{\text{next}}$

*Respond to information request*

At any time $t$ a neighbor $j \in \mathcal{N}(i)$ requests information, agent $i$ performs:

1: send new promise $X^j_i[t]$ to agent $j$

*Event-trigger information update*

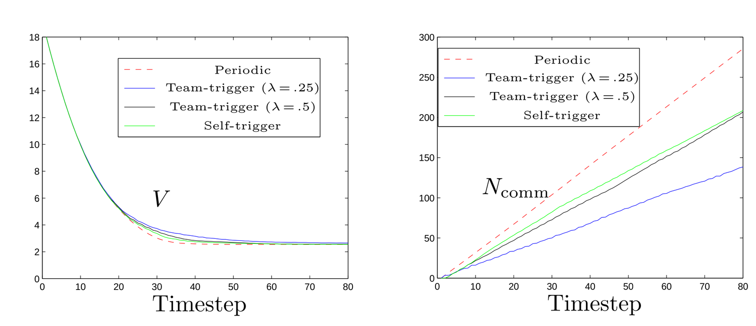
At all times $t$, agent $i$ performs:

1: **if** there exists $j \in \mathcal{N}(i)$ such that $x_i(t) \notin X^j_i[\cdot](t)$ **then**
2: send new promise $X^j_i[t]$ to agent $j$

### Results

Convergence and robustness guarantees under team-triggered coordination:

- Lyapunov function $V$ monotonically nonincreasing along network evolution with no Zeno behavior
- network trajectories asymptotically converge to $D$
- robust version (w/ warning messages) ensures asymptotic convergence w/ probability one under packet drops, bounded delayed messages, and bounded communication noise




($\lambda$ captures *tightness* of promises, 0 corresponds to exact trajectories, 1 corresponds to trivial promises –recovers self-triggered case)

### References


- Team-triggered coordination for real-time control of networked cyberphysical systems, C. Nowzari, J. Cortés, IEEE Transactions on Automatic Control, vol 61, number 1, 2016, to appear
- Team-triggered coordination of robotic networks for optimal deployment, C. Nowzari, J. Cortés, G. Pappas, Proceedings of the American Control Conference, Chicago, Illinois, USA, 2015, pp. 5744-5751