# Decentralized Robust Control of Power Grids Using LPV-Models of DAE-Systems

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**Abstract:** For large, distributed, and decentrally controlled systems, physical links between subsystems must be considered in designing local controllers to obtain global system stabilization. This manuscript addresses the task of robustly stabilizing systems with nonlinear subsystem dynamics and interconnections given by nonlinear algebraic equations, as motivated by the typical structure of power grids. The proposed approach first transforms the subsystems into a set of LPV-models, and the interconnections are represented by parameter intervals. For each subsystem, a local robustly stabilizing controller is synthesized by solution of a semidefinite program, and the stability of the overall system is implied if interval conditions for the parameters are satisfied. The method is demonstrated for a simple (yet often used) instance of a power grid.

*Keywords:* LPV, decentralized control, robust stability, energy systems, differential-algebraic.

## 1. INTRODUCTION

This paper addresses the question of how distributed systems with physical links between subsystems can be controlled robustly in a decentralized scheme. The particular focus is on systems modelled by nonlinear differential-algebraic equations (DAE), where the nonlinear dynamics of the subsystems are connected by algebraic equality constraints. The path proposed in this contribution is to map the dynamics of each subsystem into a *linear parameter varying system* (LPVS), and to represent the equality constraints locally (i.e for each subsystem) by parameter intervals. For this system structure, the local controllers can be synthesized separately by semidefinite programming, in order to robustly stabilize each subsystem as well as the global system.

The class of systems under investigation is motivated by the typical structure of power grids. While a grid can be modeled by a large set of nonlinear DAEs for which centralized control is undesirable, reliable operation in practice is achieved by separation of concerns: The stability of rotor angle, frequency, and voltage is treated separately and achieved by local controllers affecting (mainly) the synchronous generators (Kundur et al., 2004). The focus here is on rotor angle stability (the so-called *transient stability*), which refers to the ability of synchronous generators to stay in synchronism after a large disturbance. The standard controller for this purpose and to achieve a good damping of electromechanical oscillations is the so-called *Power System Stabilizer* (PSS). The classical method of PSS design is based on modeling synchronous generators as LTI-systems, restricting the operability to close vicinities of the chosen points of operation/linearization. Uncertainties arising from changing operating conditions (as typical for larger shares of renewable energies), of neglected nonlinearities, and parameter changes can deteriorate the control performance and lead to temporary shutdown of grid sections. To reduce these effects, different approaches for robustification have been proposed in the past, see (Fan, 2009) for an overview of handling nonlinearities and parameter changes. In (Gordon and Hill, 2008), *direct feedback linearization* was proposed to linearize the decentralized system behavior and to design a robust controller for transient stability. While the coupling of the generator to the grid was modeled by bounded uncertain parameters, the damping of oscillations was not considered. Measures to include damping of power systems by pole placement and LMI-based design are reported in (Rao and Sen, 2000; Rao and Paul, 2011; Werner et al., 2003), where the first reference is on synthesis of state feedback controllers, and the latter two on synthesizing output feedback controllers. The three approaches determine single robust controllers for the whole space of uncertainties, what can lead to rather conservative results. In addition, it is a drawback of these methods that they are based on linearization (and thus approximation) of the DAEs rather than formulating matrix polytopes by analytic expressions over the parameter space – (Rao and Sen, 2000) classifies finding a system description as matrix polytope containing all uncertainties as a difficult task.

An alternative to handle system nonlinearities and parameter variations is to use LPVS-based techniques, in which the complete operating range is defined by the varying parameters. The controller is not defined constant but depending on the parameters as well: In (Qiu et al., 2004), an LMI-based controller synthesis for LPVS has been proposed for designing a decentralized PSS, and in (Liu et al., 2006a,b) it is extended to the control of FACTS. The decentralized models there are also derived by linearizing around operating points and interpolation in between. In consequence, the success of this approach is depending on the underlying gridding. The method in (He et al., 2009) combines the LMI-based pole-placement technique with parametrized controllers. In contrast to the previously referenced approaches, an exact polytopic representation for a particular instance of a power grid, the so-called *single machine infinite bus system* (SMIB) was derived. The SMIB was already similarly introduced in (He et al., 2006), and sufficient conditions for stability were provided. These seem the only papers so far, which use an exact LPVS representation of a synchronous generator. However, a main disadvantage of this model is that it contains the algebraic equations of the grid: While this appears to be acceptable for a SMIB system, it does not extend to decentralized design of controllers for larger modular structures of power systems with several generators.

The main contributions of this paper are to propose first a modular and exact LPVS representation of nonlinear DAEs as appearing for power grids. An important aspect here is that the algebraic equations are not subsumed in the generator model (as in (He et al., 2006, 2009)), but kept separately to connect the parameter ranges of several grid nodes. Secondly, the paper proposes a synthesis procedure for decentralized controllers of the LPVS structure such that the overall system is robustly stabilized.

## 2. LPVS MODELS OF POWER GRIDS

Since the application focus of this work is the control of transient stability in power grids, the system model must include electro-mechanical phenomena (which are well known, of course). In abstract form, such a grid model represents a nonlinear DAE-system of index 1:

$$\dot{x}(t) = f(x(t), y(t), u(t)),\ 0 = g(x(t), y(t), u(t)) \qquad (1)$$

with time $t$, the vector $x \in \mathbb{R}^{n_d}$ of $n_d$ differential variables, the vector $y \in \mathbb{R}^{n_a}$ of $n_a$ algebraic variables, and the $m$ inputs $u \in \mathbb{R}^m$. In a classical power system, the controlled subsystem is the synchronous generator. In order to illustrate the principles of transforming a grid model of type (1) in LPVS-form, the SMIB-system is suitable. The following parts first introduce the corresponding DAE-system and then describe the representation as LPVS.

### *2.1 DAE-Model of the SMIB-System*

The model equations for general power systems and for the SMIB-system in particular are standard and can be found, e.g. in (Milano, 2010) and (Kundur, 1994). The description is dq-transformed and the dependency of the variables on the time is omitted for brevity. As can be seen in Fig. 1, the SMIB-system consists of a synchronous generator connected through a transformer and two parallel lines to the so-called *infinite bus*. The model equations can be separated into the differential equations for the generator (machine), the algebraic equations for the generator, and the algebraic equations modeling the grid. The following description does not eliminate the algebraic variables referring to the grid by insertion of explicit equations. The

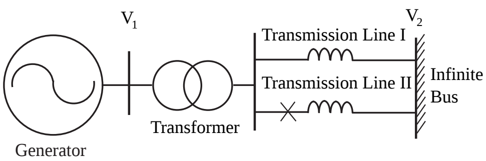


Fig. 1. SMIB-System (Milano, 2010)

differential equations of the synchronous generator with the three states rotor angle $\delta$, the angular velocity $\omega$ and the d-axis transient voltage $e'_q$ can be described by (Milano, 2010):

$$\begin{aligned} \dot{\delta} &= \Omega_b(\omega - \omega_b),\ \dot{\omega} = \frac{1}{2H}\Big(\tau_m - \tau_e - D(\omega - \omega_b)\Big) \\ \dot{e}'_q &= \frac{1}{T'_{dO}}\Big( - e'_q - (x_d - x'_d) i_d + v_f\Big). \end{aligned} \qquad (2)$$

The mechanical torque $\tau_m$ and the field voltage $v_f$ are the two inputs of the system. The machine-related algebraic variables electrical torque $\tau_e$, machine voltages (dq-transformed) $v_q$ and $v_d$, machine currents $i_q$ and $i_d$, as well as the injected active and reactive powers $p_1$ and $q_1$ of the bus 1 are given as follows:

$$\begin{aligned} \tau_e &= (v_d + r_a i_d) i_d + (v_q + r_a i_q) i_q \\ v_q &= -r_a i_q + e'_q - x'_d i_d, \quad v_d = -r_a i_d + x_q i_q \\ v_d &= v_1 \sin(\delta - \Theta_1), \quad\quad v_q = v_1 \cos(\delta - \Theta_1) \\ p_1 &= v_d i_d - v_q i_q, \quad\quad\quad q_1 = v_q i_d - v_d i_q. \end{aligned} \qquad (3)$$

The voltage $v_1$ at a bus with index 1 and its phasor $\Theta_1$ can be calculated with the help of the grid algebraic equations. These equations can be formulated in general as the injected active and reactive powers at the bus $h$:

$$\begin{aligned} p_h &= v_h \sum_{k=1}^{r} v_k (g_{hk} \cdot \cos\Theta_{hk} + b_{hk} \sin\Theta_{hk}) \\ q_h &= v_h \sum_{k=1}^{r} v_k (g_{hk} \cdot \sin\Theta_{hk} - b_{hk} \cos\Theta_{hk}). \end{aligned} \qquad (4)$$

Here, $r$ is the number of the buses of the power system, $g_{hk}$ and $b_{hk}$ are the conductances and susceptances between the buses $h$ and $k$. $v_h$ and $v_k$ are the voltages at the buses, and $\Theta_{hk}$ is the difference between the phasors of the two buses $\Theta_{hk} = \Theta_h - \Theta_k$. These equations can represent large grids with thousands of connections, and a synchronous generator may be included at each of the buses $k$. In case of the simple SMIB-system, only two buses are involved, and the equations simplify to:

$$\begin{aligned} p_1 &= \frac{v_1 v_2}{x_s} \sin(\Theta_1 - \Theta_2) \\ q_1 &= \frac{v_1^2}{x_s} - \frac{v_1 v_2}{x_s} \cos(\Theta_1 - \Theta_2) \end{aligned} \qquad (5)$$

Here, $x_s$ is the sum of the reactances between the two buses with indices 1 and 2. The voltage $v_2$ and the phasor $\Theta_2$ at the infinite bus 2 are set to constant values, since the bus represents an infinitely strong grid. The machine parameters $\Omega_b$, $\omega_b$, $D$, $H$, $r_a$, $x_d$, $x'_d$, and $T'_{dO}$ denote the base synchronous frequency, the reference frequency, the damping coefficient, the inertia constant, the armature resistance, the d-axis synchronous reactance, the d-axis transient reactance, and the d-axis open circuit transient time constant.

### *2.2 LPV-Model of the Synchronous Generator*

To prepare the controller synthesis, the synchronous generator is now transformed into an LPVS-model with the states $x \in \mathbb{R}^{n_x}$, the outputs $y \in \mathbb{R}^{n_y}$, the inputs $u \in \mathbb{R}^m$, and the parameters $\theta(t) \in \mathbb{R}^p$ (with the numbers $n_x$, $n_y$, $m$, and $p$ of the respective quantities):

$$\begin{aligned} \dot{x}(t) &= A(\theta(t)) x(t) + B(\theta(t)) u(t) \\ y(t) &= C(\theta(t)) x(t) + D(\theta(t)) u(t) \end{aligned} \qquad (6)$$

Finding an LPVS-model of a nonlinear system is a non-trivial task, and the resulting model is not unique as it depends on the choice of parameters. The transformation procedures can, in principle, be categorized in two classes: linearization around a choice of operating points and analytic transformation based on algebraic operations. The latter method has the advantage that the dynamic behaviour of the original model is preserved. Transformation procedures of this type for nonlinear differential equations are proposed in (Tóth, 2010) and (Kwiatkowski et al., 2006), but cannot immediately be transferred to the DAE-systems under consideration here. Nevertheless, the idea of "hiding" the nonlinearities described by the mentioned authors can be applied to power systems, too. Given the objective to model networked systems and to control in a decentralized scheme, the transformation must be aligned to modular system structures. Hence, the grid equations must be retained to provide means for coupling LPVS-models assigned to the grid nodes (in contrast to the inclusion of the grid equation into the LPVS, as described in (He et al., 2006, 2009)). Consequently, only the algebraic equations (3) can be eliminated or included in (2), but not the algebraic equations contained in (4) and (5).

The main idea in obtaining the LPVS-model is to insert a suitable choice of algebraic variables into the differential equations and to assign the remaining nonlinearities to the varying parameters. Thus, not the whole power system is described by the following LPV-model, but only the dynamic subsystem (i.e. the synchronous generator). When focusing on transient stability, only the input $v_f$ is used for controller actions, and $\tau_m$ remains constant. The resulting affine LPVS-model, which is exact, follows with the states $x = \left[\delta,\, \Delta\omega,\, e'_q\right]^T$ and the input $u = v_f$ for $D = 0$ to:

$$\dot{x} = \begin{bmatrix} 0 & \Omega_b & 0 \\ \frac{1}{2H}\theta_1 & 0 & -\frac{1}{2H}\theta_2 \\ -\frac{(x_d - x'_d)}{T'_{dO}}\theta_3 & 0 & -\frac{1}{T'_{dO}} \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T'_{dO}} \end{bmatrix} u \qquad (7)$$

with the parameters

$$\begin{aligned} \theta_1 &= (\tau_m - (x_q - x'_d) i_d \cdot i_q) \cdot \frac{1}{x_1} \\ \theta_2 &= i_q, \quad \theta_3 = i_d \cdot \frac{1}{x_1} \end{aligned} \qquad (8)$$

The definition of the state $x_2$ corresponds to $\Delta\omega = \omega - \omega_b$ in (2). The first state has to be restricted to $x_1 \neq 0$. The restriction does not constitute a problem, since the value zero is not in the normal operating range of the controlled rotor angle. The equations (8) show that the differential system is coupled to the rest of the system through the machine currents $i_d$ and $i_q$. The currents are not directly measurable, but can be calculated from local (with respect to bus 1) and measurable quantities:

$$\begin{aligned} i_d &= \frac{p_1}{v_1}\sin(x_1 - \Theta_1) + \frac{q_1}{v_1}\cos(x_1 - \Theta_1) \\ i_q &= \frac{p_1}{v_1}\cos(x_1 - \Theta_1) - \frac{q_1}{v_1}\sin(x_1 - \Theta_1) \end{aligned} \qquad (9)$$

These equations are derived from (3) and show that $i_d$ and $i_q$ are physically coupled. Also the parameters are inherently coupled, but are treated as independent in the differential system as indicated by (8). The implications of these dependencies will be discussed in Sec. 4.

While the derivations in this sections started from the SMIB-system, it is stressed at this point that larger grids can be easily built from (7), (8), (9) and partly (3) by connecting the LPVS-models of several synchronous generators through sets of grid equations of the type (4). Hence, the modeling scheme allows one to set up modular LPVS-models of larger power grids.

## 3. LPV CONTROLLER SYNTHESIS

The controller synthesis is based on polytopic sets of the states matrices of the LPVS. Finding a polytopic LPVS-model is difficult, and the simpler form of an LPVS in affine form may appear preferable. In the following, the connection between affine LPVS and the polytopic representation is addressed. Then, required definitions of LMI formulations for LTI-systems are provided and extended to a multi-objective control synthesis based on polytopic descriptions. The presented LMI formulations are based on results from (Chilali and Gahinet, 1996; Chilali et al., 1999; Scherer et al., 1997).

### 3.1 Affine LPV-Model

The LPVS according to (6) is called *affine* if for the matrix $A(\theta)$ and a set of matrices $\{A_0, A_1, \ldots, A_p\}$ it holds that:

$$A(\theta) = A_0 + \sum_{j=1}^{p} A_j \theta_j, \qquad (10)$$

and if the equivalent formulations apply for $B(\theta)$,$C(\theta)$ and $D(\theta)$. With parameter limits $\theta_j \in [\underline{\theta}_j, \overline{\theta}_j]$, $A(\theta)$ varies within a matrix-polytope with the vertices $\widetilde{A}_i$ corresponding to the extremal values of the parameters. Thus, $A(\theta)$ can be described as a matrix polytope in the form (Apkarian et al., 1994):

$$A(\theta) = \sum_{i=1}^{l} \alpha_i \widetilde{A}_i : \ \sum_{i=1}^{l} \alpha_i = 1, \alpha_i \geq 0. \qquad (11)$$

The matrix polytope defines the convex hull of the $l$ matrices $\widetilde{A}_i$. Using all combinations of $[\underline{\theta}_j, \overline{\theta}_j]$, the number of vertices becomes $l = 2^p$. Given a polytopic description of the LPVS, the LMI-based controller synthesis can be applied as described in the following subsections.

### 3.2 LMI Formulations for Pole Placement Design

The control synthesis based on pole placement is prepared by the following definition and lemma:

*Definition 1 – LMI Region* (Chilali and Gahinet, 1996): With symmetric matrices $\alpha, \beta \in \mathbb{R}^{m \times m}$, a subset $\mathfrak{D}$ of the complex plane is defined as (with $\bar{z}$ the conjugate complex of $z$):

$$\mathfrak{D} = \{z \in \mathbb{C} : f_D(z) := \alpha + z\beta + \bar{z}\beta^T < 0\}. \qquad (12)$$

*Lemma 1. – $\mathfrak{D}$-Stability* (Chilali and Gahinet, 1996): An LTI-system with the system-matrix $A$ is $\mathfrak{D}$-stable, i.e. the poles of $A$ are located in the LMI-region $\mathfrak{D}$, if a symmetric matrix $X_D > 0$ exists such that:

$$\alpha \otimes X_D + \beta \otimes (AX_D) + \beta^T \otimes (AX_D)^T < 0 \qquad (13)$$

applies where $\otimes$ represents the Kronecker product.

In the following, the conditions for some LMI-regions relevant in controller synthesis are specified with $X_D > 0$:

- a left half-space with $Re(z) < -\alpha$:
$$2\alpha X_D + AX_D + X_D A^T < 0, \quad (14)$$
(choosing $\alpha = 0$ leads to the Lyapunov theorem, and using alternatively the relation $>$ in (14) defines a right half-space);
- a conic sector bounded by lines through the origin and with angles $\pm\varphi$ to the negative real-axis:

$$\begin{bmatrix} \sin(\varphi)(AX_D + X_D A^T) & \cos(\varphi)(AX_D - X_D A^T) \\ -\cos(\varphi)(AX_D - X_D A^T) & \sin(\varphi)(AX_D + X_D A^T) \end{bmatrix} < 0. \quad (15)$$

### 3.3 LMI Formulations for $H_\infty$-Design

Given an LTI-system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1 w(t) + B_2 u(t) \\ z(t) &= C_1 x(t) + D_{11} w(t) + D_{12} u(t) \\ y(t) &= C_2 x(t) + D_{21} w(t) + D_{22} u(t) \end{aligned} \quad (16)$$

with the states $x$, the outputs $y$, the inputs $u$ as given in (6). The outputs for specifying control performance are $z \in \mathbb{R}^{n_z}$, and the exogenous inputs (e.g. disturbances) are $w \in \mathbb{R}^{n_w}$. The closed loop transfer function $G_{zw}(s)$ is defined as the transfer function from $w$ to $z$. The $H_\infty$-closed-loop performance $\|G_{zw}(s)\|_\infty < \gamma$ can be guaranteed, if a symmetric matrix $X_\infty$ is found which satisfies the *Bounded Real Lemma* according to the following LMI condition (Chilali et al., 1999):

$$\begin{bmatrix} A_{cl}X_\infty + X_\infty A_{cl}^T & B_{cl} & X_\infty C_{cl}^T \\ B_{cl}^T & -\gamma I & D_{cl}^T \\ C_{cl}X_\infty & D_{cl} & -\gamma I \end{bmatrix} < 0, \quad X_\infty > 0 \quad (17)$$

Here, the matrices $A_{cl}$, $B_{cl}$, $C_{cl}$, and $D_{cl}$ formulate the closed-loop system.

### 3.4 Multiobjective Design of the LPV Controller

For multiobjective control, the LMIs referring to the desired control goals have to be combined. The LMI formulations, so far specified for LTI systems, have to be extended to LPVS, i.e. tailored to the polytopic form of the model. The LMI formulations can be used either to synthesize a single invariant controller (as in (Rao and Paul, 2011; Rao and Sen, 2000; Werner et al., 2003) for linearized models), or to design a linear-parameter-varying controller. The following description will focus on the latter case, and the underlying controller structure is chosen as a linear parameter-varying feedback controller of the type $A_{cl}(\theta) = A(\theta) + BK(\theta)$. With an affine or polytopic $K(\theta)$, the resulting closed loop matrix $A_{cl}(\theta)$ is also affine and thus can be described as a polytope as in (11) with the vertices $\widetilde{A}_{cli} = \widetilde{A}_i + B\widetilde{K}_i$, $i \in \{1\ldots, l\}$. If the LMIs (14) or (15), and (17) are satisfied for any of the vertices $\widetilde{A}_{cli}$ of the polytope $A(\theta)$ with the same matrix $X$, the properties established by the constraints of the semidefinite program also hold for the complete polytopic space of the parameters $\theta$ (Apkarian et al., 1995).

The first design step is then to compute the matrix $\widetilde{A}_{cli}$, $i \in \{1\ldots, l\}$ for any vertex of the matrix polytope. For a selected set of control objectives, the referring LMIs are formulated for $\widetilde{A}_{cli}$. The obtained matrix inequalities are linearized by using the auxiliary variables $Y_i := \widetilde{K}_i X$. The solution of the following semidefinite program:

$$\begin{aligned} &\min_{\widetilde{K}_i, X} \quad \gamma \\ &s.t.: \quad (14) \text{ or } (15), \text{ and } (17) \end{aligned} \quad (18)$$

is the controller matrix $\widetilde{K}_i$. By equating $A(\theta)$ in (10) and (11), and by using the conditions for $\alpha_i$ in (11), $\alpha_i$ can be obtained from a semidefinite optimization problem, and the controller follows: $K(\theta) = K(\alpha) = \sum_{i=1}^{l} \alpha_i \widetilde{K}_i$. During operation, $\theta$ can be determined via (8) and (9) from measurable quantities. If the semidefinite optimization problem returns a feasible solution $\widetilde{K}_i$ and $X$ for any vertex $\widetilde{A}_i$ of the matrix polytope $A(\theta)$ (and likewise for $B(\theta)$, $C(\theta)$, $D(\theta)$), the LPV-controller $K(\theta)$ stabilizes the model consisting of (7) and (8) for any parameter in $\theta_j \in [\underline{\theta}_j, \bar{\theta}_j]$ with $j \in \{1, \ldots, p\}$.

For larger grids with several machines, the physical links in between the machines are modeled by (4), i.e. for any machine $h$ the currents (9) formulate the effect from the rest of the grid on $h$. If this effect is conservatively mapped into the parameter interval of machine $h$, i.e. if $\theta_j^h \in [\underline{\theta}_j^h, \bar{\theta}_j^h]$, the robustly stable behavior of any machine extends to stable behavior of the complete grid.

The intervals can be found iteratively by starting with a first guess for the interval based on simulations and a controller with a large LMI region. The parameter range can then be iteratively reduced until a sufficient system behavior is reached.

## 4. SIMULATION RESULTS

This part illustrates and validates the design procedure for the SMIB-system as described in Sec. 2.1. The considered scenario is an outage of line two in Fig. 1 one second after initialization. The line is operable again 6 seconds after the fault occurrence. The LMI-region chosen for controller synthesis is the right half plane with $0 > Re(z) > -7$ and a conic sector with the angle $\varphi = \pm 45°$ to the negative real axis. These criteria account for limiting the velocity of the closed-loop response and for sufficient damping. For power quality and stability reasons, it is essential that oscillations die down in a reasonable time (de Oliveira et al., 2010). By choosing $\varphi = \pm 45°$, a relative damping of ca, 70% is enforced. In addition, an $H_\infty$-constraint for the transfer function from an additive disturbance of $\dot{x}_2$ to the output $x_2$ is used. The constraint pushes the poles of the closed-loop system to the left-hand side of the LMI-region. The controller is synthesized by minimizing the bound $\gamma$ of the $H_\infty$-criterion considering the LMIs (14), (15), and (17) for the vertices of the considered matrix polytope of the closed-loop system.

The parameters for the simulation are taken from example 13.2 in (Kundur, 1994). This reference also contains a controller of the standard power system stabilizer (PSS) type for the example, which can serve for a comparison to the control scheme proposed here. A PSS is usually used in combination with an excitation system combined with an automatic voltage regulator (AVR). The inclusion of an exciter combined with an AVR is omitted for the LPV-controller, since the focus is on transient stability (i.e. only rotor angle and angular velocity are of interest here).

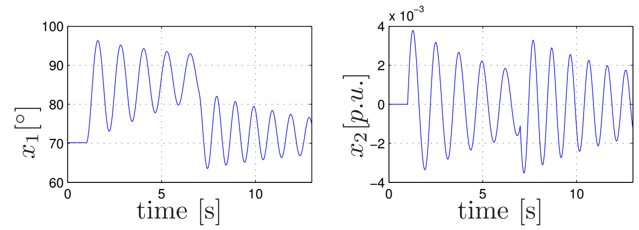


Fig. 2. Rotor angle ($x_1$) and angular velocity ($x_2$) without control.

Figure 2 first shows simulations results for the uncontrolled SMIB, revealing large and hardly damped oscillations after occurrence and removal of the fault. The two states start to oscillate after the disturbances. While not visible for the chosen time-interval, the system becomes unstable after approx. 32 seconds, i.e stabilization by controller is required. For the same fault scenario, Fig. 3 shows the simulation results obtained for an LPV-controller synthesized as proposed before (solid blue line) and for a PSS (dashed red line). The system is stabilized in both cases, but the LPV controller realizes a significantly better damping, such that the steady state values of the fault state and the original state are reached within 1.5 sec after line outage and recovery, while the behavior with PSS is much worse.

With respect to the control action, Fig. 4 reveals a maximum amplitude for the PSS that is up to 37% larger than for the LPV-controller. Figure 5 illustrates the position of the poles for the closed-loop system with LPV-controller as occurring in the simulation. It is clearly visible that the poles (blue crosses) lie within the LMI-region (dashed line) specified for synthesis. This result is implied by the choice of the limits $\theta_i \in [\underline{\theta}_i, \overline{\theta}_i]$, and the corresponding 8 vertices of the feasible parameter space. This space is indicated in Fig. 6 by the box bounded by dashed red lines. The true course of the parameters $\theta_1$ to $\theta_3$ obtained

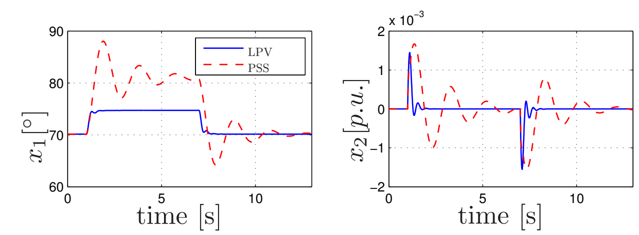


Fig. 3. Controlled behaviour upon line outage / recovery with LPV-controller and PSS.

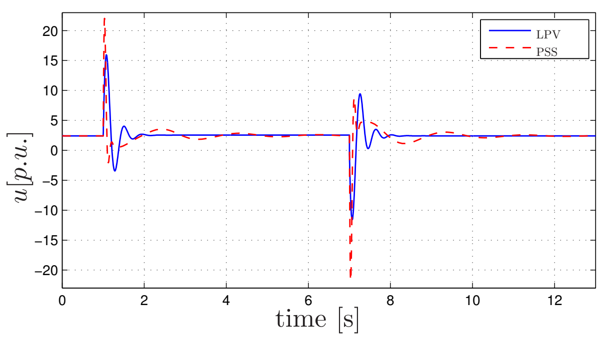


Fig. 4. Output of the LPV-controller and the PSS.

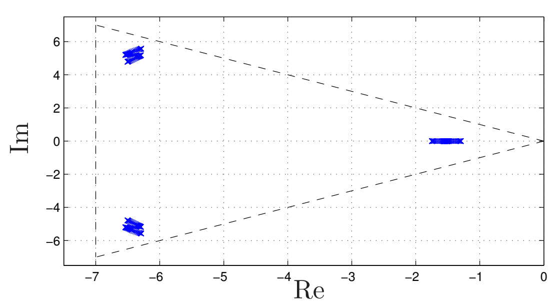


Fig. 5. LPV-control: positions of the closed-loop poles.

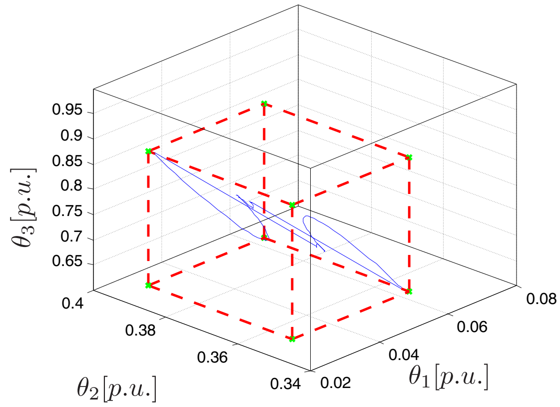


Fig. 6. Trajectory of the parameter vector of time for the scenarion and parameter box.

from the simulation is shown as well. Obviously, the parameter trajectory covers only a rather small region of the box (approximately a planar manifold with diagonal orientation inside of the box). This is due to the interdependence of the parameters and the currents $i_d$ and $i_q$ as specified by (8) and (9). Since the synthesis procedure leads to a controller which robustly stabilizes the closed-loop system for the complete box, it is more conservative than necessary for the considered scenario. Future work will consider how suitable but less conservative polytopic enclosures in the parameter space can be obtained from sets of given scenarios by considering the equations (8) and (9). The conservativeness corresponding to the box in Fig. 6 contributes to relatively high $u$-amplitude documented in Fig. 4, which are even higher for the PSS. Since the scenario represents a drastic disturbance for the power grid, a maximum amplitude of above 15 (or above 20 for the PSS) is higher than typically acceptable for grid operation. Thus, the last set of simulation tests the two controllers for typical constraints for the field voltage, i.e. for $u_{min} = -6.4 \leq u \leq u_{max} = 7$. Though the LPV controller was synthesized for the unconstrained case, the simulation in Fig. 7 shows that the controller still stabilizes the system and is well damped in presence of the input constraints while some overshoot in $x_1$ occurs upon link outage. Evidently, the LPV-controller clearly outperforms the PSS also in this respect. Figure 8 illustrates the corresponding input trajectories for the two controllers.

## 5. CONCLUSIONS AND FUTURE WORK

The proposed scheme for modular modeling of distributed systems with physically coupled subsystems represents the grid node by LPVS and the links by algebraic equations.

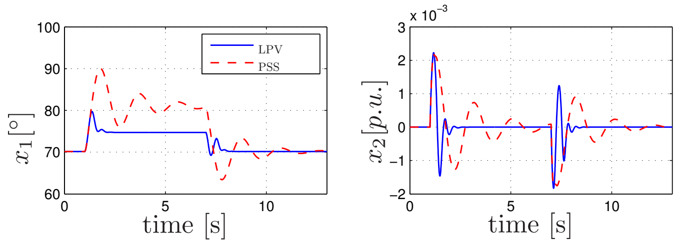


Fig. 7. States $x_1$ and $x_2$ with LPV-controller and standard PSS for input saturation.

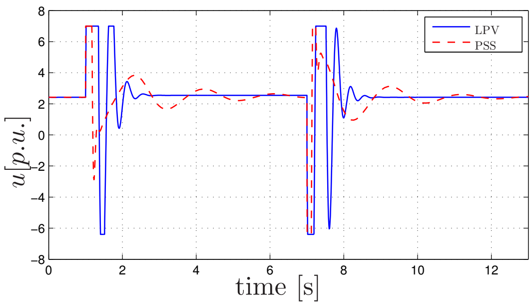


Fig. 8. Output of the LPV-controller and of the PSS for input saturation.

The modeling scheme is scalable and thus applicable to networks of arbitrary size. The model structure provides the basis for decentralized controller design: if the parameter ranges can be conservatively approximated, the proposed approach to controller synthesis based on semidefinite programming can lead to a feedback controller in LPV-form that robustly stabilizes the grid node for the polytopic parameter set. If the synthesis finds a feasible solution to the synthesis step for any network node, and if the parameter sets for any pair of nodes are consistent with respect to the grid equations, the grid is overall stabilized.

The considered (simple) example of transient stability for a SMIB-system shows that the control approach leads to better results with respects to disturbance rejection and damping than the commonly used power system stabilizer. Recent work has successfully applied the principle to larger grids with 9 to 14 buses. Future work will address the handling of constraints, the determination of admissible and conservative parameter spaces, and the inclusion of additional control objectives (like voltage control for power grids) more comprehensively.

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