Robust Decentralized LPV Control for Transient Stability of Power Systems

Konstantin Schaab * **Olaf Stursberg** *

* *Control and System Theory, EECS, University of Kassel (Germany). Email: {k.schaab, stursberg}@uni-kassel.de*

Abstract: In daily operation of power systems, transient stability is targeted by designing local standard controllers for narrow operating ranges of any generating node. If deviations due to disturbances or a higher share of (uncertain) renewable energy occur, exemption routines are necessary for returning to nominal operation. This paper proposes a method for synthesizing local robust multivariable controllers such that the interaction between the grid nodes is explicitly considered. It is shown that variably parametrized controllers in conjunction with linear-parameter-varying models of the grid nodes can consistently stabilize the system. Since the control architecture is nevertheless decentralized, the design effort grows moderately with the size of the power system.

Keywords: Power system, decentralized control, transient stability, robust control, scalability.

1. INTRODUCTION

Power grids constitute large distributed systems which can be represented by nonlinear differential algebraic equations (DAEs). Reliable operation is typically achieved by a separation of concerns, resulting in the three stability categories *rotor angle stability*, *frequency stability*, and *voltage stability* (Kundur et al., 2004). The category of concern in this work is the so-called *transient stability*, i.e. the subclass of rotor angle stability describing the ability of synchronous generators to stay in synchronism after a large disturbance. The standard controller to establish transient stability and to achieve a good damping of electromechanical oscillations is the so-called *power system stabilizer* (PSS). It represents a decentral controller and is typically based on modeling the synchronous generator as a linear time-invariant (LTI) system, i.e. the PSS is applicable only close to the chosen operating point. Uncertainties arising from changing operating conditions, neglected model nonlinearities, and parameter changes can deteriorate the controller performance – these uncertainties will become more important in the future due to increasing shares of renewable energies in power grids.

Different schemes to enhance robustness of PSS have been proposed, see (Fan, 2009) for an overview. The key to robustness is the handling of the DAE-system including the nonlinearities and parameter changes. In (Gordon and Hill, 2008), *direct feedback linearization* (DFL) was used to linearize the decentralized system. Robust controllers for transient stability were designed with considering the coupling to the grid by bounded ranges for the uncertain variables. Although the feedback controllers are decentralized, global stability of the considered power system is shown. The drawback of the DFL techniques is that the damping of the dynamic behavior may not be sufficient. Damping can be enhanced by pole-placement, as reported in (Rao and Paul, 2011; Rao and Sen, 2000; Werner et al., 2003; Shayeghi et al., 2010). The resulting robust controllers place the closed-loop poles such that a desired damping is achieved for the considered parameter range. In (de Oliveira et al., 2010) this is done by using a classical PSS structure and computerized tuning of the parameters for different operating points. The pole-placement is ensured by using formulations of linear matrix inequalities (LMI). In (Rao and Paul, 2011) and in (Werner et al., 2003), output feedback controllers were synthesized using LMI formulations as well. These approaches determine a single robust controller for the whole space of uncertainties. (Rao and Sen, 2000) uses similar techniques for synthesizing state feedback controllers, and shows the application for a relatively large system. Stability of the whole power system is only shown by simulation. The common drawback of the techniques in the last three papers referenced is that they are based on linearization of the DAEs, rather than using matrix polytopes in order to obtain a conservative system representation. As stated in (Rao and Sen, 2000), finding a system description in the form of a matrix polytope containing all uncertainties is usually difficult. Furthermore, finding a single controller for the whole range of uncertainties may be impossible for large ranges. An alternative approach to handle system nonlinearities and parameter variations is the use of *linear parameter varying* (LPV) techniques, in which the operating range is defined by varying parameters. The controller is no longer constant but is defined depending on the model parameters as well. The stability of the controlled system can be proven under certain conditions. In (Qiu et al., 2004), an LMI-based controller synthesis based on LPV-systems is proposed for designing a decentralized PSS, and in (Liu et al., 2006b,a) the approach is extended to the control of FACTS. The decentralized models are also derived by linearizing for a set of operating points and interpolation in between. The success of this approach depends on the choice of linearization points. An alternative approach which combines the positive aspects of LMI-based pole-placement technique and an LPV-controller was introduced in (He et al., 2006, 2009). In contrast to the other methods, an exact polytopic representation for the *single machine infinite bus system* (SMIB) was derived, and sufficient conditions for stability are not violated, as long as the parameters (and thus the uncertainties) stay in the range considered in synthesis. To the best of our knowledge, these are the only papers of other authors so far which use an exact LPV-model of synchronous generators. A characteristic of this approach is that the algebraic equations describing the grid are inserted into the LPV model. While this seems appropriate for the SMIB system, the extension of this principle to larger power systems with multiple generators is not possible since the grid usually has a large and complicated structure. However, stability of the whole power system must be considered and among the discussed contributions global stability was only shown in (Gordon and Hill, 2008).

In contrast, the contribution of this paper is to propose a method for designing LPV-controllers for a power system representation in which any generating node is modeled by an exact LPV model while the algebraic equations for modeling the physical coupling of the nodes and the grid are not included in the LPV model directly. Only the parameters couple the considered subsystem to the grid. The algebraic equations are used to formulate consistent parameter ranges for the node models. The synthesis of the LPV-controller for any node considers these ranges in order to guarantee transient stability as well as sufficient damping of the dynamic behavior for any permissible parameter value. In contrast to the results presented already in Schaab and Stursberg (2015), stability of the whole power system is shown. As further extension of Schaab and Stursberg (2015), the effectiveness of the approach is demonstrated on a 9-bus system.

2. POWER SYSTEMS MODEL

2.1 Differential-Algebraic Model of the Power System

Since the focus of this work is transient stability, the power system model must include the electromechanical phenomena of generators and buses as described in many standard texts, see e.g. (Milano, 2010) or (Kundur, 1994). The following description is dq-transformed and most physical values are given in per-units. A power system typically consists of many (hundreds) synchronous generators (which are the driving force for stabilizing the power system), transmission lines, and loads. The modeling equations can be separated into the differential equations of the generators (machines), the algebraic equations of the machine, and the algebraic equations of the grid and loads.

The following formulation of the generator equations does not involve any elimination of algebraic variables. The differential equations of a generator with index h comprises one ODE each for the rotor angle δ_h, the angular velocity ω_h, and the transient voltage $e'_{q,h}$ (d-axis):

$$\begin{aligned}\dot{\delta}_h &= \Omega_{b,h}(\omega_h - \omega_{b,h})\\ \dot{\omega}_h &= \frac{1}{2H_h}\Big(\tau_{m,h} - \tau_{e,h} - D_h(\omega_h - \omega_{b,h})\Big)\\ \dot{e}'_{q,h} &= \frac{1}{T'_{dO,h}}\Big(- e'_{q,h} - (x_{d,h} - x'_{d,h})i_{d,h} + v_{f,h}\Big)\end{aligned} \tag{1}$$

The mechanical torque $\tau_{m,h}$ and the field voltage $v_{f,h}$ are the two inputs of the machine. The set of machine parameters include the base synchronous frequency $\Omega_{b,h}$, the reference frequency $\omega_{b,h}$, the damping coefficient D_h, the inertia constant H_h, the armature resistance $r_{a,h}$, the d-axis synchronous reactance $x_{d,h}$, the d-axis transient reactance $x'_{d,h}$, and the d-axis open circuit transient time constant $T'_{dO,h}$. In addition, the algebraic variables of the machine with index h comprise the electrical torque $\tau_{e,h}$, the machine voltages (dq-transformed) $v_{q,h}$ and $v_{d,h}$, the machine currents $i_{q,h}$ and $i_{d,h}$ as well as the injected active and reactive powers p_h and q_h. These quantities are determined by the following algebraic equations:

$$\begin{aligned}0 &= \tau_{e,h} - (v_{d,h} + r_{a,h}i_{d,h})i_{d,h} - (v_{q,h} + r_{a,h}i_{q,h})i_{q,h}\\ 0 &= v_{q,h} + r_{a,h}i_{q,h} - e'_{q,h} + x'_{d,h}i_{d,h}\\ 0 &= v_{d,h} + r_{a,h}i_{d,h} - x_{q,h}i_{q,h}\\ 0 &= v_{d,h} - v_h \sin(\delta_h - \Theta_h)\\ 0 &= v_{q,h} - v_h \cos(\delta_h - \Theta_h)\\ 0 &= p_h - v_{d,h}i_{d,h} - v_{q,h}i_{q,h}\\ 0 &= q_h - v_{q,h}i_{d,h} + v_{d,h}i_{q,h}\end{aligned} \tag{2}$$

The voltage v_h and its phasor Θ_h can be calculated using the grid algebraic equations in matrix notation [1]:

$$\underbrace{\begin{bmatrix}\bar{s}_1\\ \bar{s}_2\\ \vdots\\ \bar{s}_r\end{bmatrix}}_{\bar{s}} = \underbrace{\begin{bmatrix}\bar{v}_1 & 0 & \dots & 0\\ 0 & \bar{v}_2 & \dots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \dots & \bar{v}_r\end{bmatrix}}_{\bar{V}} \underbrace{\begin{bmatrix}\bar{y}_{11}^* & \bar{y}_{12}^* & \dots & \bar{y}_{1r}^*\\ \bar{y}_{21}^* & \bar{y}_{22}^* & \dots & \bar{y}_{2r}^*\\ \vdots & \vdots & \ddots & \vdots\\ \bar{y}_{r1}^* & \bar{y}_{r2}^* & \dots & \bar{y}_{rr}^*\end{bmatrix}}_{\bar{Y}^*} \underbrace{\begin{bmatrix}\bar{v}_1^*\\ \bar{v}_2^*\\ \vdots\\ \bar{v}_r^*\end{bmatrix}}_{\bar{v}^*} \tag{3}$$

The connections between the buses are mapped into the *admittance matrix* $\bar{Y}$, in which a diagonal element y_{hh} is the sum of all shunt and line admittances connected to the line. The non-diagonal elements $y_{hk} = y_{kh}$ are the negative values of the sum of the admittances connecting the buses h and k, and they equal to zero if no connection exists. Thus, the admittance matrix is symmetric. One can also see that the resulting complex power $\bar{s}_h$ at the bus h is equal to the sum of all powers of lines ending at the bus. The complex value $\bar{s}_h$ can be separated in active and reactive power $\bar{s}_h = p_h + jq_h$. Loads and transformers are modeled as constant impedances.

The overall set of equations constitutes a decentralized model in which $h \in \{1, \dots, r\}$ numbers the buses contained in the grid, and (3) is specified for any bus. A subset of the buses (and thus of $\{1, \dots, r\}$) is associated with synchronous generators, and a set of equations according to (1), (2) for any generator is part of the model.

2.2 LPV-Model of a Synchronous Generator

As mentioned above, the dynamics of each generator is transformed into an LPV-model for subsequent controller synthesis. With vectors of states $x_h \in \mathbb{R}^{n_{x,h}}$, outputs $y_h \in \mathbb{R}^{n_{y,h}}$, inputs $u_h \in \mathbb{R}^{n_{u,h}}$ and parameters $\theta_h \in \mathbb{R}^{p_h}$, the general form the LPV-model is:

$$\dot{x}_h(t) = A_h(\theta_h(t))x_h(t) + B_h(\theta_h(t))u_h(t) \tag{4}$$

$$y_h(t) = C_h(\theta_h(t))x_h(t) + D_h(\theta_h(t))u_h(t) \tag{5}$$

Since the controller synthesis in the following section uses a state-feedback scheme, (5) can be omitted in the sequel.

[1] A bar $\bar{\bullet}$ indicates a phasor and an asterisk $\bullet^*$ the conjugate complex of the respective variable.

Finding an LPV-model for a given nonlinear system is a nontrivial task, and the choice of parameters is not unique in general. Procedures for transforming nonlinear differential equations into LPV-models using algebraic operation but no approximation (like linearization) are described in (Tóth, 2010) and (Kwiatkowski et al., 2006). The advantage of these procedures is that the dynamic behavior of the original model is preserved, and thus the impact of approximations on stability and robustness need not to be evaluated. Given the overall model as specified in Sec. 2.1, the objective of the transformation is to retain a decentralized structure in which any generator is mapped into a local LPV-model, while the grid algebraic equations are kept to connect the local models. Thus, the equations (3) do not become part of the LPV-model as in (He et al., 2009), but only the algebraic equations (2) are partially inserted into (1) during the transformation. Another important criterion for the transformation is that the transformed model is controllable.

The main idea of the transformation for any generator is to insert some of the algebraic variables into the differential equations and to move remaining nonlinearities and algebraic variables from the differential equations into the varying parameters [2] $\theta_h(t)$. When focussing on transient stability, only the input $v_{f,h}$ is used for control action, and $\tau_{m,h}$ can be considered as given (i.e. as constant value). For the generator with index h, the affine LPV-model of the type (4) is formulated for a state vector $x_h := \left[\delta_h \, \Delta\omega_h \, e'_{q,h}\right]^T$, and for the input $u_h := v_{f,h}$. The model is obtained as:

$$\dot{x}_h = \begin{bmatrix} 0 & \Omega_{b,h} & 0 \\ \frac{1}{2H_h}\theta_{1,h} & \frac{-D_h}{2H_h} & -\frac{1}{2H_h}\theta_{2,h} \\ -\frac{(x_{d,h}-x'_{d,h})}{T'_{dO,h}}\theta_{3,h} & 0 & -\frac{1}{T'_{dO,h}} \end{bmatrix} x_h + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T'_{dO,h}} \end{bmatrix} u_h \tag{6}$$

with the following definition of parameters:

$$\theta_{1,h} = \frac{(\tau_{m,h} - (x_{q,h} - x'_{d,h}) i_{d,h} i_{q,h})}{x_{1,h}}$$
$$\theta_{2,h} = i_{q,h}, \quad \theta_{3,h} = \frac{i_{d,h}}{x_{1,h}}. \tag{7}$$

Thus, $A_h(\theta_h)$ depends linearly on the parameters and B_h is independent of θ_h. The state $x_{2,h}$ corresponds to $\Delta\omega_h = \omega_h - \omega_{b,h}$ in (1). This LPV-model is an exact substitute of the original model of the generator, but since $x_{1,h}$ occurs in the denominator of two parameters, the range of this variable has to be restricted to $x_{1,h} \neq 0$. This, however, is no significant restriction since the value zero of δ_h can be avoided in the controlled case. [3]

Given the model according to (6) and (7), it is obvious that a coupling to other parts of the power system is established through the parameters θ_h and thus through the machine currents $i_{d,h}$ and $i_{q,h}$. While the parameters in (7) are inherently coupled through the variables $x_{1,h}$, $i_{d,h}$, and $i_{q,h}$, they are treated as independently varying in the subsequent controller synthesis, what may lead to conservativity.

[2] For notational convenience the time dependency in $\theta_h(t)$ is omitted and denoted by θ_h.

[3] Other choices for the parameters than in (7) were considered, but were rejected due to the controllability requirement.

3. LPV CONTROLLER SYNTHESIS AND STABILITY

The approach to synthesizing LPV-controllers for the LPV-models derived in the previous sections is based on polytopic sets of the parameters in θ_h. A convenient way to formulate a polytopic set for the matrix $A_h(\theta_h)$ given the structure of (6) is an affine representation for interval-bounded parameters. An LPV-model of type (4) with B_h independent of θ_h (as applies for (6)) is called *affine* if for the matrix $A_h(\theta_h)$ applies:

$$A_h(\theta_h) = A_{0,h} + \sum_{j=1}^{p_h} \theta_{j,h} \cdot A_{j,h} \tag{8}$$

with matrices $A_{j,h} \in \mathbb{R}^{n_{x,h} \times n_{x,h}}$. For parameter limits $\theta_{j,h} \in [\underline{\theta}_{j,h}, \overline{\theta}_{j,h}]$, $A_h(\theta_h)$ varies within a matrix-polytope with vertices $\widetilde{A}_{i,h} \in \mathbb{R}^{n_{x,h} \times n_{x,h}}$, which correspond to the parameter bounds. Thus, $A_h(\theta_h)$ can be described as a matrix polytope by (Apkarian et al., 1995):

$$\mathcal{A}_h = \left\{ A_h = \sum_{i=1}^{l_h} \alpha_{i,h} \cdot \widetilde{A}_{i,h}, \sum_{i=1}^{l_h} \alpha_{i,h} = 1, \alpha_{i,h} \geq 0. \right\} \tag{9}$$

and $A_h(\theta_h) \in \mathcal{A}_h$. The matrix polytope is the convex hull of the l_h matrices $\widetilde{A}_{i,h}$. For all combinations of interval bounds $\underline{\theta}_{j,h}$ and $\overline{\theta}_{j,h}$, the number of polytope vertices is $l_h = 2^{p_h}$. Given a polytopic description of the LPV-system, LMI based controller synthesis can be applied.

The objective is to obtain an LPV state feedback controller $K_h(\theta_h)$ for any generator with index h. This controller is chosen to satisfy criteria of $\mathfrak{D}$-stability and robustness in the H_∞-sense (Apkarian et al. (1995)). For the particular model structure of (6), tailored LMI conditions are specified for synthesis. By imposing an affine controller structure $K_h(\theta_h)$, the closed-loop matrix:

$$A_{cl,h}(\theta_h) = A_h(\theta_h) + B_h \cdot K_h(\theta_h) \tag{10}$$

follows to be affine as well, and can thus be formulated as a matrix polytope as in (9). For vertices of the polytope:

$$\widetilde{A}_{cli,h} = \widetilde{A}_{i,h} + B_h \cdot \widetilde{K}_{i,h}, \tag{11}$$

the LMIs need to be specified and solved only for the l_h many vertices $\widetilde{A}_{cli,h}$. For H_∞-design, the LPV model (4) has to be extended by appropriate variables to measure control performance. For the particular structure (6), the extended model is chosen to be:

$$\dot{x}_h(t) = A_h(\theta_h)x_h(t) + B_h u_h(t) + B_{\infty,h} w_h(t)$$
$$z_h(t) = C_{\infty,h} x_h(t) \tag{12}$$

with x_h and u_h as in (4), and in addition outputs $z_h \in \mathbb{R}^{n_{z,h}}$ as well as exogenous inputs $w_h \in \mathbb{R}^{n_{w,h}}$ (e.g. disturbances); $n_{z,h}$ and $n_{w,h}$ denote the number of the outputs z_h and of the exogenous inputs. The closed loop transfer function $G_{zw,h}(s)$ is defined as the transfer function from w_h to z_h.

Lemma 1. Vertex property of quadratic H_∞ performance (Apkarian et al., 1995): For (12) and a parameter space Θ_h, H_∞ closed-loop performance $\|G_{zw,h}(s)\|_\infty < \gamma_h$ is guaranteed for all possible parameter trajectories $\theta_h \in \Theta_h$, if a symmetric matrix X_h can be found that satisfies the *bounded real lemma* for each of the l_h vertices of the polytope h. For the particular form of (12) with only $A_{cl,h}(\theta_h)$ being parameter-varying, the vertices for the closed-loop system are defined by $\widetilde{A}_{cli,h}$ while the constant other matrices are set to $B_{cl,h} = B_{\infty,h}$, $C_{cl,h} = C_{\infty,h}$ and $D_{cl,h} = 0$. The bounded real lemma thus can be established by the following LMI [4]:

$$\begin{bmatrix} \widetilde{A}_{cli,h}X_h + X_h\widetilde{A}_{cli,h}^T & B_{cl,h} & X_hC_{cl,h}^T \\ B_{cl,h}^T & -\gamma_h I & D_{cl,h}^T \\ C_{cl,h}X_h & D_{cl,h} & -\gamma_h I \end{bmatrix} < 0,\ X_h > 0 \quad (13)$$

A solution X_h for any vertex implies asymptotic stability of the closed-loop with the Lyapunov function $V_h(x) = x_h^T P_h x_h$ and $P_h = X_h^{-1}$. □

According to (Chilali and Gahinet, 1996), an *LMI region* can be defined as a subset $\mathfrak{D}$ of the complex plane for symmetric matrices $\alpha, \beta \in \mathbb{R}^{m\times m}$:

$$\mathfrak{D} = \{z \in \mathbb{C} : f_D(z) = \alpha + z\beta + \bar{z}\beta^T < 0\}. \quad (14)$$

Lemma 2. – Quadratic $\mathfrak{D}$-*Stability* (Chilali and Gahinet, 1996): A polytopic system with system-matrix $A_{cl,h}(\theta_h)$ as in (10) is $\mathfrak{D}$-stable with poles contained in the LMI-region $\mathfrak{D}_\mathfrak{h}$, if there exists a symmetric matrix $X_h > 0$ such that:

$$M(\widetilde{A}_{cli,h}, X_h) = \alpha_h \otimes X_h + \beta_h \otimes (\widetilde{A}_{cli,h}X_h) + \dots \\ \beta_h^T \otimes (\widetilde{A}_{cli,h}X_h)^T < 0, \quad (15)$$

for all l_h vertices of the polytope [5]. □

For $X_h > 0$, the form of the LMI-regions $\mathfrak{D}_\mathfrak{h}$ chosen here combine specifications of a *half plane* and a *conic sector*. The half-plane with $Re(z) < -\alpha_h$ is realized by:

$$2\alpha_h X_h + \widetilde{A}_{cli,h}X_h + X_h\widetilde{A}_{cli,h}^T < 0, \quad (16)$$

Obviously, $\alpha_h = 0$ leads to the Lyapunov theorem. Furthermore, to realize a half plane $Re(z) > -\alpha_h$, the relation in (16) changes to $>$. A conic sector with the angle φ_h between the bounding line and the real-axis is realized by the following LMI:

$$\begin{bmatrix} \sin(\varphi_h)(T_1) & \cos(\varphi_h)(T_2) \\ -\cos(\varphi_h)(T_2) & \sin(\varphi_h)(T_1) \end{bmatrix} < 0, \quad (17)$$

using the abbreviations $T_1 := \widetilde{A}_{cli,h}X_h + X_h\widetilde{A}_{cli,h}^T$ and $T_2 := \widetilde{A}_{cl,h}X_h - X_h\widetilde{A}_{cli,h}^T$.

For multiobjective control, the named LMIs are used in conjunction. Convexity is enforced by solving the LMIs with a common matrix X_h. Linearization of the resulting matrix inequalities is achieved by using the auxiliary variables $Y_{i,h} := \widetilde{K}_{i,h}X_h$. Eventually, the semi-definite optimization problem to be solved is:

$$\min_{\widetilde{K}_{i,h}, X_h} \quad \gamma_h \quad (18) \\ s.t.: \quad (16) \text{ or } (17), \text{ and } (13).$$

If a feasible solution $\widetilde{K}_{i,h}$ and X_h for any vertex $\widetilde{A}_{cli,h}$ of the matrix polytope $A_{cl,h}(\theta_h)$ is obtained, the LPV-controller $K_h(\theta_h)$ stabilizes the subsystem consisting of (6) and (7) for any parameter in $\theta_{j,h} \in [\underline{\theta}_{j,h}, \bar{\theta}_{j,h}]$ with $j \in \{1, \dots, p_h\}$. In real-time control, θ_h is measured or determined from measurable quantities, and $K_h(\theta_h)$ is calculated. By equating $A_h(\theta_h)$ in (8) and (9) as well as using the conditions for α_h in (9), α_h can be determined by solving a linear optimization problem with p_h+1 equations and l_h unknown variables $\alpha_{i,h}$. The controller follows from $K_h(\theta_h) = K_h(\alpha_h) = \sum_{i=1}^{l_h} \alpha_{i,h}\widetilde{K}_{i,h}$ (Apkarian et al., 1994).

The following assumption introduces the extension of the stability result to the complete power system:

Assumption 1. For a system with q generators and a grid as represented by (3), assume that Θ_h over-approximates the set of parameters $\Theta_{h,real}$, which results for generator h from the effects imposed by all nodes with $\bar{y}_{hk}^* \neq 0$. □.

Lemma 3. If, for all buses associated with generators $h \in \{1, \dots, q\}$, Assumption 1 holds and the controller $K_h(\theta_h)$ with $\theta_h \in \Theta_h$ is synthesized according to the decentralized solution of (18) for the model (6) and (7), then the power system (1), (2), and (3) is asymptotically stabilized. □

The proof of this lemma is sketched as follows: The assumption $\Theta_h \supseteq \Theta_{h,real}$ implies that θ_h conservatively represents the effects of the power system on the generator h. The coupling according to (3) is thus replaced by the local robustly parametrized models. The separate synthesis of $K_h(\theta_h)$ for each generator leads to the following block-diagonal structure:

$$\underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_q \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} A_{cl1}(\theta_1) & 0 & \dots & 0 \\ 0 & A_{cl2}(\theta_2) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & A_{clq}(\theta_q) \end{bmatrix}}_{\mathbf{A}(\theta)} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_q \end{bmatrix}}_{\mathbf{x}} \quad (19)$$

If for any h, (18) has a solution, i.e. $K_h(\theta_h)$ and $X_h > 0$ exist, then lemma 2 implies asymptotic stability and the existence of a local Lyapunov function $V_h(x_h)$ for the generator. A global Lyapunov function is obtained by constructing $V(x) = \mathbf{x}^T P \mathbf{x}$ with $P = diag(X_1^{-1},\ X_2^{-1}, \cdots, X_q^{-1})$: $X_h > 0$ implies $V(x) > 0$, and $\frac{\mathrm{d}V(x)}{\mathrm{d}t} = \mathbf{x}^T(\mathbf{A}(\theta)^T P + P\mathbf{A}(\theta))\mathbf{x} < 0$ follows from the vertex property of the polytopic description of $\mathbf{A}(\theta)$ for any $\theta(t)$. Since the LPV-model (19) was obtained from (1), (2) by exact transformation, the latter system is asymptotically stable . ■

4. SIMULATION RESULTS

The controller synthesis procedure is applied to a 9-bus-system which is sketched in Fig. 1 and taken from Sec. 2.10 in (Anderson, 2003). Three generators $G1$, $G2$ and $G3$ are connected to the triangle-structured grid through the transformers $T1$, $T2$ and $T3$. The loads are denoted by A, B and C. Each generator is modeled according to (1) and (2), and the grid structure is cast into an admittance matrix as in (3). To investigate controller performance, a particular (challenging) disturbance/fault sequence is chosen: The system starts from a steady state and has to change into a second (the nominal one in the reference) which results from doubling the admittance of the line between bus 5 and 7 at time 1 sec, i.e. y_{57}, y_{75}, y_{55} and y_{77} of the admittance matrix change. After 9 more seconds, a severe fault occurs: the shunt admittance of the bus 9 is changed by $-100i$, i.e. y_{99} is reduced by $-100i$. This change emulates a large current flow to ground, close to a short circuit. This condition is kept for the rest of the simulation. First, the scenario is simulated without control of the generators, Fig. 2: after the first disturbance at 1 sec, the angular velocities $x_{2,h}$ of the three generators drift

[4] The LMI in (13) is equivalent to the one in the reference, but uses $P_h = X_h^{-1}$ as a more suitable notation.

[5] $\otimes$ represents the Kronecker product.

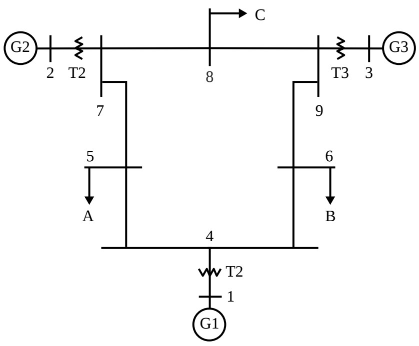

Fig. 1. Structure of the 9-bus-system.

away from the steady state, and the powers p_h start to oscillate but the system remains synchronous. After the second change at 10 sec the generators lose synchronism, since $G2$ and $G3$ (blue and red lines) start to accelerate while $G3$ decelerates. This is due to the fact that the fault is introduced at bus 9 (i.e. close to $G3$) causing that the active power of this generator falls to almost zero (red line) – thus a stabilizing controller is necessary.

A possible controller choice, used to contrast it to our proposed scheme, is the use of a standard PSS in combination with an excitation system and an automatic voltage regulator (AVR) to control the voltage at the terminal of the generator. Fig. 3 shows simulations results obtained with classical AVR-PSS controllers for parameters taken from (Shayeghi et al., 2010) (same parametrization of the controllers of $G1$ and $G2$). The system remains transiently stable after the first disturbance, since the angular velocities of the generators remain synchronous (though the courses of $x_{2,h}$ drift away from zero). The power values oscillate but are damped quickly by the PSS. However, when the second fault is introduced, the system becomes unstable, as $G3$ loses synchronism with $G1$ and $G2$ (see red line after 10 sec). The power values enter into an unstable oscillation and never recover, i.e. classic controllers (typically designed for a small operating range) become ineffective for this type of fault. Next, the controller structure and synthesis procedure proposed in this paper is investigated: First, appropriate LMI regions have to be specified, which depend on the impact of the disturbances/faults on the respective generators – the objective here is to cope with the two types of effect contained in the scenario with a single controller for each generator. The LMI regions are specified such that for any subsystem the location of the closed-loop poles are: (i) placed in the left half-plane, (ii) are bounded to the left by $Re(z_h) > -\alpha_h$ to limit the speed of response (and thus to avoid numerical problems), and (iii) to introduce sufficient damping. To enhance damping

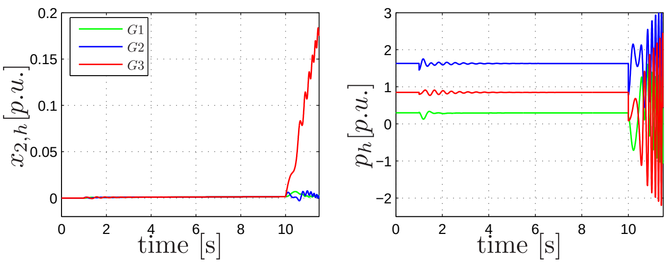

Fig. 3. Simulation for PSS-AVR control structures.

(see also (de Oliveira et al., 2010)), the conic sectors are defined to have an angle of $\varphi_h = 45°$, leading to a relative damping of around 70%. Furthermore, an H_∞ constraint is imposed on the transfer function from an additive disturbance on $\dot{x}_{2,h}$ to the output $x_{2,h}$. Though, this kind of disturbance is not consistent with the LPV description as given in (6) because the coupling to the grid is completely realized by the parameters. However, this definition is used for the controller synthesis to push the poles to the left hand side of the considered LMI-region.

By using these specifications and transforming the generator dynamics according to (6), (7), a set of semi-definite programs (18) can be formulated. The solution by minimizing γ_h for the vertices of the considered matrix polytopes of the closed loop system for each of the subsystems h leads to the desired controllers. It is stressed that the LMI solutions X_h and the resulting controllers $K_h(\theta)$ differ depending on the considered generator h. Simulations for $x_{2,h}$ and p_h using these controllers are contained in Fig. 4. It is apparent that the system remains stable during the complete simulation (in contrast to the previous plots). For both faults, the angular velocity oscillated but with a very small amplitude of less than 0.001, and it is damped down within 1 sec. Furthermore, the amplitudes of p_h are much faster damped than with the PSS-AVR controller. The largest difference to the PSS-AVR structure occurs for the second fault: The power amplitudes are not only stable, but also damped completely within 1 sec. The control actions for the conventional controller (PSS+AVR) and the LPV are compared in Fig. 5. The maximum amplitude of u_h for the first transition (after 1 sec) are comparable for both types of controllers (i.e. for $G2$ and $G3$), but the oscillations last longer for the conventional scheme. For the LPV scheme, the maximum amplitude of 15 for G_1 is relatively high. Note that the synthesis procedure in the presented form does not include input constraints; this is subject of future work. For the second fault (after 10 sec), the control actions of the conventional scheme are unsatisfactory, as the input of $G3$ increases above 80. In

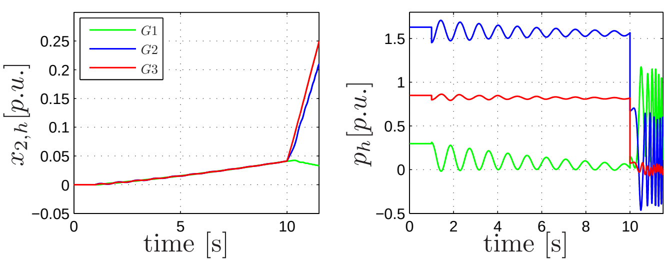

Fig. 2. Simulation for generators without control.

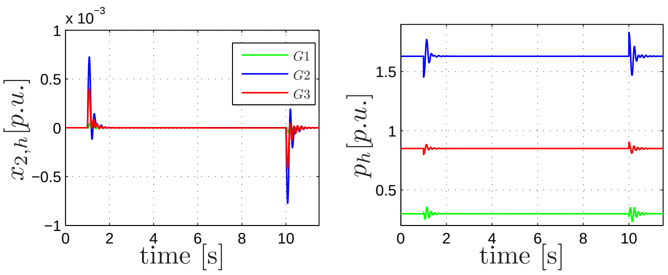

Fig. 4. Simulation for the proposed LPV controllers.

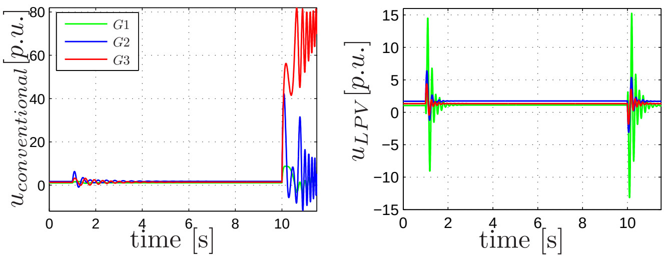

Fig. 5. Control actions for the conventional (left) and the LPV control scheme (right).

contrast, the control actions for the LPV controller are of the same order than for the first transition. (Note the different scaling of the ordinates in the two parts). For the implementation of the feedback LPV-controller, the parameters (i.e. $i_{q,h}$ and $i_{d,h}$) and the states (δ_h, ω_h and $e'_{q,h}$) have to be known. All of these quantities can be measured, calculated, or estimated (Guo et al., 2000).

5. CONCLUSION AND FUTURE WORK

The proposed synthesis technique establishes a decentralized control structure for power systems. The partition of the overall DAE-model of the power system into nodes modeled by linear-parameter-varying systems and the grid algebraic equations to represent the coupling enables one to synthesize the controllers for the generators separately. Thus, the local synthesis problems are of moderate size, and in addition the overall design effort grows moderately with the size of the power system. For any node, a synthesis procedure based on LMI-formulation and semi-definite programming was proposed which leads to robustly stable and well-damped behavior of the generator (in the sense of transient stability). If ranges for the model parameters of any node are determined conservatively, the stabilization of the power system can be concluded. Future work will address the inclusion of input constraints and the rigorous criteria to determine conservative parameter set.

ACKNOWLEDGEMENTS

Partial financial support by the DFG through the project ROCS-Grid and by the EU through the H2020-project UnCoVerCPS is gratefully acknowledged.

REFERENCES

Anderson, P. M.and Fouad, A.A. (2003). *Power system control and stability*. Wiley-Interscience.

Apkarian, P., Gahinet, P., and Becker, G. (1994). Self-scheduled H_∞ of linear parameter-varying systems. In *American Control Conf., Vol. 1*, 856–860. IEEE.

Apkarian, P., Gahinet, P., and Becker, G. (1995). Self-scheduled h_∞ control of linear parameter-varying systems: a design example. *Automatica*, 31(9), 1251–1261.

Chilali, M. and Gahinet, P. (1996). H_∞ design with pole placement constraints: an LMI approach. *IEEE Tr. on Automatic Control*, 41(3), 358–367.

de Oliveira, R.V., Ramos, R.A., and Bretas, N.G. (2010). An algorithm for comp. automatic tuning of power system stabilizers. *Control Eng. Practice*, 18(1), 45–54.

Fan, L. (2009). Review of robust feedback control applic. in power systems. In *IEEE Power Systems Conf.*, 1 –7.

Gordon, M. and Hill, D. (2008). Global transient stability and voltage regulation for multimachine power systems. In *Conversion and Delivery of Electrical Energy in the 21st Century*, 1–8.

Guo, Y., Hill, D.J., and Wang, Y. (2000). Nonlinear decentralized control of large-scale power systems. *Automatica*, 36(9), 1275–1289.

He, R., Liu, K.Z., and Mei, S. (2009). LPV modelling and gain-scheduled control approach for the transient stabilization of power systems. In *4th IEEE Conf. on Ind. Electronics and Applications, 2009.*, 29 –34.

He, R., Liu, K.Z., Mei, S., and Gui, X. (2006). A gain-scheduled state feedback control method for transient stability control of a single-machine infinite-bus power system. In *IEEE Chinese Control Conf.*, 2147–2152.

Kundur, P. (1994). *Power System Stability and Control*. McGraw-Hill.

Kundur, P., Paserba, J., Ajjarapu, V., Andersson, G., Bose, A., Canizares, C., Hatziargyriou, N., Hill, D., Stankovic, A., Taylor, C., Van Cutsem, T., and Vittal, V. (2004). Definition and classification of power system stability. *IEEE Tr. on Power Systems*, 19(2), 1387–1401.

Kwiatkowski, A., Boll, M.T., and Werner, H. (2006). Automated generation and assessment of affine LPV models. In *45th IEEE Conference on Decision and Control*, 6690–6695.

Liu, Q., Vittal, V., and Elia, N. (2006a). Expansion of system operating range by an interpolated LPV FACTS controller using multiple Lyapunov functions. *IEEE Tr. on Power Systems*, 21(3), 1311–1320.

Liu, Q., Vittal, V., and Elia, N. (2006b). LPV supplementary damping controller design for a thyristor controlled series capacitor device. *IEEE Tr. on Power Systems*, 21(3), 1242 –1249.

Milano, F. (2010). *Power System Modelling and Scripting*. Power Systems. Springer.

Qiu, W., Vittal, V., and Khammash, M. (2004). Decentralized power system stabilizer design using linear parameter varying approach. *IEEE Tr. on Power Systems*, 19(4), 1951 – 1960.

Rao, K. and Paul, S. (2011). Design of robust power system stabilizer using mixed sensitivity based H_∞ output-feedback control in LMI framework. In *Conf. on Sustainable Energy and Intelligent Systems*, 403–408.

Rao, P.S. and Sen, I. (2000). Robust pole placement stabilizer design using linear matrix inequalities. *IEEE Tr. on Power Systems*, 15(1), 313–319.

Schaab, K. and Stursberg, O. (2015). Decentralized robust control of power grids using LPV-models of DAE-systems. In 1^{st} *IFAC Workshop on Linear-Parameter-Varying Systems*.

Shayeghi, H., Shayanfar, H., Jalilzadeh, S., and Safari, A. (2010). Multi-machine power system stabilizers design using chaotic optimization algorithm. *Energy Conversion and Management*, 51(7), 1572–1580.

Tóth, R. (2010). *Modeling and identification of linear parameter-varying systems, Vol. 403*. Springer Science & Business Media.

Werner, H., Korba, P., and Yang, T.C. (2003). Robust tuning of power system stabilizers using LMI-techniques. *IEEE Tr. on Control Systems Techn.*, 11(1), 147–152.