Hybrid Modelling of a Wind Turbine (Benchmark Proposal)

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Abstract

This paper proposes a simplified nonlinear model of a wind turbine equipped with a switching controller. The composition of wind turbine and controller results in a hybrid system. The control of such a system and the enforcement of numerous safety and performance constraints constitute a relevant benchmark to evaluate tools for proving safety requirements in hybrid systems.
Category: industrial **Difficulty:** high

1 Introduction

Wind turbine systems are considered as one of the fastest-growing source of electricity in the world. The demand for more power has set a trend for larger offshore turbines that can operate reliably and efficiently with minimum maintenance. Increasing power efficiency and reducing mechanical stress is therefore very important. To guarantee that a specific design meets the aforementioned requirements, usually extensive simulations with different wind conditions acting on the turbine are performed [7]. However, this is very time consuming and can never cover all possible cases. Replacing these simulations by formal analysis can help to significantly reduce simulation time while at the same time giving a more comprehensive evaluation of a new wind turbine design. Additionally, wind turbines are inherently nonlinear and therefore an interesting problem for existing verification tools.

Due to increasing mechanical stress on the structural components for increasing wind speeds, the control strategies for wind turbines change according to the current wind conditions. Formally describing this interaction of the wind turbine and its controller requires a hybrid model, which poses a challenging problem from a control and verification perspective. Even though this is standard in control of wind turbines, the problem of the hybrid controller has, to the best of our knowledge, not been approached in a systematic way in the literature. For simulation purposes, high fidelity models of wind turbines are available (see e.g. [8]), however, these models are composed of detailed structural and aerodynamic subsystems and have usually several hundred states. For the purpose of this paper, we propose a simplified nonlinear model that still reflects the complexity and hybrid nature of the system and can be used for verification.

This paper describes the physical background of power capture from wind, the nonlinear modelling of the wind turbine and its actuator dynamics. State of the art control algorithms are presented that result in a hybrid system, consisting of turbine and controller. Performance goals and requirements for verification are described in addition.

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2 Power capture in wind turbines

The maximum power that can be theoretically extracted from the wind is given by

$$P = \frac{1}{2} c_p \rho A v^3,$$

where c_p is the power coefficient, ρ is the air density, A is the area covered by the rotor and v is the wind speed [2]. Wind power is thus proportional to the area covered by the rotor blades and proportional to the third power of the wind speed (see also Figure 1, dashed line). Thus, there are two possibilities to extract more power from the wind, either by using larger turbine rotors, or by installing the turbines at places with higher average wind speed. However, with larger turbines, mechanical loads on the structure increase. Therefore, the actual power capture of the turbine is limited above a certain wind speed due to increasing mechanical stress on the structure and for very large wind speed, the turbine is turned off completely.

The different modes of operation of a wind turbine are defined as follows (see also Figure 1 for details):

- **Region 1** The wind speed is too low and the turbine is turned off.
- **Region 2** Below rated power, power capture from the wind should be maximised.
- **Region 3** Above rated power, the turbine blades are pitched out off the wind such that the power capture is constant and the loads on the turbine are minimized.

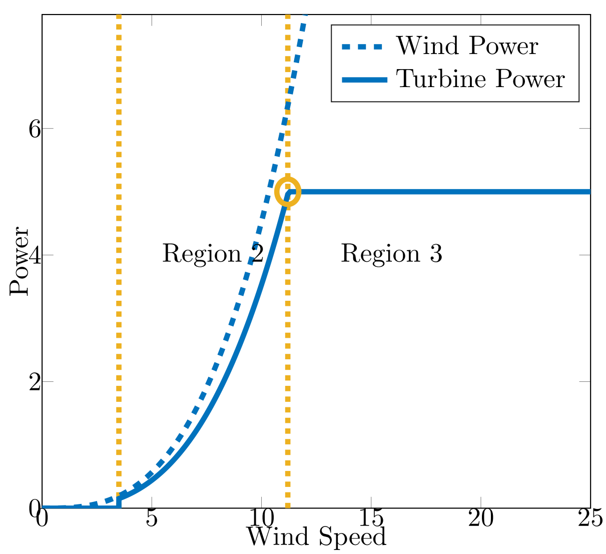

Figure 1: Different operating regions of wind turbines. Regions are separated by dashed yellow lines, the circle corresponds to region 2.5.

The circle in Figure 1 corresponds to the so called region 2.5 (or at-rated), an interpolation between region 2 and 3. A turbine that is optimally sized for the site where it is installed, operates for a significant amount of time around region 2.5. This also implies that the controller is constantly switching between the different control modes and that the switching behaviour cannot be neglected.

3 Wind disturbance

In general, the wind field impacting the turbine is three-dimensional and stochastic. For wind turbine certification, wind fields have to be generated according to the IEC standards [7] with a certain wind class. For the simplified turbine model considered in this paper, wind fields were created according to Class A with TurbSim [10] using a Kaimal turbulence model. These three dimensional wind fields were then reduced to one-dimensional rotor effective wind speeds v_0 acting as a scalar disturbance input to the simplified model.

One of the challenges in validation and verification of wind turbines is the generalisation of the wind disturbance. While for simulation purposes specific realisations (so called seeds) of the stochastic disturbance are considered, verification tools can analyse the system behaviour for a

complete class of disturbances. Here a trade-off has to be found between covering for instance all possible representations of a specific turbulence class and not being too conservative, such that the results do not reflect a realistic behaviour anymore.
Additionally to the stochastic wind disturbance, deterministic wind disturbances (e.g. gusts) are considered in the IEC standards, covering extreme events (1 year gust, 50 year gust, ...).

4 Simplified nonlinear wind turbine model

We derive a simplified nonlinear model for a wind turbine that considers drive train shaft dynamics, tower fore-aft motion, and blade pitch dynamics. The model is mainly based on [12] and is compared to the model given in [4]. The wind turbine dynamics are highly nonlinear functions of the operating point defined by the rotor speed, wind speed and blade pitch angle.

Servo-elastic subsystem

In the servo elastic part, tower fore-aft bending and rotational motion are considered

$$J\dot{\Omega} + M_g/i = M_a(\dot{x}_T, \Omega, \theta, v_0) \tag{1a}$$
$$m_{Te}\ddot{x}_T + c_{Te}\dot{x}_T + k_{Te}(x_T - x_{T0}) = F_a(\dot{x}_T, \Omega, \theta, v_0). \tag{1b}$$

Equation (1a) describes the rotor dynamics with rotor speed Ω, blade pitch angle θ, tower position x_T and rotor effective wind speed v_0, where M_a is the aerodynamic torque, M_g is the generator torque, i is the gearbox ratio and J is the sum of moments of inertia at the rotational axis of hub J_H, blades J_B and generator J_G with

$$J = J_H + 3J_B + J_G/i^2.$$

Equation (1b) describes the tower fore-aft dynamics, where F_a is the aerodynamic thrust, x_{T0} is the static tower top position without aerodynamic thrust, m_{Te}, c_{Te} and k_{Te} are the tower equivalent model mass, structural damping and bending stiffness, respectively. They were calculated according to [5] as

$$\begin{aligned} m_{Te} &= 0.25m_T + m_N + m_H + 3m_B \\ c_{Te} &= 4\pi m_{Te} d_s f_0 \\ k_{Te} &= m_{Te}(2\pi f_0)^2. \end{aligned}$$

Aero-elastic subsystem

The nonlinearity of the model is contained in the aerodynamic torque and thrust acting on the rotor

$$M_a = \frac{1}{2}\rho\pi R^3 \frac{c_P(\lambda,\theta)}{\lambda} v_{rel}^2, \qquad F_a = \frac{1}{2}\rho\pi R^2 c_T(\lambda,\theta) v_{rel}^2 \tag{2}$$

with $\lambda = \frac{\Omega R}{v_{rel}}$, $v_{rel} = (v_0 - \dot{x}_T)$, where R is the rotor radius, ρ is the air density , c_T is the thrust coefficient and λ is the tip speed ratio. The tip speed ratio λ is the ratio between the tangential speed of the tip of the blade and the relative velocity of the wind. The relative wind speed v_{rel} is computed as a superposition of the tower top speed $\dot{x}_T$ and the rotor effective wind speed v_0.
Usually, the c_P and c_T coefficients are included in the model as two-dimensional look-up tables, obtained from steady state simulations using e.g. WT_Perf [11]. For this simplified nonlinear model, we approximated the c_P and c_T look-up tables by two-dimensional polynomials using a regression model.

Pitch actuator subsystem

For large wind turbines, the pitch actuator dynamics cannot be neglected. Usually, a second order system is considered. The dynamics are given by

$$\ddot{\theta} + 2\xi\omega\dot{\theta} + \omega^2(\theta - \theta_c) = 0, \tag{3}$$

with θ_c the demanded pitch angle, ω the undamped natural frequency of the blade pitch actuator and ξ the damping factor of the blade pitch actuator.
We can organize the subsystems (1), (2) and (3) in the usual nonlinear state space form

$$\dot{x} = f(x, u, d) \tag{4a}$$
$$y = h(x, u, d) \tag{4b}$$

with

$$x = [\Omega, x_T, \dot{x}_T, \theta, \dot{\theta}]^T$$
$$u = [M_g, \theta_c]^T, \qquad d = v_0, \qquad y = [\Omega, \theta]^T.$$

In some setups, we could also assume that the tower accelerations $\ddot{x}_T$ is available for measurement.

Model discussions and possible extensions

The proposed model is the most simplistic model that still exhibits the characteristic nonlinear behaviour of a wind turbine. Several extensions to this model are possible:

- In [4], mechanical losses on the shaft bearings are considered ($M_l(\Omega)$). Furthermore, first order lag dynamics are considered for the generator $\dot{M}_g + \frac{1}{\tau}_g(M_g - M_{g,c}) = 0$, with τ_g the time constant and $M_{g,c}$ the demanded generator torque.
- In addition to the tower fore-aft motion, the tower side-to-side motion can be added to the model for a more accurate model of the tower motion.
- The pitch actuator dynamics are sometimes approximated by either a first order lag element or a time delay for simplification.
- The current model does not consider individual blade actuation nor any blade dynamics or blade loads. However, most state-of-the-art turbines have individual blade actuation.

5 Hybrid/Switched controller

As discussed in Section 2, the regions of operation of a wind turbine controller are defined based on the power constraint. *Below-rated* is defined as the region of operation when the power constraint is not reached (region 2). Similarly, *above-rated* is the region of operation when power has to be *shed* by pitching the blades in order to limit electrical power to its constraint value (region 3). The transitioning point between below-rated and above rated is called *at-rated* (region 2.5). Region 2.5 is not just a single point (as illustrated in Figure 1) but a region to limit tip speed ratio (and hence acoustic noise emissions) at rated speed.
In variable-speed wind turbines, the conventional approach for controlling power production relies on the design of two basic control systems: a generator-torque controller and a full-span blade pitch controller. The two control systems are designed to work independently for the most part, in the below-rated and above-rated wind speed range, respectively. The control approaches for torque and pitch discussed below are taken from the reference turbine [9], where also a more detailed description is available. A slightly more advanced control scheme for pitch

and torque control loops is discussed in [3]. These control algorithms show where the hybrid nature of the controller enters the system.

In the considered set up, we assume the wind turbine always to be perfectly oriented towards the mean wind direction, i.e. yaw control is assumed to be out of scope within this effort.

Generator-Torque Controller

Below rated, the control goal is to extract maximum power from the wind. Therefore, the torque is used to control the speed to a set-point. This set-point is determined such that optimal tip-speed ratio (λ_{opt}) is tracked, unless the speed constraint is violated. Above rated, the generator power is held constant so that the generator torque is inversely proportional to the generator speed. The generator-torque controller given in [9] consists of a lookup table on the filtered generator speed incorporating five control regions: 1, 1.5, 2, 2.5 and 3.

In *region 1* the generator torque is zero and no power is extracted from the wind: instead, the wind is used to accelerate the rotor for start-up, i.e.

$$M_{g,d} = 0 \quad \text{for} \quad \Omega_g < \Omega_{g,1\,\max}. \tag{5a}$$

In *region 2*, the generator torque is proportional to the square of the filtered generator speed to maintain constant (optimal) tip-speed ratio (see [3] for details)

$$M_{g,d} = \frac{\pi \rho R^5 c_p}{2\lambda^3 i^3} \Omega_g^2 \quad \text{for} \quad \Omega_{g,2\,\min} < \Omega_g \leq \Omega_{g,2\,\max}. \tag{5b}$$

In *region 3* the generator power is held constant, so that the generator torque is inversely proportional to the filtered generator speed.

$$M_{g,d} = 1/\Omega_g \quad \text{for} \quad \Omega_g > \Omega_{g,3\,\min} \vee \theta > \theta_3. \tag{5c}$$

Region 1.5 is a linear transition between region 1 and region 2 and *region 2.5* a linear interpolation between region 2 and 3.

Collective Blade-Pitch Controller

Below region 3, the blade pitch angle is set to the optimal value θ^{opt}, corresponding to the maximum in the c_P curves. In region 3, the full-span rotor-collective blade-pitch-angle commands are computed using gain-scheduled proportional-integral (PI) control on the speed error between the filtered generator speed and the rated generator speed. The gains K_p and K_I are chosen such that the linearised closed loop responds as a second-order system with user defined natural frequency ω_{cl} and damping ratio ζ_{cl}. The blade sensitivity $\partial P/\partial\theta$ is an aerodynamic property of the rotor that depends on the wind speed, rotor speed and blade pitch angle. It can be computed by linearisation analysis for different wind speeds at rated rotor speed such that the blade angle produces the rated mechanical power. As a result, the pitch sensitivity varies nearly linearly with the blade pitch angle. Therefore, the gains of the PID controller can be gain-scheduled by the blade pitch angle, i.e.

$$K_p(\theta) = \frac{2J\Omega\zeta_{cl}\omega_{cl}}{i\left(-\frac{\partial P}{\partial\theta}|_{\theta=0}\right)} f(\theta), \qquad K_i(\theta) = \frac{J\Omega\omega_{cl}^2}{i\left(-\frac{\partial P}{\partial\theta}|_{\theta=0}\right)} f(\theta), \tag{6}$$

where $f(\theta)$ is the gain scheduling factor.

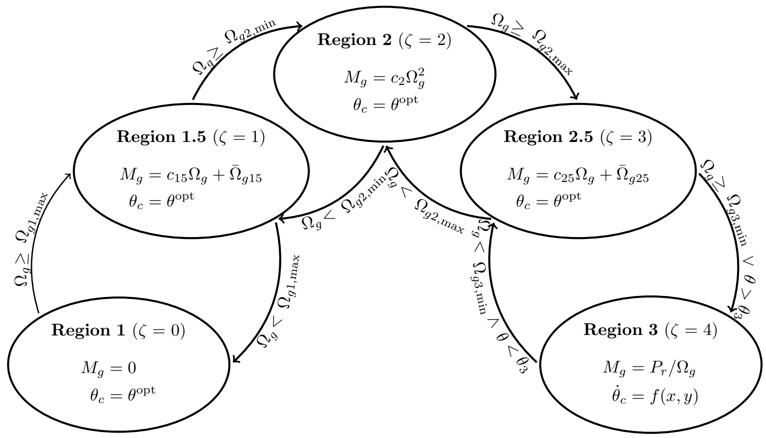

Figure 2: Wind turbine as a hybrid automaton.

Additional control objectives

The above described controller is the most basic turbine controller. Additional control objectives considered in conventional wind turbine controllers are

- tower damping to reduce tower fore-aft and tower side-to-side fatigue loads,
- drive train damping to attenuate vibrations in the gear box,
- individual pitch control to generate independent pitch commands for all blades to compensate asymmetric blade loads.
- The static torque controller is often replaced by a dynamic controller with an online set-point calculation based on the rotor speed.

6 Combined hybrid model

As the system described in the previous sections includes continuous-time dynamics and discrete-time events due to controller switching, we can rewrite the system as an impulsive system, using the formalism of [6]. The system composed of the joint continuous and discrete state $q = [x_{cl}^T, \zeta]^T$ as given in (4), (5) and (5) can be written as follows

$$\frac{d}{dt}\begin{bmatrix} x_{cl} \\ \zeta \end{bmatrix} = \begin{bmatrix} F(x_{cl}, \zeta) \\ 0 \end{bmatrix} \quad \text{for } q \in C, \tag{7a}$$

$$\begin{aligned}
\begin{bmatrix} x_{cl}^+ \\ \zeta^+ \end{bmatrix} &= \begin{bmatrix} x_{cl} \\ 1 \end{bmatrix} \quad \text{for } q \in D_1, & \begin{bmatrix} x_{cl}^+ \\ \zeta^+ \end{bmatrix} &= \begin{bmatrix} x_{cl} \\ 0 \end{bmatrix} \quad \text{for } q \in D_2, \\
\begin{bmatrix} x_{cl}^+ \\ \zeta^+ \end{bmatrix} &= \begin{bmatrix} x_{cl} \\ 2 \end{bmatrix} \quad \text{for } q \in D_3, & \begin{bmatrix} x_{cl}^+ \\ \zeta^+ \end{bmatrix} &= \begin{bmatrix} x_{cl} \\ 1 \end{bmatrix} \quad \text{for } q \in D_4, \\
\begin{bmatrix} x_{cl}^+ \\ \zeta^+ \end{bmatrix} &= \begin{bmatrix} x_{cl} \\ 3 \end{bmatrix} \quad \text{for } q \in D_5, & \begin{bmatrix} x_{cl}^+ \\ \zeta^+ \end{bmatrix} &= \begin{bmatrix} x_{cl} \\ 2 \end{bmatrix} \quad \text{for } q \in D_6, \\
\begin{bmatrix} x_{cl}^+ \\ \zeta^+ \end{bmatrix} &= \begin{bmatrix} x_{cl} \\ 4 \end{bmatrix} \quad \text{for } q \in D_7, & \begin{bmatrix} x_{cl}^+ \\ \zeta^+ \end{bmatrix} &= \begin{bmatrix} x_{cl} \\ 3 \end{bmatrix} \quad \text{for } q \in D_8,
\end{aligned} \tag{7b}$$

$$D_1 : \{\zeta = 0; \quad \Omega_g \geq \Omega_{g1,\max}\}, \qquad D_2 : \{\zeta = 1; \quad \Omega_g < \Omega_{g1,\max}\},$$

$$D_3 : \{\zeta = 1; \quad \Omega_g \geq \Omega_{g2,\min}\}, \qquad D_4 : \{\zeta = 2; \quad \Omega_g < \Omega_{g2,\min}\},$$
$$D_5 : \{\zeta = 2; \quad \Omega_g \geq \Omega_{g2,\max}\}, \qquad D_6 : \{\zeta = 3; \quad \Omega_g < \Omega_{g2,\max}\},$$
$$D_7 : \{\zeta = 3; \quad \Omega_g \geq \Omega_{g3,\min}\}, \qquad D_8 : \{\zeta = 4; \quad \Omega_g < \Omega_{g3,\min}\},$$

where $C \in \mathbb{R}^n$ defines the flow set and, in virtue of Figure 2, the jump set D can be defined as $D = \bigcup_{i=1}^{8} D_i$.

For the two basic control loops described in this paper, the combined model can be formalised in a simpler way than the one described above. However, for more advanced control strategies (e.g. a dynamic torque controller), the formal description is necessary to capture the switching behaviour correctly. Additionally, the described turbine switches into region 3 for $\theta \geq \theta_3$ or $\Omega_g \geq \Omega_{g3,\min}$. However, most commercial wind turbines switch from region 2.5 to region 3 if the measured electrical power exceeds the power reference; and switch back to region 2.5 if the measured electrical power is lower than the power reference and the collective pitch control command is less than or equal to the fine pitch limit θ_3.

7 Requirements for verification

As described above, increasing electrical power production and decreasing mechanical loads on the structure are the main design drivers in wind turbine control. The electrical power P_{el} is calculated by $P_{el} = \eta M_g \Omega / i$, where η represents the efficiency of the electro-mechanical energy conversion. For the previously defined turbine model, only the tower base fore-aft bending moment M_{yT} can be considered, with $M_{yT} = h_H(c_T \dot{x}_T + k_T x_T)$.

Usually, in addition to the dynamic loads, damage equivalent loads (DELs) are also calculated for the turbine. The basic idea is a reduction of the complicated load distribution over the whole lifetime of a turbine to a single load indicator using rain flow counting [1] together with a Weibull distribution. The DEL is the amplitude of the reference loading cycle causing in theory the same damage as the corresponding load cycle distribution.

In the following, we summarise important requirements to be considered for verification:

- The pitch angle shall be larger than the stall pitch angle (only below region 3).
- The pitch rate shall be smaller than the maximal pitch rate.
- The generator torque and torque rate shall be between the minimum and the maximum torque (rate).
- The rotor speed shall be smaller than the maximal rotor speed.
- The tower base moment shall be between the minimal and the maximal tower base moment.
- Damage equivalent loads (DEL) of tower base moment shall be less than the max. DEL.
- The absolute difference between the commanded pitch angle and the measured pitch angle can only be larger than the max. difference for less than c time units (i.e. blades are not moving synchronous).
- The absolute difference between two individual pitch angles can only be larger than the max. difference for less than c time units (i.e. one blade is not moving).

8 Implementation

The NREL turbine considered for parametrization of the model has a rated power output of $P_r = 5$MW, a rated rotor speed of $\Omega = 12.1$rpm at a rated wind speed of $v_r = 11.2$m/s. The model is implemented in Matlab/Simulink (version 2014b) with `SimplifiedWTModel.slx` containing the model itself, `SimplifiedWTModel_main.m` the main file to be run and `SimplifiedWTModel_config.m` contains user specified configuration. Two basic options with different post processing are provided: The option `SingleRun` runs the model for one user-defined average wind speed, while `AllCases` runs a full fatigue load analysis with three different seeds per wind speed and wind speeds ranging from $4:2:24\text{m/s}$. For the full run, also a damage equivalent load analysis is performed in addition to the dynamic load analysis. A description of the individual files is given in the file `ReadMe.txt`. To calculate the damage equivalent loads for the global post processing, the WAFO toolbox (version 2.5) [13] is needed.

9 Summary and Outlook

In this paper we described the model of a wind turbine and its controller as a switched system. The switching is caused by a change in the control strategies based on the current wind conditions. We presented a simplified nonlinear model of the turbine, a baseline controller and requirements for verification. We believe that wind turbines pose an interesting example for verification, since usually only specific realisations of the stochastic disturbance (i.e. wind) can be simulated, while verification tools offer the possibility to generalise this to a complete class of disturbances.

We described how the presented model and especially the controllers can be augmented and extended and we encourage the users of this benchmark example to do so.

References

[1] "ASTM E 1049-85 Standard practices for cycle counting in fatigue analysis," ASTM, Reapproved 2005.

[2] A. Betz, "The maximum of the theoretically possible exploitation of wind by means of a wind motor," *Wind Engineering*, vol. 37, no. 4, pp. 441–446, 2013, Translation of: Das Maximum der theoretisch möglichen Ausnützung des Windes durch Windmotoren, Zeitschrift für das gesamte Turbinenwesen, Heft 26, 1920.

[3] E. A. Bossanyi, "Individual blade pitch control for load reduction," *Wind energy*, vol. 6, no. 2, pp. 119–128, 2003.

[4] C. L. Bottasso, A. Croce, B. Savini, W. Sirchi, and L. Trainelli, "Aero-servo-elastic modeling and control of wind turbines using finite-element multibody procedures," *Multibody System Dynamics*, vol. 16, no. 3, pp. 291–308, Nov. 2006.

[5] R. Gasch and J. Twele, Eds., *Windkraftanlagen - Grundlagen, Entwurf, Planung und Betrieb*, 8th ed. Springer Vieweg, 2013.

[6] R. Goebel, R. G. Sanfelice, and A. R. Teel, "Hybrid dynamical systems," *IEEE Control Systems*, vol. 29, no. 2, pp. 28–93, 2009.

[7] "IEC 61400-1 Wind turbines - Part 1: Design requirements," IEC, August 2005.

[8] J. M. Jonkman and M. L. Buhl, "FAST users's guide," National Renewable Energy Laboratory, Tech. Rep. NREL/EL-500-38230, 2005.

[9] J. M. Jonkman, S. Butterfield, W. Musial, and G. Scott, "Definition of a 5-MW reference wind turbine for offshore system development," National Renewable Energy Laboratory, Tech. Rep. NREL/TP-500-38060, 2009.

[10] N. D. Kelley and B. J. Jonkman, "Overview of the TurbSim stochastic inflow turbulence simulator," National Renewable Energy Laboratory, Tech. Rep. NREL/TP-500-41137, 2007.

[11] A. D. Platt and M. L. Buhl, "WT_perf users guide," National Renewable Energy Laboratory, Tech. Rep. NREL/TP-XXXXX, 2012.

[12] D. Schlipf, D. J. Schlipf, and M. Kühn, "Nonlinear model predictive control of wind turbines using LIDAR," *Wind Energy*, vol. 16, no. 7, pp. 1107–1129, Oct. 2013.

[13] WAFO Group, "WAFO - a matlab toolbox for analysis of random waves and loads," Math. Stat., Center for Math. Sci., Lund University, Tech. Rep., 2011. [Online]. Available: https://code.google.com/archive/p/wafo/