UNIVERSITY OF CAMBRIDGE

$\ell_{asso} - MPC$ for Over-actuated Systems

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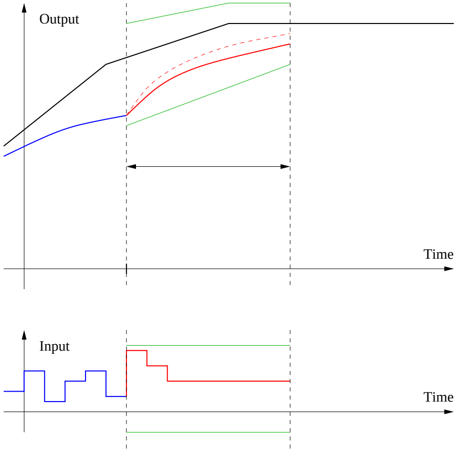
Cambridge University Engineering Department

Model Predictive Control (MPC) — the basic idea

Plan over a future horizon



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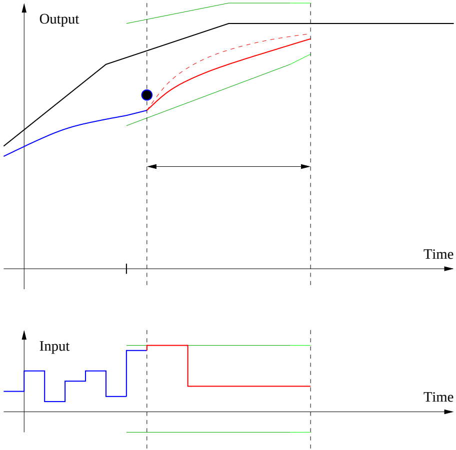
The receding horizon concept



The optimisation problem

If:

Model: *Linear* & **Cost:** *Convex* & **Constraints:** *Convex*

Then:

Convex optimisation problem (QP, LP, . . .)

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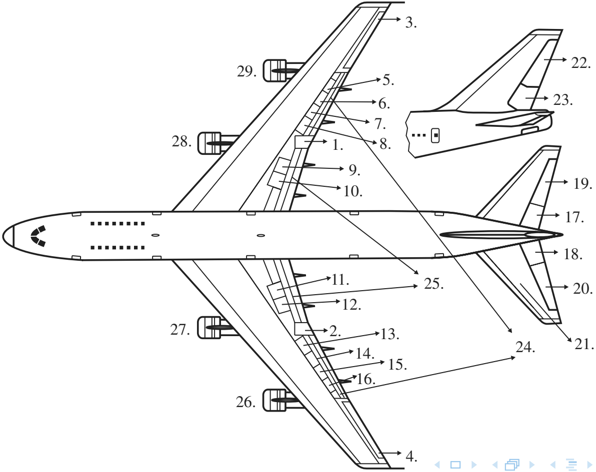
So:

Formulate convex problem if possible.

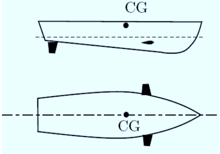
Over-actuated systems

Aircraft example: 12 states, nearly 30 actuators



Over-actuated systems

Ship roll stabilisation: fins and rudder



Cruise ship
Michelangelo
(1962)

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- If control actions are expensive, we may want sparse in time solutions — like *Statistical Process Control*.

How to get sparse solutions?

'Regularise' by adding $\|u\|_q$ (or $\|\Delta u\|_q$) penalty term

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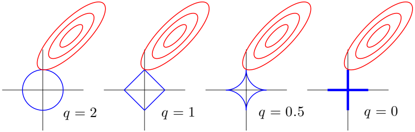

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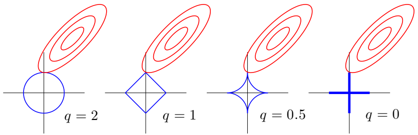


$q = 1$ is the smallest q that gives a convex problem.

ℓ_{asso}-MPC gives sparse solutions for large enough λ

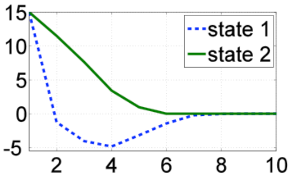
Example: Unstable toy plant

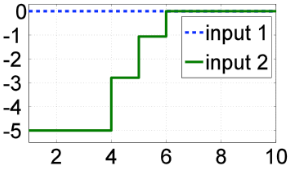
$$A = \begin{bmatrix} 0.15 & 0.1 \\ 0 & 1.1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$Q = \begin{bmatrix} 20 & 0 \\ 0 & 60 \end{bmatrix} \quad R = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

$$\|x\|_\infty \leq 20 \qquad \|u\|_\infty \leq 5$$

$$\lambda = 300$$





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Example: Ship roll control

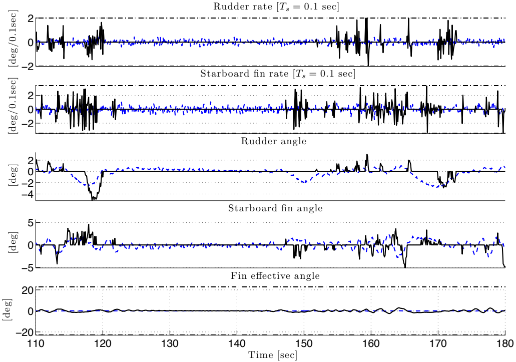

Figure: Solid: ℓ_{asso}-MPC ($\lambda = 1.8$). Dashed: Standard MPC.

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  wavelet expansions, . . .
- It’s not ‘ℓ_2-MPC’ or ‘ℓ_1-MPC’.

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- Offset-free tracking: Use disturbance estimator and target calculator (modified for ℓ_1 term).

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  - Tuning to select preferred actuators, enhance pre-existing controllers.
  - Design to maximise region of attraction.

References

- M. Gallieri and J.M. Maciejowski, The ℓ_{asso} MPC: Smart regulation of over-actuated systems, *Proc. American Control Conference*, Montreal, July 2012.
- H. Ohlsson, F. Gustafsson, L. Ljung, and S. Boyd, Trajectory generation using sum-of-norms regularization, *Proc. IEEE Conference on Decision and Control*, Atlanta, December 2010.