

# Stability and Predictability in Dynamically Complex Physical Interactions

Salah Bazzi<sup>1</sup>, Neville Hogan<sup>2</sup>, and Dagmar Sternad<sup>1</sup>

<sup>1</sup>Northeastern University

<sup>2</sup>Massachusetts Institute of Technology

## How do humans manipulate dynamically-complex objects?

- Dynamically-complex objects: nonlinear, chaotic, underactuated dynamics, large number of degrees of freedom
- Long delays imply heavy reliance on predictive (feed-forward) control based on an internal model
- But complex internal models seem unlikely

**Hypothesis: Humans adopt control strategies that make physical interactions with the object predictable. Predictability simplifies the required internal model.**



## Predictability and Stability

- Predictability is mathematically operationalized as stability
- Movements in these physically-interactive tasks do not occur around a fixed point of the system, and are non-periodic
- Interested in evaluating stability of a trajectory, irrespective of final behavior
- **Contraction analysis** is proposed as an appropriate tool for assessing (exponential) stability in these complex tasks [1]

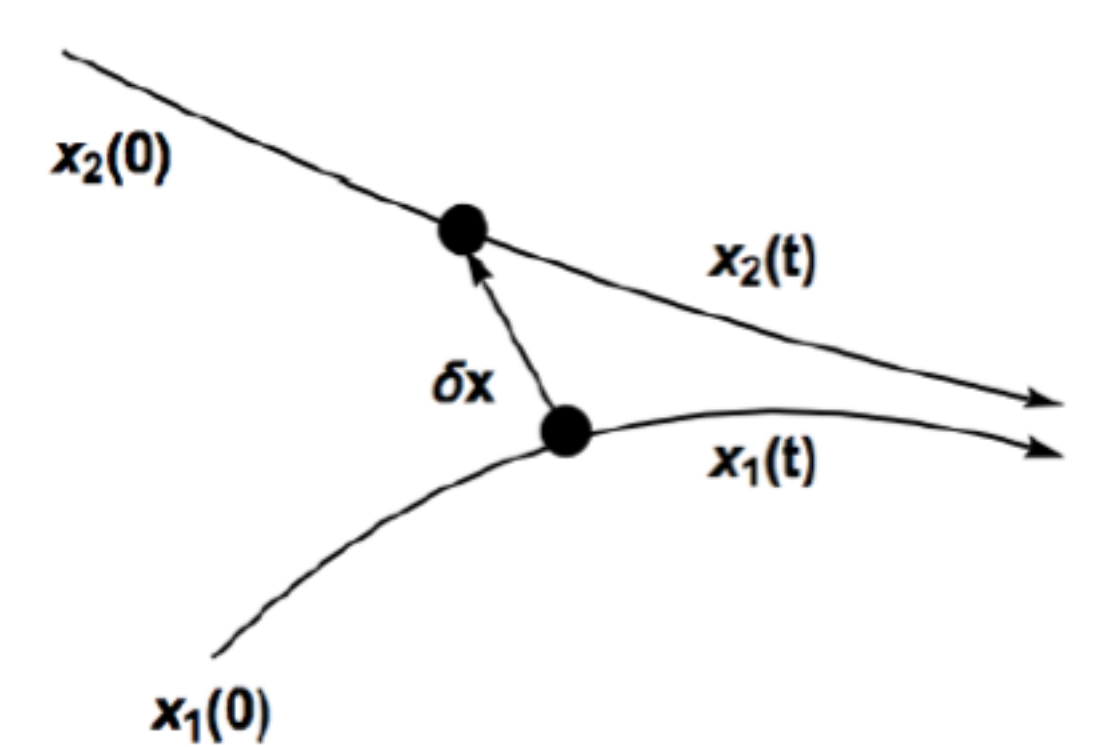
## Contraction Analysis

**Contraction condition:**

$$\frac{1}{2}(F + F^T) < 0 \text{ where } F = (\dot{\Theta} + \Theta \frac{\partial f}{\partial x})\Theta^{-1}$$

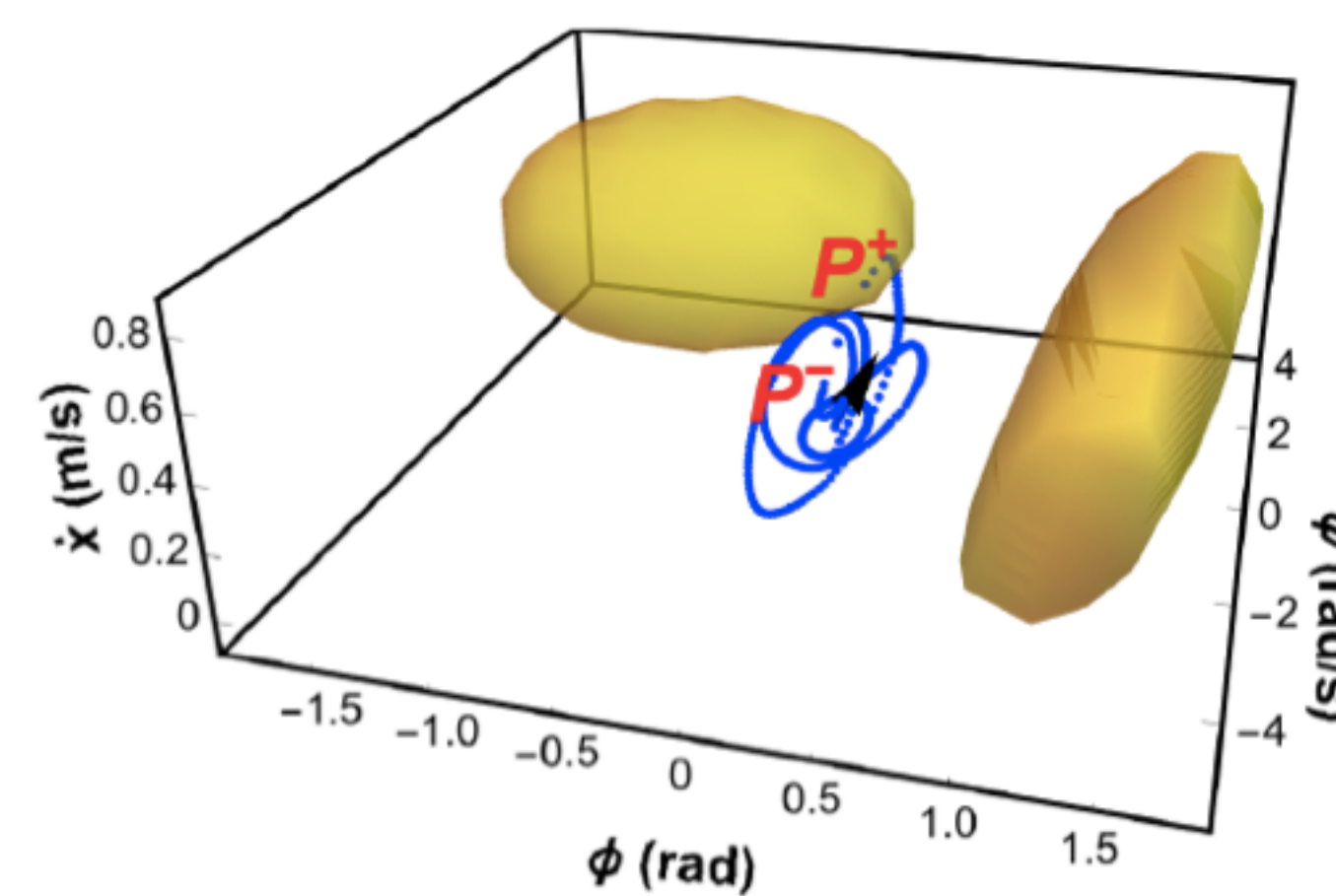
**Contraction metric:**

$$\frac{\partial \Theta}{\partial X} f + \Theta J = -\Theta$$

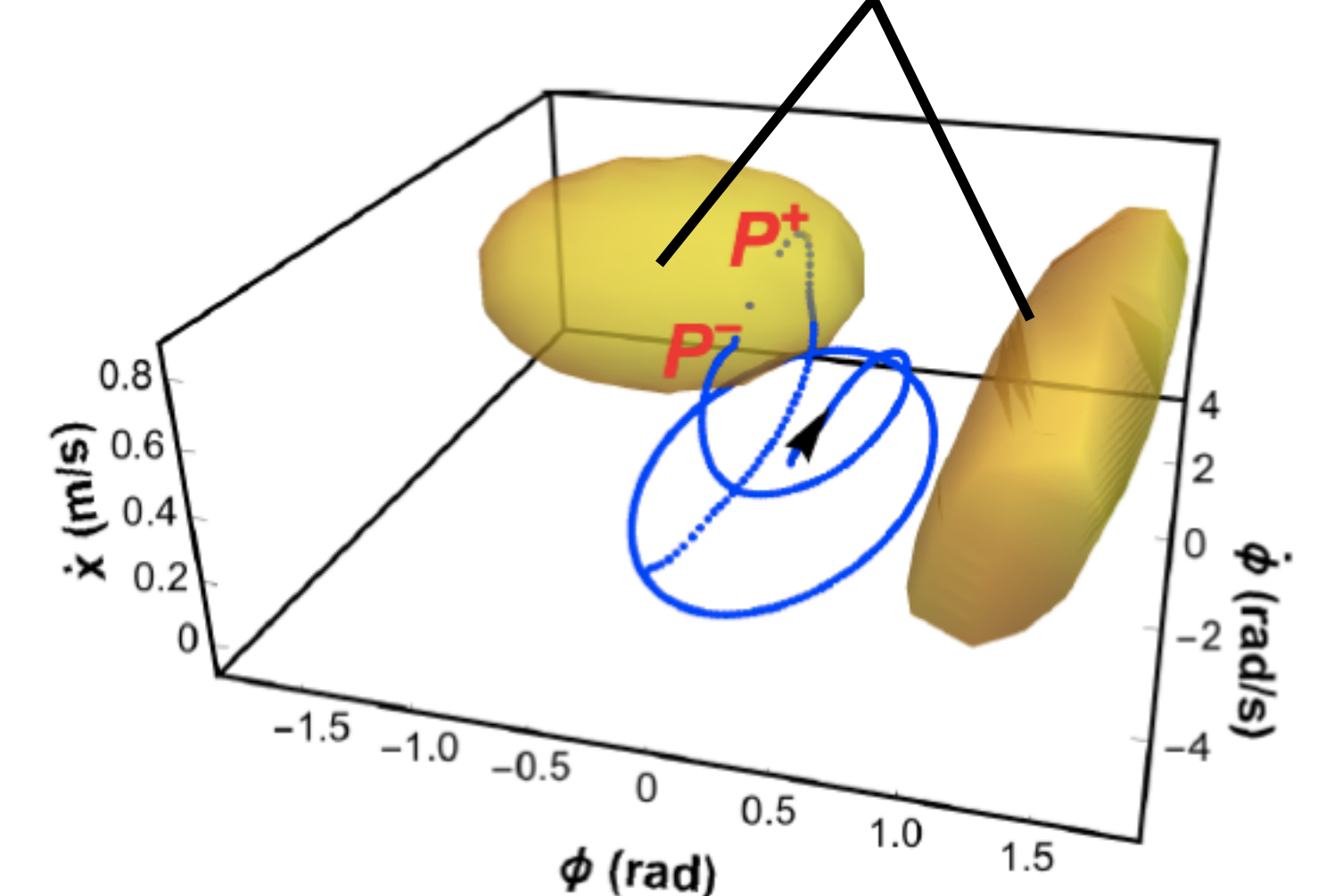


## Results

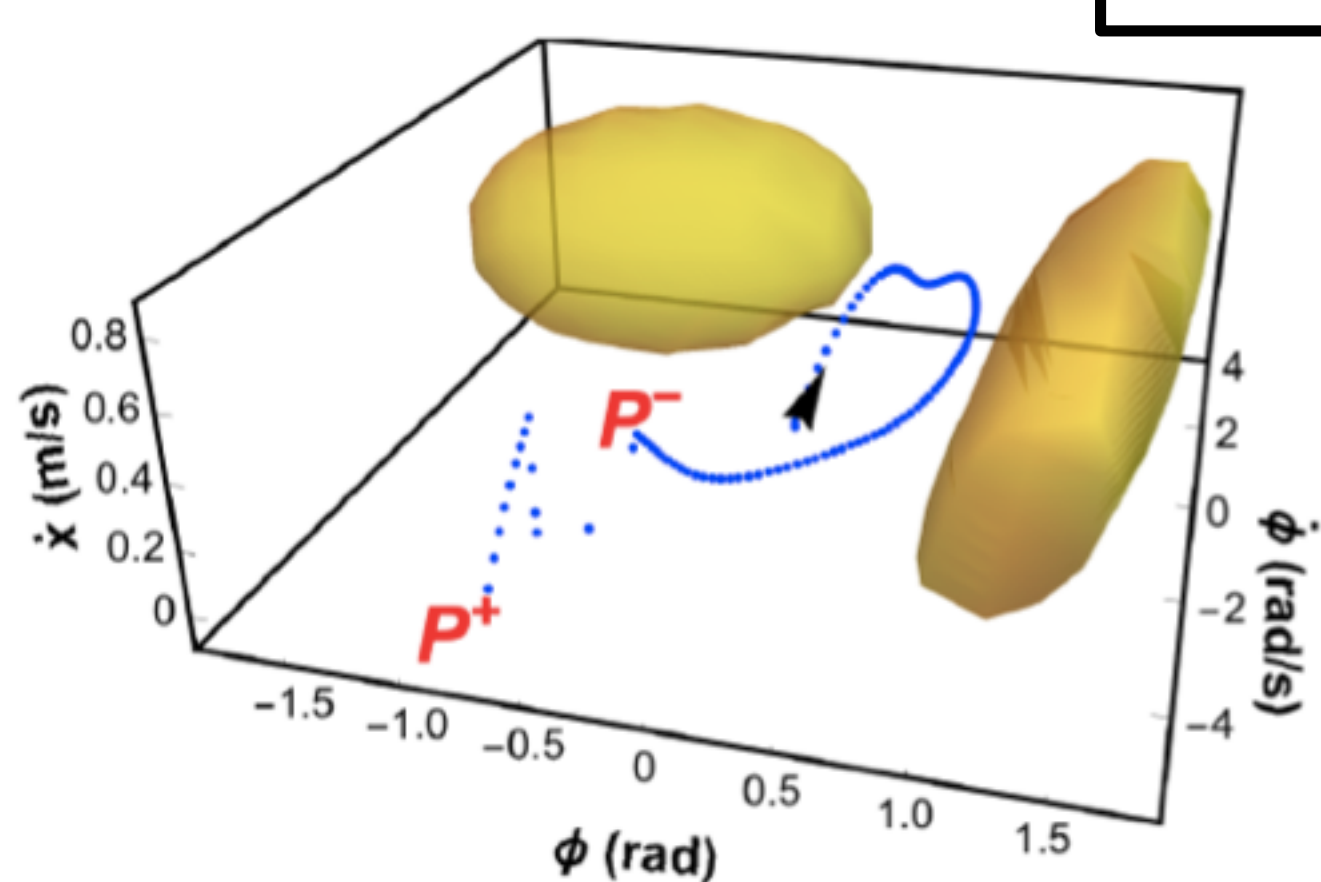
### Assistive Perturbations



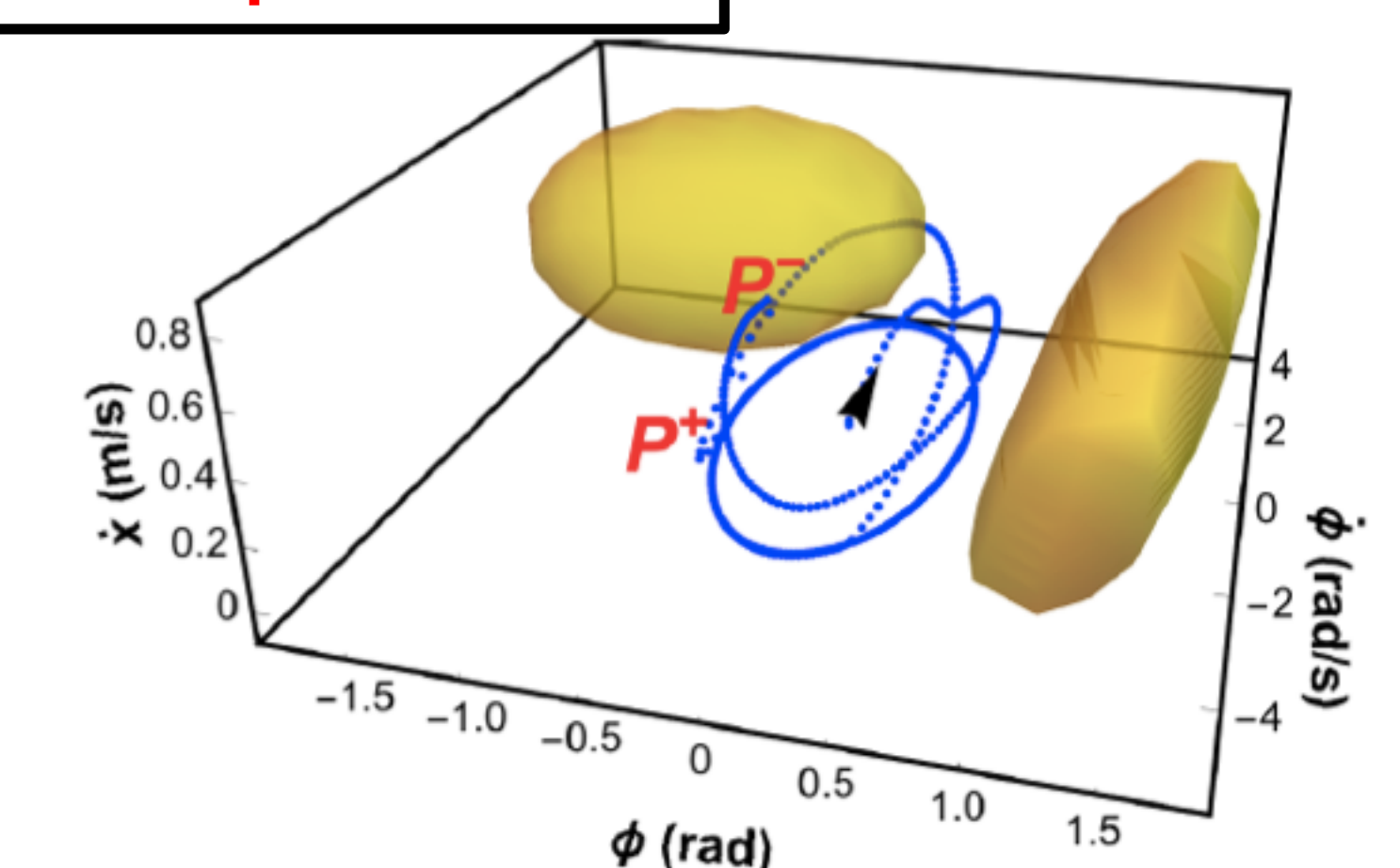
### Contracting regions



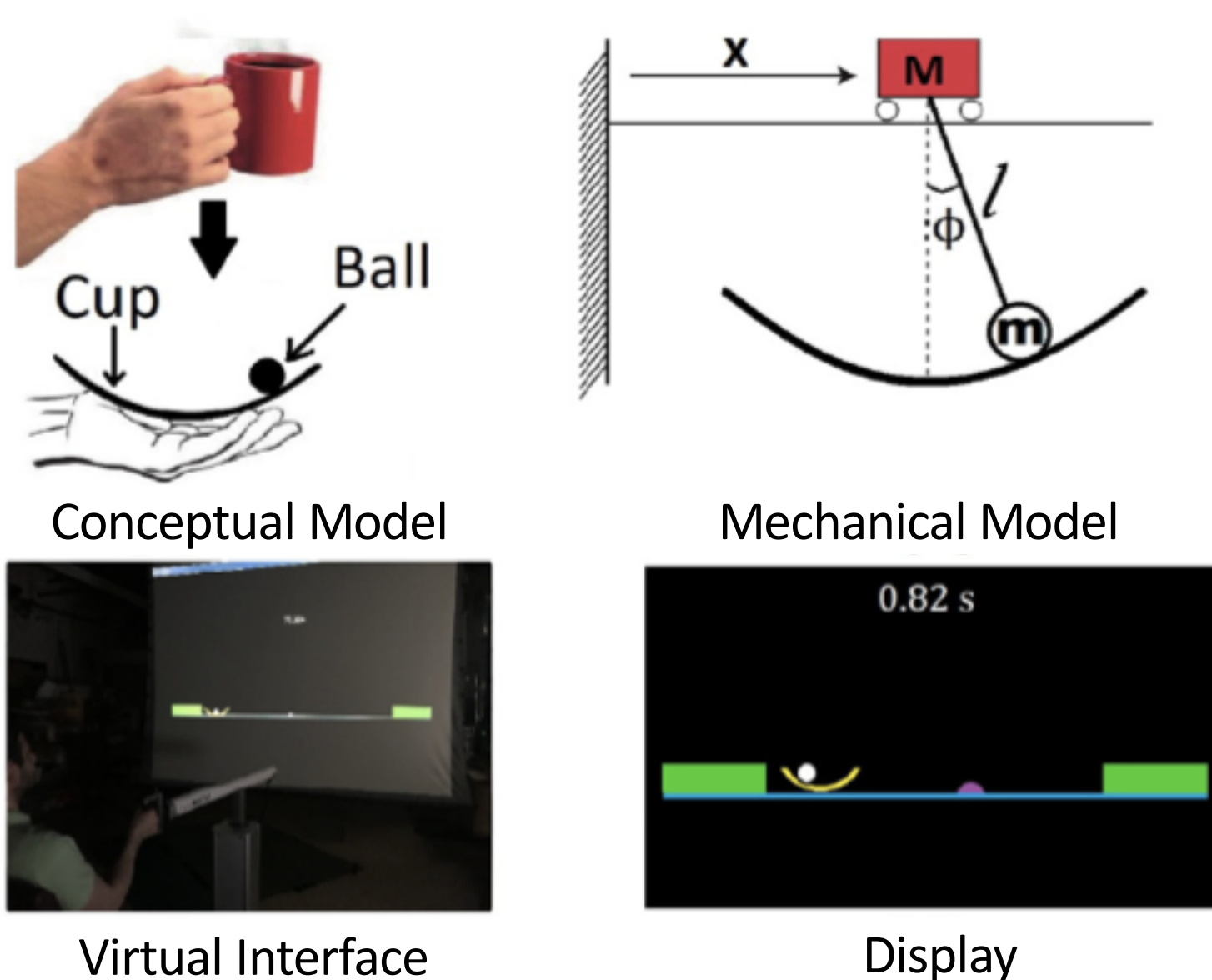
### Resistive Perturbations



**P<sup>-</sup> Instant before perturbation**  
**P<sup>+</sup> Instant after perturbation**



## The Cup-of-Coffee Task



- **Task goal:** Move the cup from start box to target box as fast as possible without losing the ball
- A perturbation is applied at a fixed visible location acting either with (assistive) or against (resistive) the cup's direction of motion

Blocks	Baseline 1	Perturbation 1	Baseline 2	Perturbation 2
Trials	60	60	10	60

### Literature

- [1] Lohmiller, W., & Slotine, J. J. E. (1998). On contraction analysis for non-linear systems. *Automatica*, 34(6), 683-696.
- [2] Bazzi, S., Ebert, J., Hogan, N., Sternad, D. (2018). Stability and predictability in dynamically complex physical interactions. *IEEE International Conference on Robotics and Automation (ICRA)*, 5540-5545.
- [3] Bazzi, S., Ebert, J., Hogan, N., Sternad, D. (2018). Stability and predictability in human control of complex objects. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 28(10), 103103.

## Summary and Conclusions

- Humans exploit contraction regions to accommodate perturbations during dynamically complex physical interactions [2, 3].
- This enables perturbed trajectories to converge to their desired behavior without active error correction, making the trajectory more predictable.
- These findings may inform the development of more robust control strategies for robotic manipulation.