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Stabilization of Large-scale Distributed Control Systems using I/O Event-driven Control and Passivity

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***Abstract*— This paper examines a passivity-based I/O approach for stabilization of large scale networked control systems (NCSs) with event-driven communication. We use a cellular model to model the large scale NCSs and assume that each subsystem is an output feedback passive (OFP) system. We propose a distributed event-driven communication strategy, where each subsystem broadcasts its output information to its neighbors only when the subsystem's local output novelty error exceeds a specified threshold. Based on the proposed event-driven communication strategy, the studied large scale NCSs is finite-gain $\mathcal{L}_2$ stable in the presence of bounded external disturbances. The triggering condition is related to the topology of the underlying communication graph. We also provide a way to analyze the time interval between two consecutive communication broadcasts(the inter-event time). Simulation results are shown at the end.**

I. INTRODUCTION

Important aspects in the implementation of distributed algorithms for control of multi-agent systems are communication transmissions and actuation update schemes. Most of the work in the literature assumes that the execution of the distributed controller and the scheduling of the communication transmission are implemented in a conservative way, where a tight bound is selected as the maximal allowable inter-transmission time to guarantee the performance of the interconnected systems for all possible operating points. This traditional methodology may lead to inefficient implementation of distributed control algorithms in terms of processor usage or available communication bandwidth.

To overcome this drawback, several researchers have suggested the idea of event-based control. In a typical event-based implementation, the control signals are kept constant until the violation of a condition on certain signals triggers the re-computation of the control signals. The possibility of reducing the number of re-computations, and thus of transmissions, while guaranteeing desired levels of performance makes event-based control very appealing in networked control systems(NCSs). A comparison of time-driven and event-driven control for stochastic systems favoring the latter can be found in [4]; a deterministic event-triggered strategy was introduced in [6]; an event-triggered real-time scheduling approach for stabilization of passive and output feedback passive (OFP) systems has been proposed in [13]. All of those works apply to sensor-actuator NCSs.

Event-driven communication in large scale deployment of distributed NCSs is of interest because of the potential of reducing communication load for control. In the present paper, we propose a distributed event-driven communication strategy for stabilization of large scale networked control systems with finite-gain $\mathcal{L}_2$ stability. We use a cellular model to model the large scale NCSs, where the locally interconnected subsystems can be modeled as cells, the interconnections between subsystems in the same cell can be modeled as species coupling while the interconnections between subsystems in different cells can be modeled as coupling between cells. The purpose of employing a cellular model is for the study of "scalability" of the event-driven control strategy: for example, in control of multi-agent systems with event-driven communication, when additional groups of agents are added into the system, how does this affect the communication frequency in the network? In our event-driven communication strategy, each subsystem broadcasts its output information to its neighbors only when the subsystem's local output novelty error exceeds a specified threshold. The triggering condition is related to the topology of the underlying communication graph. We also provide an analysis of the time interval between two consecutive communication broadcasts(the inter-event time). Although in general, the "zeno" inter-event time cannot be avoided unless a lower bound of the inter-event time is imposed in the communication network, our analysis shows that the topology of the underlying communication graph plays an important role on the performance of the NCSs with event-driven communication, which to the best of our knowledge, has not been explicitly discussed in the literature of event-based distributed control systems. This conclusion is also verified through simulations. Related work can be found in [10],[7], however the role of network topology in event-driven control is not explicitly studied.

The rest of this paper is organized as follows: we introduce some background in section II; the problem is stated in section III; our main results are provided in section IV followed by the examples shown in section V; concluding remarks are made in section VI.

II. BACKGROUND MATERIAL

We first introduce some background on passive systems and graph theory which will be used to derive the results presented in the current paper.

A. Graph Theory

We consider finite weighted directed graphs $G := (V, E)$ with no self-loops and *adjacency matrix* A, where V denotes the set of all vertices, E denotes the set of all edges, and

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$A := [a_{ij}]$ with $a_{ij} > 0$ if there is a directed edge from vertex i into vertex j, and $a_{ij} = 0$ otherwise. The *in-degree* and *out-degree* of vertex k are given by $d_i(k) = \sum_j a_{jk}$ and $d_o(k) = \sum_j a_{kj}$ respectively.

The *Laplacian* matrix of a directed graph is defined as $L = D - A$, where D is the diagonal matrix of vertex out-degrees.

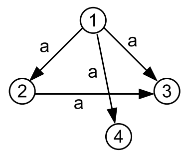

Fig. 1: graph of Example 1

Example 1: Consider a graph as shown in Fig.1, where we define

$$a_{ij} = \begin{cases} a, & \text{if vertex } i \text{ sends information to vertex } j; \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

$a > 0$ represents the coupling strength between vertices. Then we can get

$$A = \begin{bmatrix} 0 & a & a & a \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 3a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{2}$$

and the graph Laplacian is given by

$$L = \begin{bmatrix} 3a & -a & -a & -a \\ 0 & a & -a & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{3}$$

Definition 1(algebraic connectivity)[9]: Let $\mathcal{P}$ be the set $\{x \in \mathbb{R}^n | x \perp \mathbf{1}_n, \|x\| = 1\}$, where $\mathbf{1}_n := [1, 1, \ldots, 1]^T \in \mathbb{R}^n$. For a directed graph G with Laplacian matrix L, the algebraic connectivity is the real number defined as

$$a(G) = \min_{x \in \mathcal{P}} x^T L x = \min_{x \in \mathcal{P}} \frac{x^T L x}{x^T x}. \tag{4}$$

For a graph with n vertices, $a(G)$ can be efficiently computed as

$$a(G) = \lambda_{min}\{\frac{1}{2} Q(L + L^T) Q^T\}, \tag{5}$$

where $Q \in \mathbb{R}^{(n-1)\times n}$ and $Q\mathbf{1}_n = 0$.

Definition 2(strongly connected graph)[8]: A directed graph is strongly connected if for any pair of distinct vertices ν_i and ν_j, there is a directed path from ν_i to ν_j.

Definition 3(balanced graph)[8]: A vertex is balanced if its in-degree is equal to its out-degree. A directed graph is balanced if every vertex is balanced.

Lemma 1 [8]: For a balanced graph G with nonnegative weights, $a(G) > 0 \Leftrightarrow G$ is strongly connected.

B. Passivity

Consider the following dynamic system which can be used to describe both linear and nonlinear systems:

$$H : \begin{cases} \dot{x} = f(x, u) \\ y = h(x) \end{cases} \tag{6}$$

where $x \in \mathbb{X} \subset \mathbb{R}^n$, $u \in \mathbb{U} \subset \mathbb{R}^m$ and $y \in \mathbb{Y} \subset \mathbb{R}^m$ are the state, input and output variables, respectively, and $\mathbb{X}$, $\mathbb{U}$ and $\mathbb{Y}$ are the state, input and output spaces, respectively. The representation $\phi(t, t_0, x_0, u)$ is used to denote the state at time t reached from the initial state x_0 at t_0.

Definition 4(supply rate)[1]: The supply rate $\omega(t) = \omega(u(t), y(t))$ is a real valued function defined on $\mathbb{U} \times \mathbb{Y}$, such that for any $u(t) \in \mathbb{U}$ and $x_0 \in \mathbb{X}$, $y(t) = h(\phi(t, t_0, x_0, u), u)$, $\omega(t)$ satisfies

$$\int_{t_0}^{t_1} |\omega(\tau)| d\tau < \infty. \tag{7}$$

Definition 5(Dissipative System)[1]: System H with supply rate $\omega(t)$ is said to be dissipative if there exists a nonnegative real function $V(x) : \mathbb{X} \to \mathbb{R}^+$ (set of nonnegative real numbers), called the storage function, such that, for all $t_1 \geq t_0 \geq 0$, $x_0 \in \mathbb{X}$ and $u \in \mathbb{U}$,

$$V(x_1) - V(x_0) \leq \int_{t_0}^{t_1} \omega(\tau) d\tau. \tag{8}$$

Passive systems are special cases of dissipative systems as defined below.

Definition 6(Passive System)[1]: System H is said to be **passive** if there exists a storage function $V(x)$ such that

$$V(x_1) - V(x_0) \leq \int_{t_0}^{t_1} u(\tau)^T y(\tau) d\tau. \tag{9}$$

If $V(x)$ is $\mathcal{C}^1$, then we have

$$\dot{V}(x) \leq u(t)^T y(t), \ \forall t \geq 0. \tag{10}$$

Definition 7(Output Feedback Passive System)[2]: System H is said to be **Output Feedback Passive**(OFP) if it is dissipative with respect to the supply rate

$$\omega(u, y) = u^T y - \rho y^T y, \tag{11}$$

for some $\rho \in \mathbb{R}$.

Remark 1: We denote an output feedback passive system with supply rate $\omega(u, y) = u^T y - \rho y^T y$ as OFP(ρ). Note that if $\rho > 0$, then H is strictly output passive; if $\rho < 0$, then H is not passive and H is said to lack output feedback passivity. Also note that if a system is OFP(ρ), then it is also OFP($\rho - \varepsilon$), $\forall \varepsilon > 0$.

III. Problem Statement

Consider a large scale interconnected NCSs as shown in Fig.2, where we have M cells and each cell is composed of N subsystems as species. Let H_{kj} denote species k in cell j. We assume that H_{kj} is an OFP(ρ_k) system with $\mathcal{C}^1$ storage function satisfying the dissipative inequality given by

$$\dot{V}_{kj} \leq u_{kj}^T y_{kj} - \rho_k y_{kj}^T y_{kj}, \quad \rho_k \in \mathbb{R}, \ \ \forall t \geq 0, \tag{12}$$

where $u_{kj}, y_{kj} \in \mathbb{R}^m$ are the input and output of H_{kj}, V_{kj} is the storage function of H_{kj}.

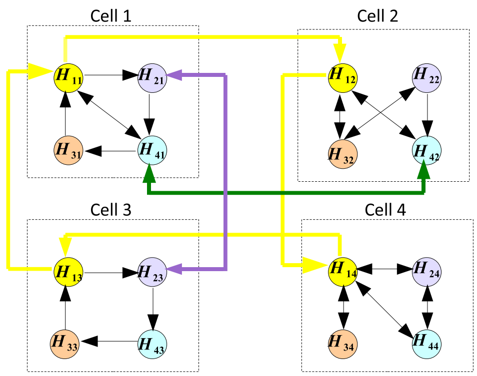

Fig. 2: Example of A Large Scale Interconnected NCSs

The control input to H_{kj} is given by

$$u_{kj} = w_{kj} + \sum_{i=1}^{N} s_{ki}^{j}(y_{ij} - y_{kj}) + \sum_{h=1}^{M} c_{jh}^{k}(y_{kh} - y_{kj}), \quad (13)$$

for $k = 1, \ldots, N$ and $j = 1, \ldots, M$, where $w_{kj} \in \mathcal{L}_{2e}$ is the external disturbance input to H_{kj}, the scalar s_{ki}^{j} is nonnegative and represents the coupling between species i and k in cell j, and we have

$$s_{ki}^{j} = \begin{cases} s_j, & \text{if } H_{ij} \text{ sends its output information to } H_{kj} \\ 0, & \text{otherwise;} \end{cases} \quad (14)$$

the scalar c_{jh}^{k} is nonnegative and represents the coupling among the species k in cell j and cell h , and we have

$$c_{jh}^{k} = \begin{cases} c_k, & \text{if } H_{kh} \text{ sends its output information to } H_{kj} \\ 0, & \text{otherwise.} \end{cases} \quad (15)$$

Let $\mathcal{N}_k^j$ denote the set of species in cell j that send output information to H_{kj}; let $\mathcal{Z}_k^j$ denote the set of species in cell j that receive output information from H_{kj}; let $\mathcal{N}_{kj}$ denote the set of species k in the other cells(excluding cell j) that send output information to H_{kj}; and let $\mathcal{Z}_{kj}$ denote the set of species k in the other cells(excluding cell j) that receive output information from H_{kj}. For example, in Fig. 2, one can see that $\mathcal{N}_1^1 = \{H_{31}, H_{41}\}$, $\mathcal{Z}_1^1 = \{H_{21}, H_{41}\}$, $\mathcal{N}_{11} = \{H_{13}\}$, and $\mathcal{Z}_{11} = \{H_{12}\}$. Let $|\cdot|$ denote the cardinality of a set, then in this example we have $|\mathcal{N}_1^1| = 2$. Note that the control law (13) is widely used in cooperative control of multi-agent systems where continuous communications between agents are assumed. Results concern the characterization and design of the information exchange structure for stabilization of dissipative multi-agent systems with continuous communication are reported in [14].

We are interested in the case when each subsystem only transmits its current output information to its neighbors when a triggering condition is satisfied. Assume at $t = t_{kj}^n$, $n = 0, 1, 2, \ldots$, H_{kj} sends out its output information $y_{kj}(t_{kj}^n)$ to its *neighbors* $\{\mathcal{Z}_k^j \cup \mathcal{Z}_{kj}\}$ and updates its control input based on $y_{kj}(t_{kj}^n)$, then we have for $t \in [t_{kj}^n, t_{kj}^{n+1})$,

$$u_{kj}(t) = w_{kj}(t) + \sum_{i \in \mathcal{N}_k^j} s_j(\widehat{y}_{ij} - \widehat{y}_{kj}) + \sum_{h \in \mathcal{N}_{kj}} c_k(\widehat{y}_{kh} - \widehat{y}_{kj}), \quad (16)$$

where $\widehat{y}_{ij}$ represents the latest output information received by H_{kj} from H_{ij} ($H_{ij} \in \mathcal{N}_k^j$), and $\widehat{y}_{kj} = y_{kj}(t_{kj}^n)$; $\widehat{y}_{kh}$ represents the latest output information received by H_{kj} from H_{kh} ($H_{kh} \in \mathcal{N}_{kj}$). Now the problems are: with event-driven communication and the control law (16), under what triggering condition we can achieve $\mathcal{L}_2$ stability of the entire system? How does the topology of the underlying communication graph impact on the overall performance of the system?

IV. Main Result

Before we state the main results of this paper, we make the following assumptions:

A1. The communication delays between each coupled subsystems are negligible;

A2. Let $a(G_j)$ denote the algebraic connectivity of the underlying communication graph in cell j and let $a(\widehat{G}_k)$ denote the algebraic connectivity of the underlying communication graph among species k in different cells, we assume $a(G_j) + a(\widehat{G}_k) > 0$, for $k = 1, \ldots, N$, $j = 1, \ldots, M$, and all the communication graphs are balanced;

A3. Each subsystem H_{kj} is OFP(ρ_k) with

$$\frac{1}{2}\min_k\{\rho_k\} \geq \max_j\{a(G_j)\} \text{ and } \frac{1}{2}\rho_k \geq a(\widehat{G}_k),$$

for $k = 1, \ldots, N$, $j = 1, \ldots, M$.

Theorem 1. Consider the model of a large scale NCSs as discussed in Section III, let assumptions **A1-A3** be satisfied. With the control input given in (16), if H_{kj} broadcasts its output information to its neighbors at the time implicitly determined by the triggering condition

$$\|e_{kj}(t)\|_2 = \sqrt{\gamma\sigma\big(|\mathcal{Z}_k^j|, |\mathcal{Z}_{kj}|\big)}\|y_{kj}(t)\|_2, \ \forall t \geq 0, \quad (17)$$

with

$$\sigma\big(|\mathcal{Z}_k^j|, |\mathcal{Z}_{kj}|\big) = \frac{a(G_j) + a(\widehat{G}_k) - (\frac{1}{2\alpha} + \frac{1}{2\beta})|\mathcal{Z}_k^j| - (\frac{1}{2\widehat{\alpha}} + \frac{1}{2\widehat{\beta}})|\mathcal{Z}_{kj}|}{\frac{(\alpha+\beta)|\mathcal{Z}_k^j|}{2} + \frac{(\widehat{\alpha}+\widehat{\beta})|\mathcal{Z}_{kj}|}{2}} \quad (18)$$

where $\gamma \in (0, 1)$, $\alpha, \beta, \widehat{\alpha}, \widehat{\beta}$ are positive scalars such that

$$a(G_j) + a(\widehat{G}_k) - (\frac{1}{2\alpha} + \frac{1}{2\beta})|\mathcal{Z}_k^j| - (\frac{1}{2\widehat{\alpha}} + \frac{1}{2\widehat{\beta}})|\mathcal{Z}_{kj}| > 0, \quad (19)$$

then the entire NCSs is $\mathcal{L}_2$ stable from $W = col\{W_1, W_2, \ldots, W_M\}$ to $Y = col\{Y_1, Y_2, \ldots, Y_M\}$, with $W_j = [w_{1j}^T, w_{2j}^T, \ldots, w_{Nj}^T]^T$ denoting the disturbance vector of cell j and $Y_j = [y_{1j}^T, y_{2j}^T, \ldots, y_{Nj}^T]^T$ denoting the output vector of cell j, for $j = 1, 2, \ldots, M$.

The proof of Theorem 1 can be found in [16].

Remark 2: A straightforward way to reduce the communication frequency among the interconnected systems is by maximizing the communication triggering threshold for each subsystem. In view of the the triggering condition (17), if we choose $\alpha = \beta = \hat{\alpha} = \hat{\beta}$ and $c_k = s_j = 1$, then one can verify that

$$\max_{\alpha}\left\{\sqrt{\gamma\sigma\big(|\mathcal{Z}_k^j|,|\mathcal{Z}_{kj}|\big)}\right\} = \frac{\sqrt{\gamma}\big[a(G_j)+a(\widehat{G}_k)\big]}{2\big(|\mathcal{Z}_k^j|+|\mathcal{Z}_{kj}|\big)}, \quad (20)$$

and in this case, we can trigger a communication broadcast whenever

$$\|e_{kj}(t)\|_2 = \frac{\sqrt{\gamma}\big[a(G_j)+a(\widehat{G}_k)\big]}{2\big(|\mathcal{Z}_k^j|+|\mathcal{Z}_{kj}|\big)}\|y_{kj}(t)\|_2. \quad (21)$$

Moreover, in view of (21), one can conclude that communication frequency can be reduced with a larger value of the ratio $\frac{a(G_j)+a(\widehat{G}_k)}{|\mathcal{Z}_k^j|+|\mathcal{Z}_{kj}|}$, which can be achieved by improving the connectivity of the communication graph while reducing the number of neighbors of each node in the graph.

Remark 3: With the cellular model employed in this paper, one can observe from the analysis above that adding additional cells (i.e., groups of agents) into the system may not increase the communication frequency in the network significantly if the network topology is optimally designed to maximize the ratio $\frac{a(G_j)+a(\widehat{G}_k)}{|\mathcal{Z}_k^j|+|\mathcal{Z}_{kj}|}$. Also, adding additional cells may not impact the communication frequency within other cells.

Remark 4: Note that **A3** requires that $\rho_k > 0$. For the case when the subsystem has $\rho_k < 0$, we could design a local controller to render the subsystem strictly output passive(i.e., implement the subsystem with a large output feedback gain $K > 0$) so that **A3** can be satisfied.

Another question need to be answered is how often should each subsystem broadcast its output information to its neighbors based on the triggering condition shown in Theorem 1? In general, it is not easy to give a common lower bound on the inter-event time since we are dealing with heterogeneous multi-agent systems, and in many situations, zeno inter-event time cannot be avoided unless a specified lower bound on the inter-event time is imposed. In the following proposition, we give an analysis of the inter-event time. Although we cannot get a common lower bound on the inter-event time, but our analysis shows that the inter-event time is related to the topology of the underlying communication graph which is important from the design perspective of large scale NCSs.

Note that since $\|e_{kj}(t)\|_2 = \|y_{kj}(t) - y_{kj}(t_{kj}^n)\|_2$ for $t \in [t_{kj}^n, t_{kj}^{n+1}], \forall n$, we have

$$\begin{aligned} &\|e_{kj}(t)\|_2 \geq \|y_{kj}(t_{kj}^n)\|_2 - \|y_{kj}(t)\|_2 \\ &\Rightarrow \|y_{kj}(t)\|_2 \geq \|y_{kj}(t_{kj}^n)\|_2 - \|e_{kj}(t)\|_2, \end{aligned} \quad (22)$$

so a sufficient condition for

$$\|e_{kj}(t)\|_2 \leq \sqrt{\gamma\sigma\big(|\mathcal{Z}_k^j|,|\mathcal{Z}_{kj}|\big)}\|y_{kj}(t)\|_2, \forall t \geq 0 \quad (23)$$

to hold is given by

$$\|e_{kj}(t)\|_2 \leq \frac{\sqrt{\gamma\sigma\big(|\mathcal{Z}_k^j|,|\mathcal{Z}_{kj}|\big)}}{\sqrt{\gamma\sigma\big(|\mathcal{Z}_k^j|,|\mathcal{Z}_{kj}|\big)}+1}\|y_{kj}(t_{kj}^n)\|_2, \quad (24)$$

for $t \in [t_{kj}^n, t_{kj}^{n+1}], \forall n$. For the following analysis, we will consider the triggering condition given by

$$\|e_{kj}(t)\|_2 = \frac{\kappa\sqrt{\gamma\sigma\big(|\mathcal{Z}_k^j|,|\mathcal{Z}_{kj}|\big)}}{\sqrt{\gamma\sigma\big(|\mathcal{Z}_k^j|,|\mathcal{Z}_{kj}|\big)}+1}\|y_{kj}(t_{kj}^n)\|_2, \quad (25)$$

where $\kappa \in (0,1)$. One can see that this is a tighter triggering condition compared with the triggering condition (17) since (24) is a sufficient condition for (23). We use it for analysis purpose to reveal some important interactions between the topology of the underlying communication graph and the inter-event time.

Proposition 1: Consider the dynamics of H_{kj} given by

$$H_{kj}: \begin{cases} \dot{x}_{kj} = f_{kj}(x_{kj}, u_{kj}) \\ y_{kj} = h_{kj}(x_{kj}), \end{cases} \quad (26)$$

Let the following assumptions be satisfied

1) $f_{kj}: \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}^m$ is Lipschitz continuous on compact set;
2) $h_{kj}: \mathbb{R}^m \to \mathbb{R}^m$ is Lipschitz continuous on compact set and it belongs to a sector $[a_{kj}, b_{kj}]$ such that $a_{kj}\|x_{kj}\|_2^2 \leq x_{kj}^T h_{kj}(x_{kj}) \leq b_{kj}\|x_{kj}\|_2^2$, where $0 < a_{kj}b_{kj} < \infty$;
3) $\|\frac{\partial h_{kj}(x_{kj})}{\partial x_{kj}}\|_2 \leq \gamma_{kj}$, where $0 < \gamma_{kj} < \infty$;
4) $\|w_{kj}(t)\|_2 \leq d, 0 < d < \infty, \forall t \geq 0$;

then the inter-transmission time $[t_{kj}^{n+1} - t_{kj}^n]$ implicitly determined by the triggering condition (25) is lower bounded by a monotone increasing function with respect to $\sigma\big(|\mathcal{Z}_k^j|,|\mathcal{Z}_{kj}|\big)$.

Proof: Since for $t \in [t_k^i, t_{k+1}^i]$, we have

$$\begin{aligned} &\frac{d}{dt}\|e_{kj}(t)\|_2 = \frac{d(e_{kj}(t)^T e_{kj}(t))^{\frac{1}{2}}}{dt} = \frac{e_{kj}(t)^T\dot{e}_{kj}(t)}{\|e_{kj}(t)\|_2} \\ &\leq \|\dot{e}_{kj}(t)\|_2 = \|\dot{y}_{kj}(t)\|_2 \\ &= \Big\|\frac{\partial h_{kj}(x_{kj})}{\partial x_{kj}}\dot{x}_{kj}\Big\|_2 \leq \Big\|\frac{\partial h_{kj}(x_{kj})}{\partial x_{kj}}\Big\|_2\|\dot{x}_{kj}\|_2 \\ &= \gamma_{kj}\Big\|f_{kj}\big(x_{kj}, -[w_{kj}(t) + \sum_{i\in\mathcal{N}_k^j} s_j(\widehat{y}_{ij} - \widehat{y}_{kj}) \\ &+ \sum_{h\in\mathcal{N}_{kj}} c_k(\widehat{y}_{kh} - \widehat{y}_{kj})]\big)\Big\|_2 \\ &\leq \gamma_{kj}L_{kj}\|x_{kj}\|_2 + \gamma_{kj}L_{kj}\Big\|w_{kj}(t) + \sum_{i\in\mathcal{N}_k^j} s_j(\widehat{y}_{ij} - \widehat{y}_{kj}) \\ &+ \sum_{h\in\mathcal{N}_{kj}} c_k(\widehat{y}_{kh} - \widehat{y}_{kj})\Big\|_2, \end{aligned} \quad (27)$$

where L_{kj} is the Lipschitz constant of $f_{kj}(x_{kj}, u_{kj})$. Moreover, since $a_{kj}\|x_{kj}\|_2^2 \leq x_{kj}^T h_{kj}(x_{kj}) \leq b_{kj}\|x_{kj}\|_2^2$, where $0 < a_{kj}b_{kj} < \infty$, one can verify that

$$\frac{\|x_{kj}\|_2}{\|y_{kj}\|_2} \leq \max\left\{\frac{1}{|a_{kj}|}, \frac{1}{|b_{kj}|}\right\} = \zeta_{kj}, \quad (28)$$

we can get

$$\frac{d}{dt}\|e_{kj}(t)\|_2 \le \gamma_{kj}L_{kj}\zeta_{kj}\big(\|e_{kj}(t)\|_2 + \|y_{kj}(t_{kj}^n)\|_2\big) + \gamma_{kj}L_{kj}d + \gamma_{kj}L_{kj}\Big\|\sum_{i\in\mathcal{N}_k^j} s_j(\widehat{y}_{ij}-\widehat{y}_{kj}) + \sum_{h\in\mathcal{N}_{kj}} c_k(\widehat{y}_{kh}-\widehat{y}_{kj})\Big\|_2, \tag{29}$$

So the evolution of $\|e_{kj}(t)\|_2$ for $t \in [t_{kj}^n, t_{kj}^{n+1}]$ is bounded by the solution to

$$\dot{p}_{kj}(t) = \gamma_{kj}L_{kj}\zeta_{kj}\big(p_{kj}(t) + \|y_{kj}(t_{kj}^n)\|_2\big) + \gamma_{kj}L_{kj}d + \gamma_{kj}L_{kj}\Big\|\sum_{i\in\mathcal{N}_k^j} s_j(\widehat{y}_{ij}-\widehat{y}_{kj}) + \sum_{h\in\mathcal{N}_{kj}} c_k(\widehat{y}_{kh}-\widehat{y}_{kj})\Big\|_2, \tag{30}$$

with $p_{kj}(t_{kj}^n) = 0$ (since at $t = t_{kj}^n$, we have $e_{kj}(t_{kj}^n) = y_{kj}(t_{kj}^n) - y_{kj}(t_{kj}^n) = 0$), the corresponding solution to (30) during $[t_{kj}^n, t_{kj}^{n+1}]$ is given by

$$p_{kj}(t) = \frac{\hat{C}_2}{\hat{C}_1}\big(e^{\hat{C}_1(t-t_{kj}^n)} - 1\big) \tag{31}$$

where $\hat{C}_1 = \gamma_{kj}L_{kj}\zeta_{kj}$, $\hat{C}_2 = \gamma_{kj}L_{kj}\zeta_{kj}\|y_{kj}(t_{kj}^n)\|_2 + \gamma_{kj}L_{kj}d + \gamma_{kj}L_{kj}\big\|\sum_{i\in\mathcal{N}_k^j} s_j(\widehat{y}_{ij}-\widehat{y}_{kj}) + \sum_{h\in\mathcal{N}_{kj}} c_k(\widehat{y}_{kh} - \widehat{y}_{kj})\big\|_2$. So we can get a lower bound of the time for $\|e_{kj}(t)\|_2$ to evolve from 0 to $\frac{\kappa\sqrt{\gamma\sigma\left(|\mathcal{Z}_k^j|,|\mathcal{Z}_{kj}|\right)}}{\sqrt{\gamma\sigma\left(|\mathcal{Z}_k^j|,|\mathcal{Z}_{kj}|\right)+1}}\|y_{kj}(t_{kj}^n)\|_2$ based on (31) which is given by

$$t_{kj}^{n+1} - t_{kj}^n \ge \tau_{kj}^n = \frac{1}{\hat{c}_1}\ln\left(1 + \frac{\hat{c}_1}{\hat{c}_2}\frac{\kappa\sqrt{\gamma\sigma\left(|\mathcal{Z}_k^j|,|\mathcal{Z}_{kj}|\right)}}{\sqrt{\gamma\sigma\left(|\mathcal{Z}_k^j|,|\mathcal{Z}_{kj}|\right)+1}}\|y_{kj}(t_{kj}^n)\|_2\right), \tag{32}$$

the proof is completed. ■

Remark 5: In view of (32), one can verify that when $\big\|\sum_{i\in\mathcal{N}_k^j} s_j(\widehat{y}_{ij}-\widehat{y}_{kj}) + \sum_{h\in\mathcal{N}_{kj}} c_k(\widehat{y}_{kh}-\widehat{y}_{kj})\big\|_2$ is large, τ_{kj}^n will be relatively small, which implies more frequent communication updates between coupled subsystems are needed when their outputs are far from agreement.

One should be aware that while our analysis of the inter-event time follows the analysis shown in [6] by restricting the output belonging to a bounded sector of the state, there are other ways in the literature to estimate the inter-event time based on different assumptions adopted in the analysis. It is possible to obtain a less conservative analysis on the inter-event time. We just use a straight-forward way to show how the topology of the communication graph can impact on the inter-event time with event-driven communication.

V. EXAMPLE

Example: Consider a large scale NCSs which is composed of five cells and each cell has five interconnected subsystems. Assume that each subsystem's dynamic is given by

$$H_{kj}: \begin{cases} \dot{x}_{kj} = -a_{kj}x_{kj} + u_{kj} \\ y_{kj} = x_{kj}, \end{cases} \tag{33}$$

for $j = 1,\ldots,5$ and $k = 1,\ldots,5$, where a_{kj} is randomly chosen from [4,10], one can verify that in this case $\min_k\{\rho_k\} = 4$. The coupling of the outputs inside a cell is described by the Laplacian given by

$$L_j = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \text{ for } j = 1,\ldots,5. \tag{34}$$

There is also output coupling among subsystem H_{11}, H_{12}, $\ldots, H_{15}$, and the coupling is described by the following Laplacian

$$\widehat{L}_k = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ 0 & 2 & -1 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}, \ k = 1. \tag{35}$$

In this case, we can obtain $a(G_j) = 0.6910$ for $j = 1,\ldots,5$; $a(\widehat{G}_k) = 0.5$, for $k = 1$, and $a(\widehat{G}_k) = 0$ for $k = 2,\ldots,5$. Based on Theorem 1, with $\gamma = 1$ (assuming no external disturbances) one can calculate that the triggering condition for each subsystem which is given by

$$\begin{aligned} &\|e_{1j}(t)\|_2 = 0.1985\|y_{1j}(t)\|_2, \text{ for } j = 1,2; \\ &\|e_{1j}(t)\|_2 = 0.2977\|y_{1j}(t)\|_2, \text{ for } j = 3,4,5; \\ &\|e_{kj}(t)\|_2 = 0.3455\|y_{kj}(t)\|_2, \text{ for } \\ &\qquad k = 2,\ldots,5, j = 1,\ldots,5. \end{aligned} \tag{36}$$

We add external disturbance into each subsystem which is an uniformly distributed random signal on the interval $[0, 0.5]$, the simulation results for subsystem H_{23} are shown in Fig.3, with σ_{kj} denoting the evolution of $\frac{\|e_{kj}(t)\|_2}{\|y_{kj}(t)\|_2}$. The evolutions of the outputs in each cell are shown in Fig.4. If we change the output coupling among subsystem H_{11}, H_{12}, $\ldots, H_{15}$ to be

$$\widehat{L}_k = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \text{ for } k = 1, \tag{37}$$

then one can verify that in this case the triggering condition for each subsystem is given by

$$\|e_{kj}(t)\|_2 = 0.3455\|y_{kj}(t)\|_2, \ \forall k, j, \tag{38}$$

in this case, we get a larger triggering threshold by changing the topology of the underlying communication graph. We compare the simulation results of H_{11} in these two different communication configuration, which is shown in Fig. 5: column (a) shows the simulation result for the first configuration, column (b) shows the simulation result for the second configuration when we have a larger triggering threshold for each subsystem. A significant reduction of the communication frequency can be observed.

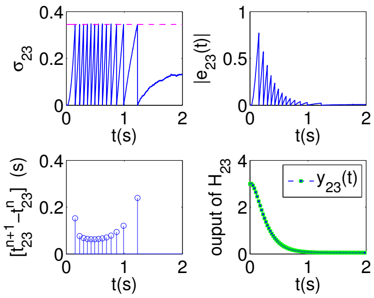

Fig. 3: simulation result of H_{23}

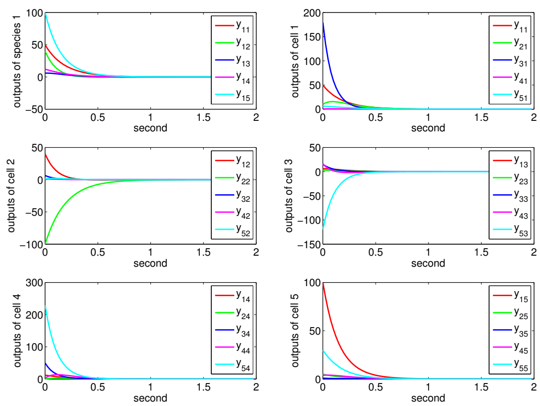

Fig. 4: evolutions of the outputs in each cell

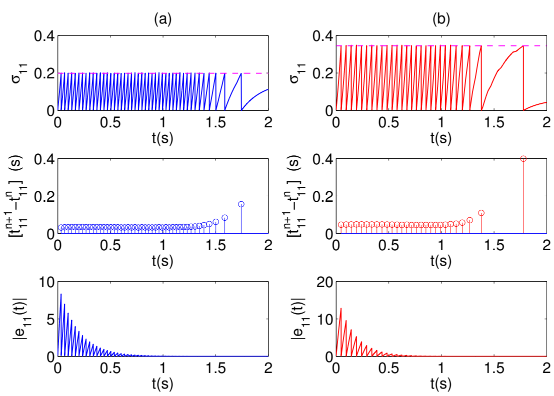

Fig. 5: comparison of different configurations

VI. Conclusion

In this paper, we propose a distributed event-driven communication strategy for stabilization of large-scale NCSs, where each subsystem broadcasts its output information to its neighbors only when the subsystem's local output novelty error exceeds a specified threshold. The triggering condition is related to the topology of the underlying communication graph. If the triggering condition derived in this paper is guaranteed, the NCSs is finite-gain $\mathcal{L}_2$ stable. We also provide a way to analyze the inter-event time. The results shown in this paper are useful in the study of the interactions between the communication frequency, the topology of the underlying communication graph and the performance for large scale deployment of distributed networked heterogeneous multi-agent systems with event-driven communication. Although only stability problem has been investigated in the current paper, extensions to output synchronization problem of multi-agent system with event-driven communication(consider both communication delay and signal quantization) can be found in [17].

VII. Acknowledgments

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References

[1] J. C. Willems, "Dissipative dynamical systems part I: General theory", *Archive for Rational Mechanics and Analysis*, Springer Berlin, Volume 45, Number 5, Pages 321-351, 1972.

[2] R. Sepulchre, and M. Jankovic and P. Kokotovic, *Constructive Nonlinear Control*, Springer-Verlag, 1997.

[3] K. E. Årzén, "A simple event based PID controller", *Proceedings of 14th IFAC World Congress,* vol.18, pp.423-428, 1999.

[4] K. J. Aström and B. M. Bernhardsson, "Comparison of Riemann and Lebesgue sampling for first order stochastic systems (I)", *Proceedings of the 41st IEEE Conference on Decision and Control*, vol.2, pp.2011-2016, 2002.

[5] M. Velasco, J. Fuertes, and P. Marti, "The self triggered task model for real-time control systems", *Progress Proceedings of the 24th IEEE Real-Time Systems Symposium*, Pages 67-70, 2003.

[6] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks", *IEEE Transaction on Automatic Control*, vol.52, no.9, pp.1680-1685, September 2007.

[7] D. V. Dimarogonas and K. H. Johansson, "Event-triggered Control for Multi-Agent Systems", *Proceedings of 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference*, pp.7131-7136, Shanghai, R.R.China, December 16-18, 2009.

[8] C. Godsil and G. Royle, *Algebraic Graph Theory*. Springer Graduate Texts in Mathematics 207, 2001.

[9] C. W. Wu, "Algebraic connectivity of directed graphs", *Linear and Multilinear Algebra*, Volume 53, Number 3, pages 203-223, 2005.

[10] X. Wang and M. D. Lemmon, "Event-Triggering in Distributed Networked Systems with Data Dropouts and Delays", *Hybrid Systems: Computation and Control*, Lecture Notes in Computer Science, Volume 5469, pages 366-380, 2009.

[11] J. P. LaSalle, "Some extensions of Liapunov's second method", *IRE Transactions on Circuit Theory*, CT-7, pp.520-527, 1960.

[12] P. G. Otanez, J. R. Moyne, D. M. Tilbury, "Using deadbands to reduce communication in networked control systems", *Proceedings of the American Control Conference*, pp.3015-3020, 2002.

[13] H. Yu and P. J. Antsaklis, "Event-Triggered Real-Time Scheduling For Stabilization of Passive/Output Feedback Passive Systems", *Proceedings of the 2011 American Control Conference*, pp.1674-1679, San Francisco, CA, June 29-July 1, 2011.

[14] S. Hirche and S. Hara, "Stabilizing interconnection characterization for multi-agent systems with dissipative properties", *Proceedings of the 17th IFAC World Congress*, pp.1571-1577, Seoul, Korea, July 6-11,2008.

[15] L. Scardovi, M. Arcak, E. D. Sontag, "Synchronization of Interconnected Systems With Applications to Biochemical Networks: An Input-Output Approach", *IEEE Transactions on Automatic Control*, Volume 55, Issue 6, pp.1367-pp.1379, June 2010.

[16] H. Yu and P. J. Antsaklis, "Stabilization of Large-scale Distributed Control Systems using I/O Event-driven Control and Passivity", compelet version with proves(http://www.nd.edu/ isis/tech.html).

[17] H. Yu and P. J. Antsaklis, "Output Synchronization of Multi-Agent Systems with Event-Driven Communication: Communication Delay and Signal Quantization", *ISIS Technical Report ISIS-2011-001*, University of Notre Dame, July 2011, (http://www.nd.edu/ isis/tech.html).