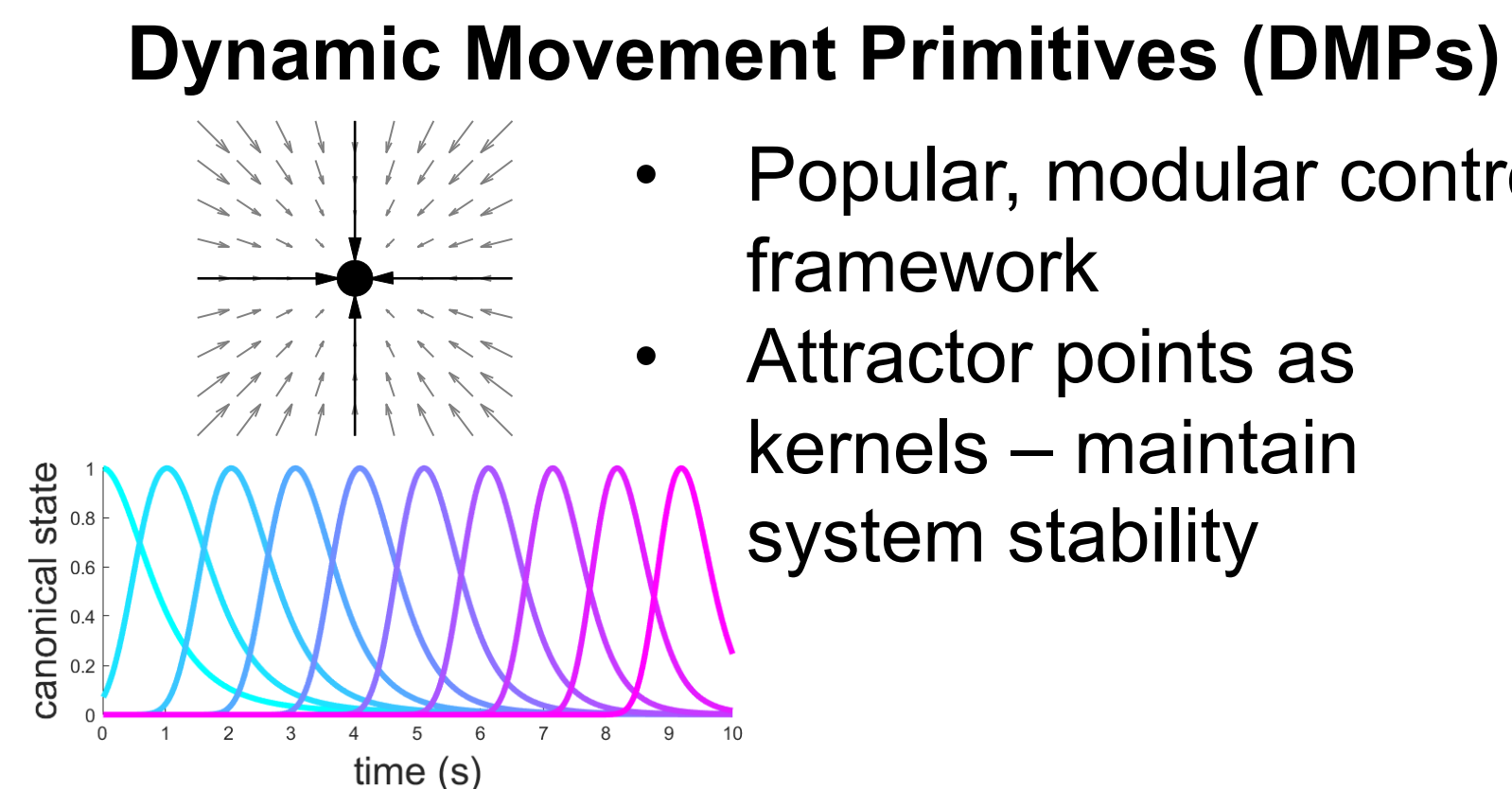


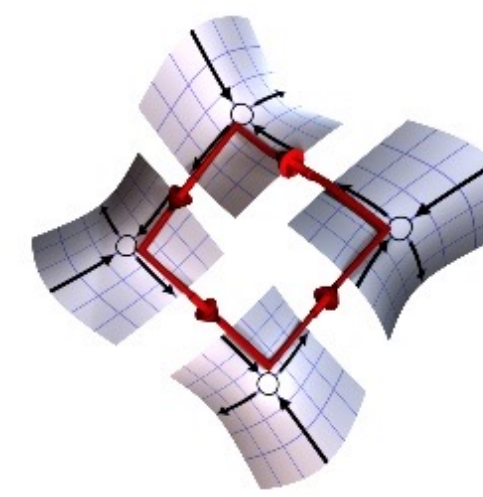
Stable Heteroclinic Channel-based Movement Primitives for Robotic Control

Movement primitives are a well-established, modular approach to robot motion planning. Dynamic movement primitives (DMPs) are a popular control framework based on nonlinear differential equations [1]. Stable heteroclinic channels (SHCs) are trajectories that connect saddle equilibria in phase space [2]. When DMP attractors are replaced with SHC saddles, stable heteroclinic channel-based movement primitives (SMPs) are formed [3].



Background

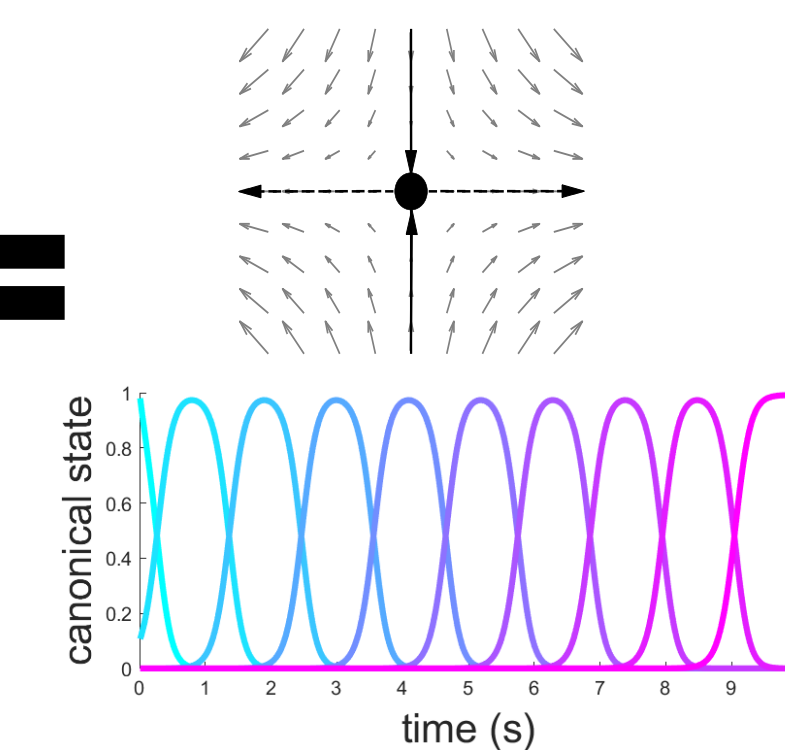
Stable Heteroclinic Channels (SHCs)



- Neural activation model
- Saddle points connected in phase space

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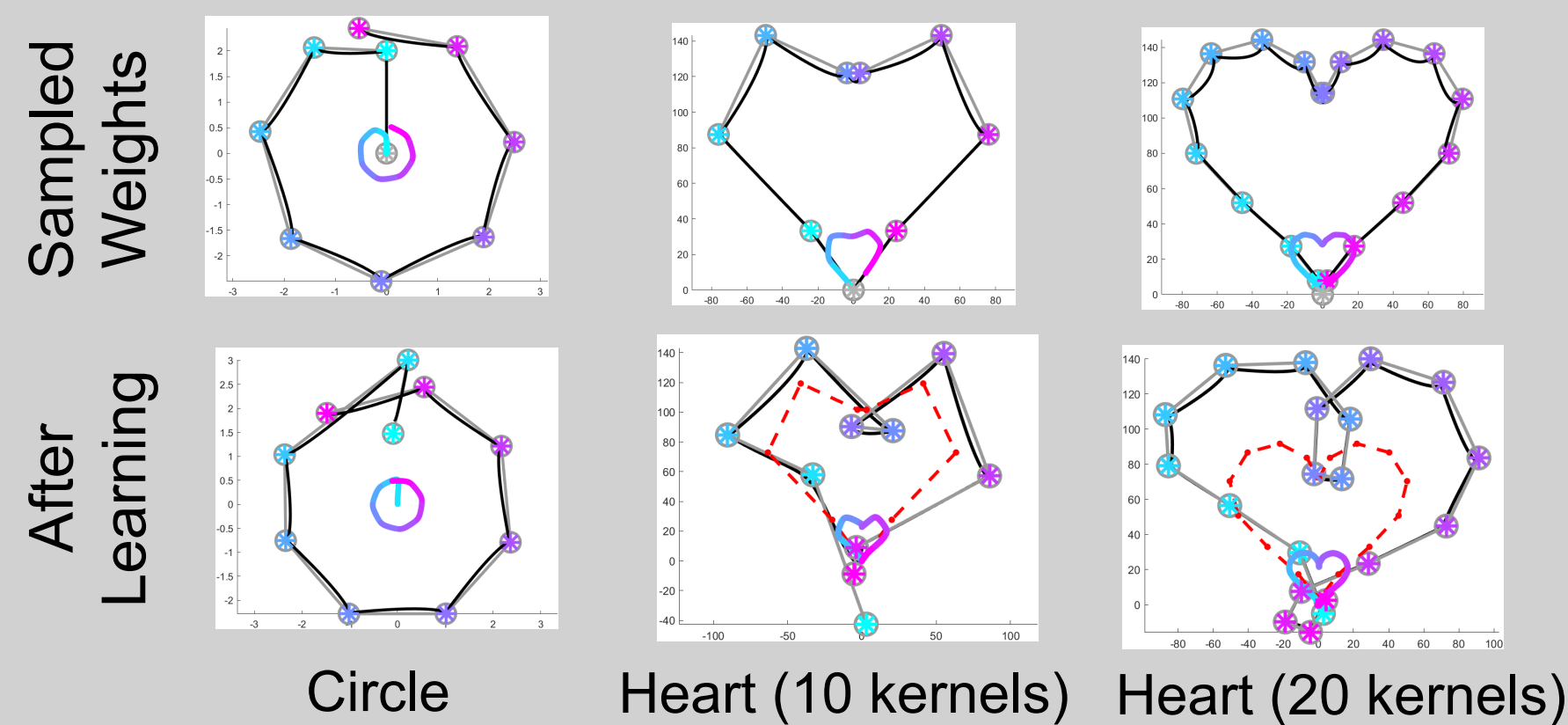
Stable Heteroclinic Channel-based Movement Primitives (SMPs)



- Maintains system stability
- Time independent
- Visualization feature: user-friendly system initialization
- Characterizable system parameters

SMP Features

Visualization



- In the task space, the SMP forcing function follows the weighted kernel locations
- Kernel weights can be initialized spatially – even sampled from the trajectory itself.

Legend:
 ○ Learned weights
 — Forcing function
 — Weights sampled from forcing function

System Parameters

Horchler *et al* [4] characterize the SHC system parameters α , β , ν as:

- $\alpha \rightarrow$ growth rate of the kernel (how fast the intrinsic excitation grows the kernel dimension)
- $\beta \rightarrow$ magnitude (maximum amplitude of the waveform)
- $\nu \rightarrow$ saddle value (insensitivity to noise)

SMP Structure

$$\tau \ddot{y} = \alpha_y (\beta_y (g - y) - \dot{y}) + f$$

$$f(x_i) = \sum_{i=1}^K x_i w_i$$

$$\tau dx_i = x_i \left(\alpha_i - \sum_{j=1}^K \rho_{ij} x_j \right) dt + \sum_{j=1}^N C_{ij} z_j$$

$$\rho_{ij} = \begin{cases} \alpha_i / \beta_i, & \text{if } i = j \\ \frac{\alpha_i - \alpha_j / \nu_j}{\beta_j}, & \text{if } i = j \oplus 1 \\ \frac{\alpha_i + \alpha_j}{\beta_j}, & \text{otherwise} \end{cases}$$

τ – time-scaling term
 x – canonical state of system
 y – relevant system variable (e.g. end-effector position)
 α_y, β_y – system damping & stiffness
 σ_i – factor for width of i^{th} kernel
 c_i – center of i^{th} kernel
 N – number of sensors
 f – external force applied to system by controller
 K – total number of underlying kernel functions
 $\alpha_i, \beta_i, \nu_i, \rho_{ij}$ – system parameters
 C_{ij} – coupling matrix
 z_j – noise
 w_i – weight of i^{th} kernel

References

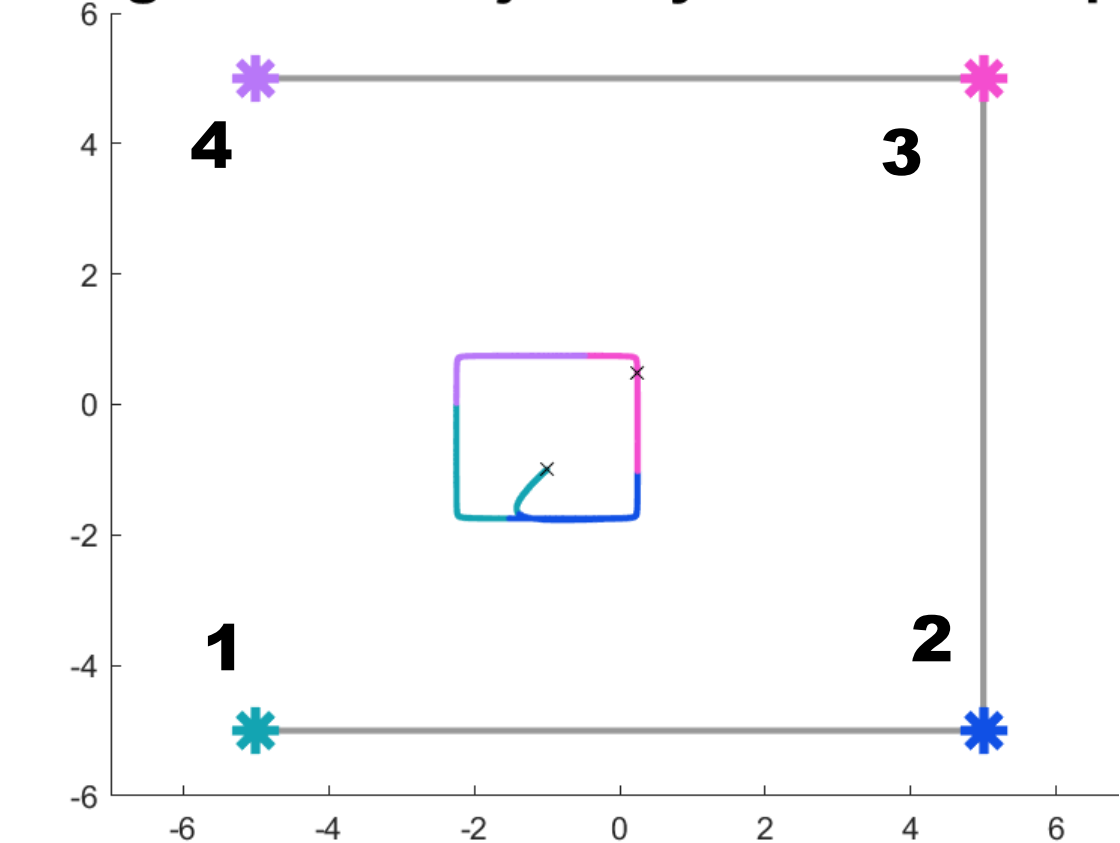
[1] A. J. Ijspeert, J. Nakanishi, H. Hoffmann, P. Pastor, and S. Schaal. "Dynamical Movement Primitives: Learning Attractor Models for Motor Behaviors," *Neural Comput.*, vol. 25, pp. 328–373, 2013.
 [2] M. Rabinovich, R. Huerta, and G. Laurent, "Neuroscience: Transient dynamics for neural processing," pp. 48–50, Jul 2008. doi: 10.1126/science.1155564
 [3] N. A. Rouse and K. A. Daltorio, "Visualization of Stable Heteroclinic Channel-Based Movement Primitives," in *IEEE Robotics and Automation Letters*, vol. 6, no. 2, pp. 2343–2348, April 2021, doi: 10.1109/LRA.2021.3061382.
 [4] Horchler, A. D., Daltorio, K. A., Chiel, H. J., & Quinn, R. D. (2015). Designing responsive pattern generators: stable heteroclinic channel cycles for modeling and control. *Bioinspiration & Biomimetics*, 10(2), 026001. <https://doi.org/10.1088/1748-3190/10/2/026001>

Acknowledgements

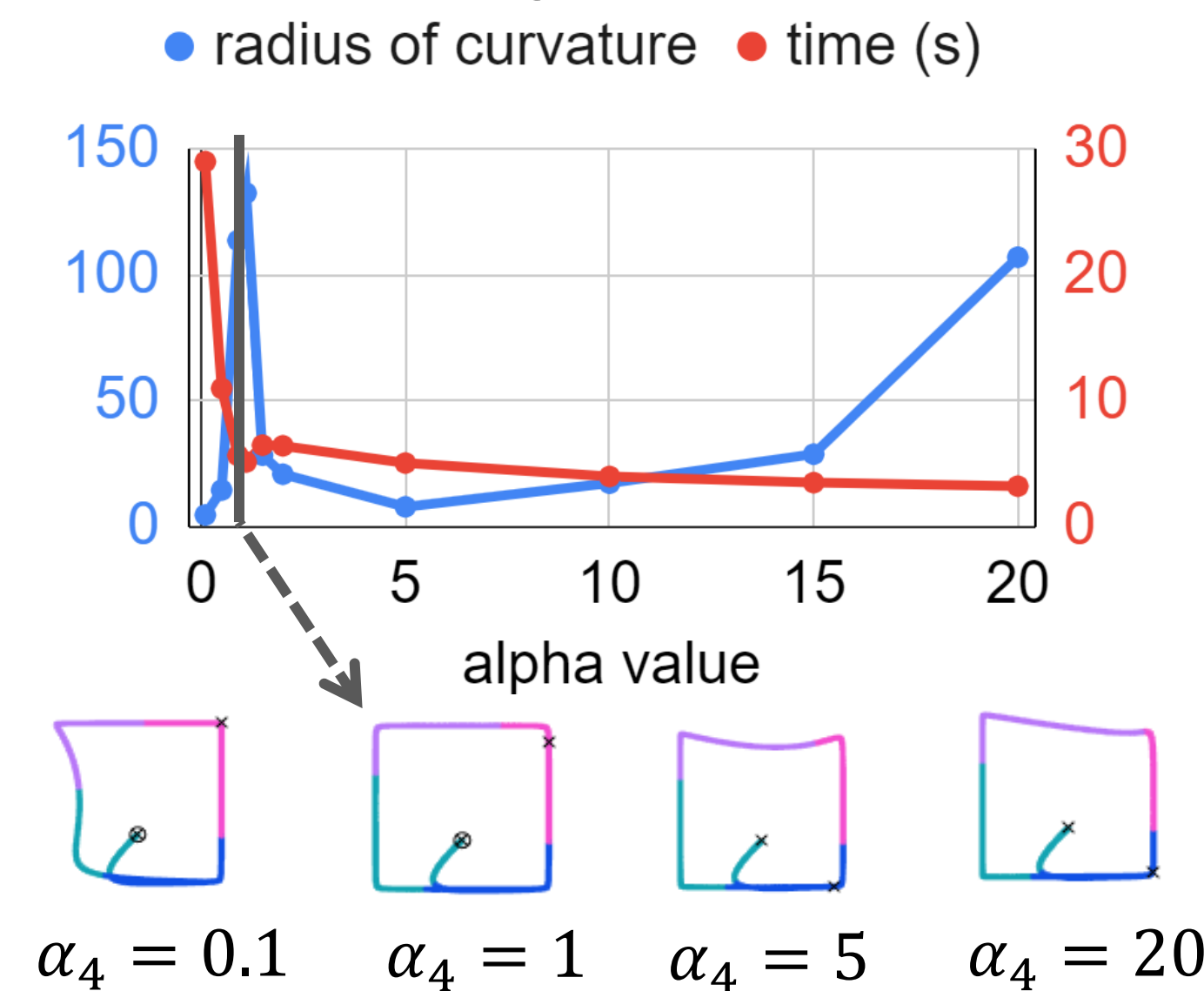
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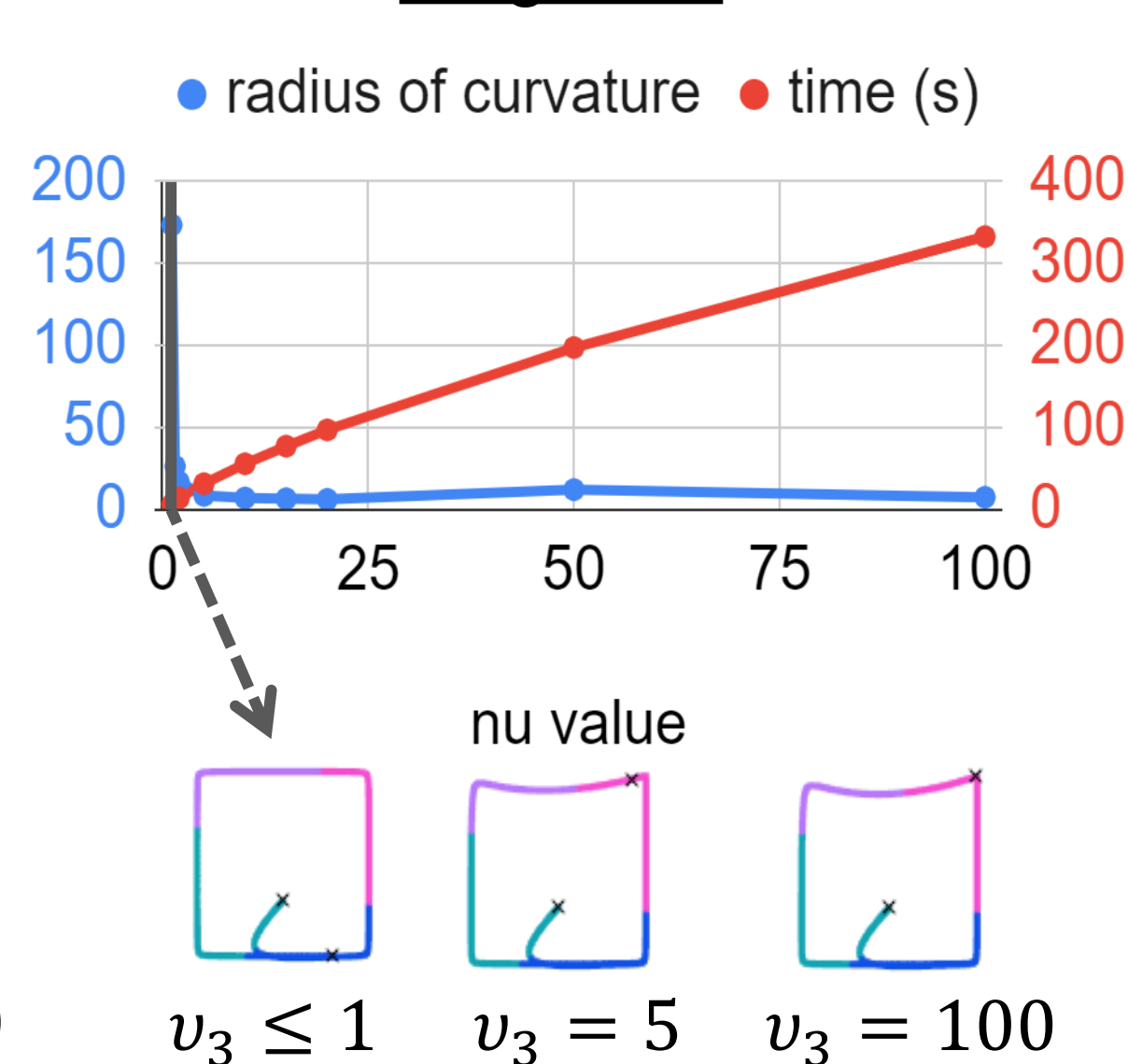
Weights and Trajectory in the Taskspace



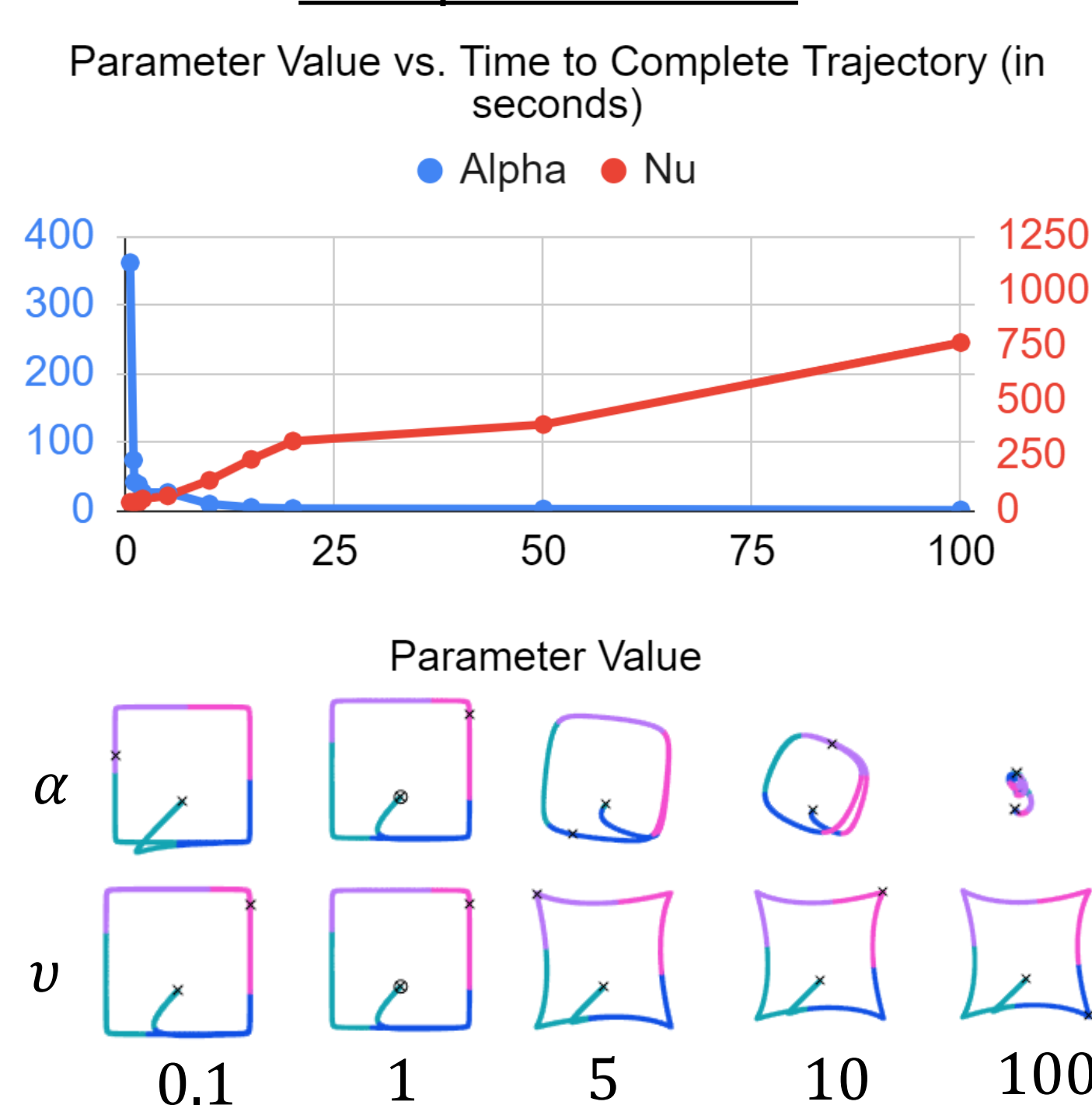
Single Alpha



Single Nu



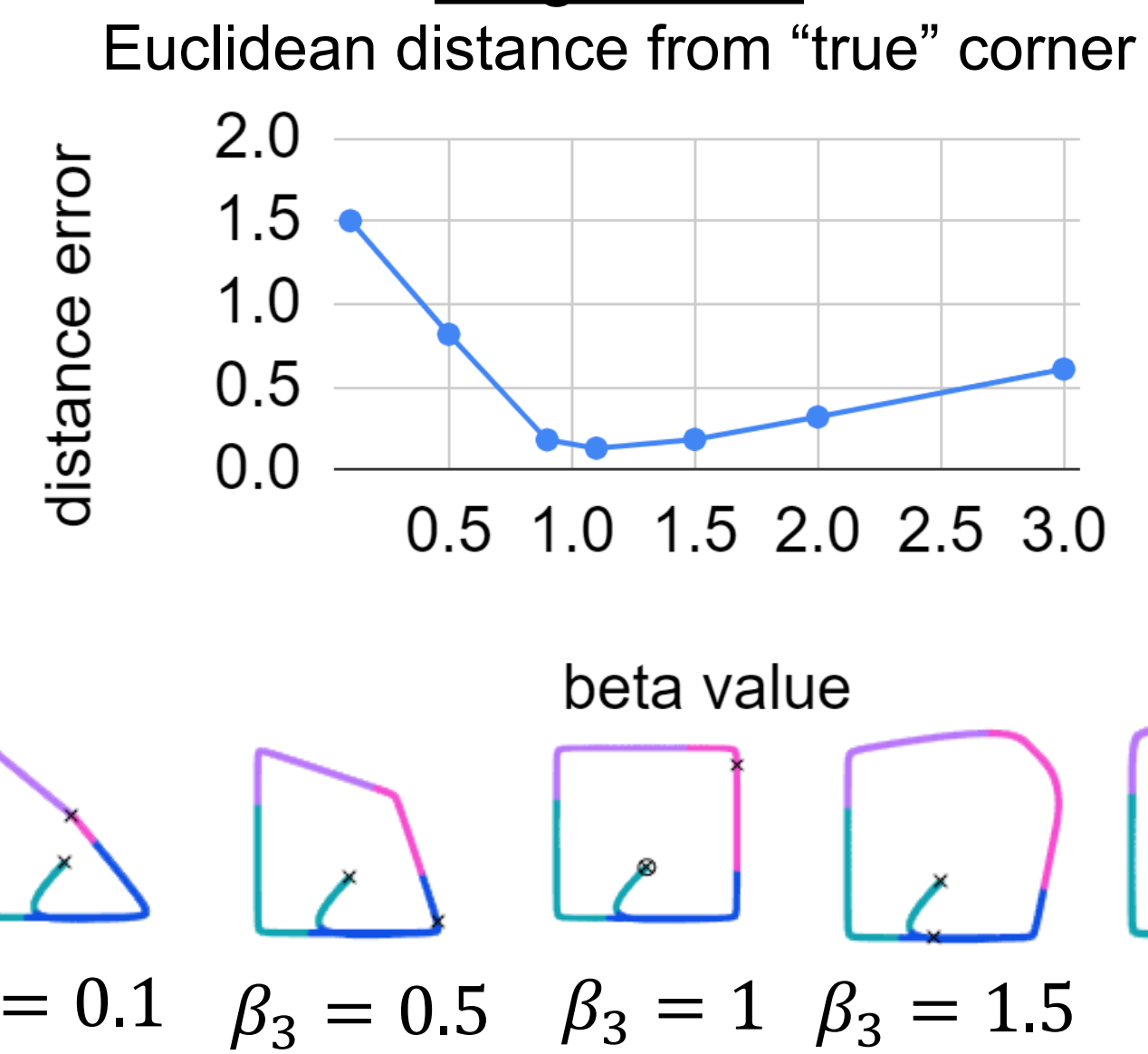
All Alpha vs All Nu



Conclusions

- α affects trajectories before it ($\alpha > 1$) and after it ($\alpha < 1$). Increasing all α rotates and shrinks the trajectory. Increasing α decreases time spent around that kernel.
- β affects the kernel-specific trajectory. Scaling ($\beta < 1$) and rotation ($\beta > 1$) occur when varying all β . Increasing β increases the time spent around the entire trajectory.
- ν_{SMP} affects the trajectory after it ($\nu > 1$); no change is observed when $\nu < 1$. Increasing all ν , increases the time around the trajectory, especially in the kernel vicinities.

Single Beta



All Beta

