Symmetries in Nonlinear Cyber-Physical Systems for Compositionality

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Introduction and Motivation

- Overall goal: results for system integration through compositionality.
- Compositionality: system-level properties can be computed from local properties of components
  - System-level properties *preserved* when expanding system.
- Initial focus: invariant properties for symmetric systems.
- Emphasis on general results, not limited to specific system dynamics.
- Initial results: symmetric systems and stability.
- These results are Lyapunov-based, so the natural extension is to passivity.
- Also working toward use of *approximate* symmetries.
- [9, 12, 10, 11, 14, 13, 4, 19]

Outline of Presentation

- Nonlinear Symmetric Systems and Compositionality
- Nonlinear Compositional Stability Results (V-based)
  - stability for “growing” systems
  - robustness for “shrinking” and failing systems
- Optimal Control for Nonlinear Symmetric Systems
- Mechanical Cyber-Physical Systems
- Ongoing and Future work

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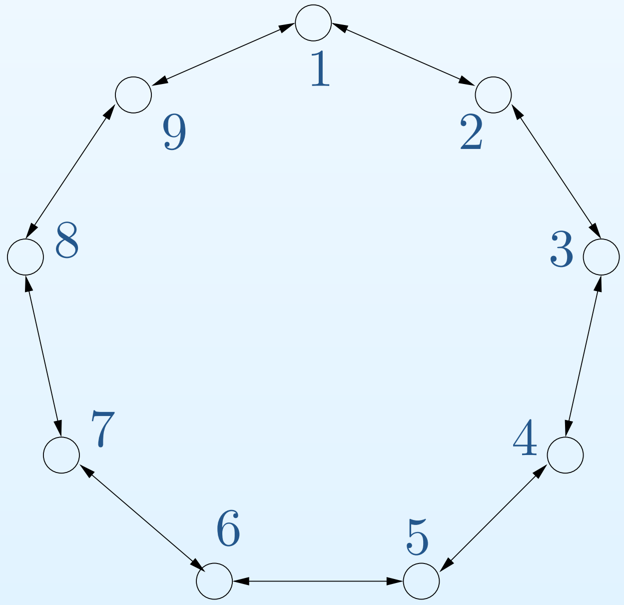

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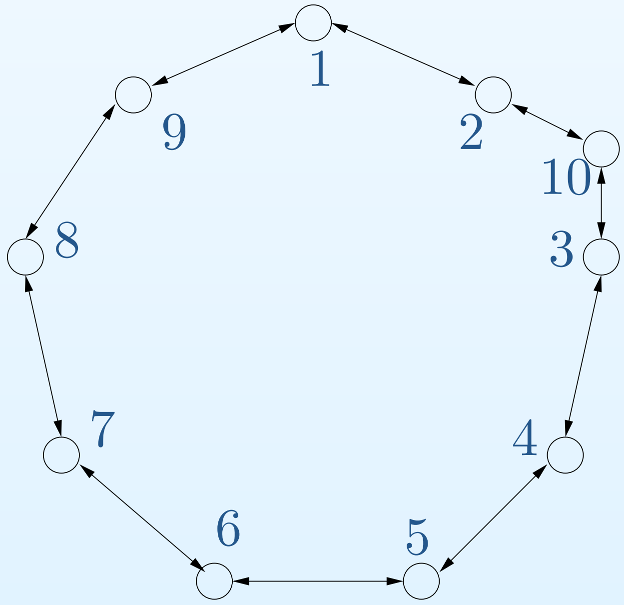

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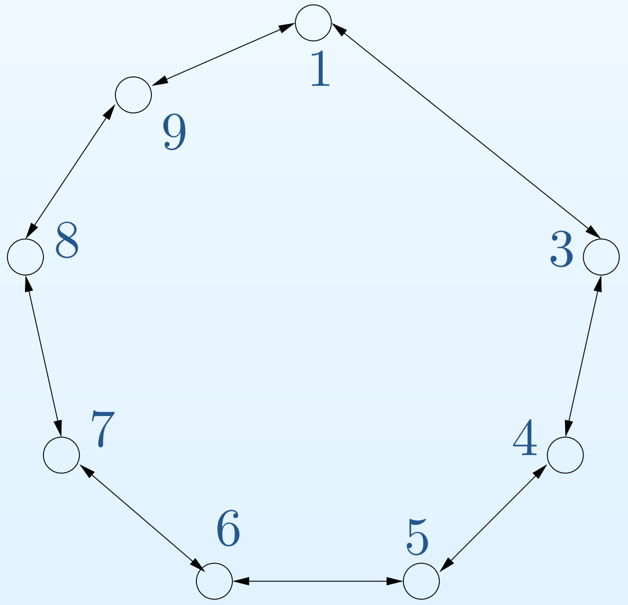
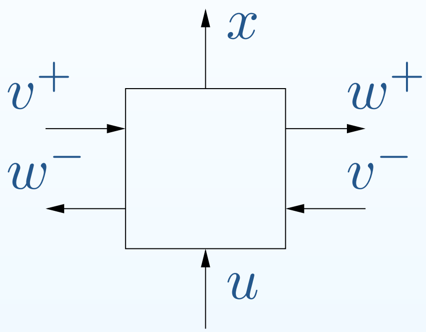


Symmetric Systems

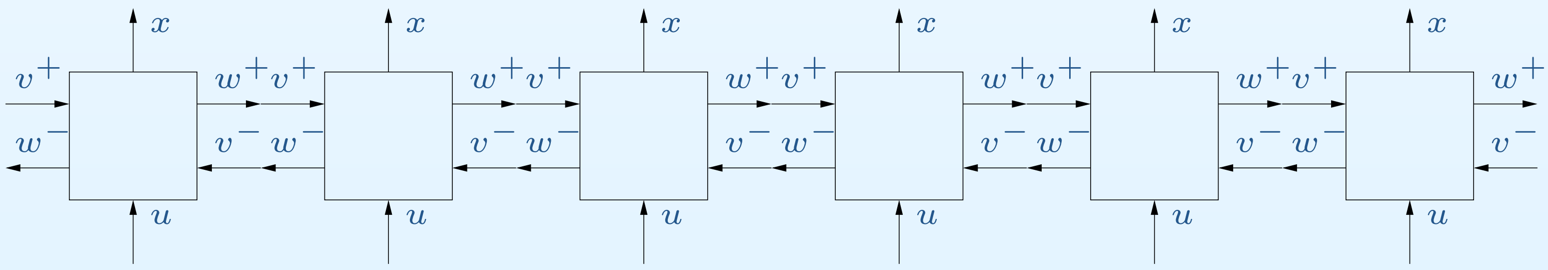
Consider a *basic building block element*:



where

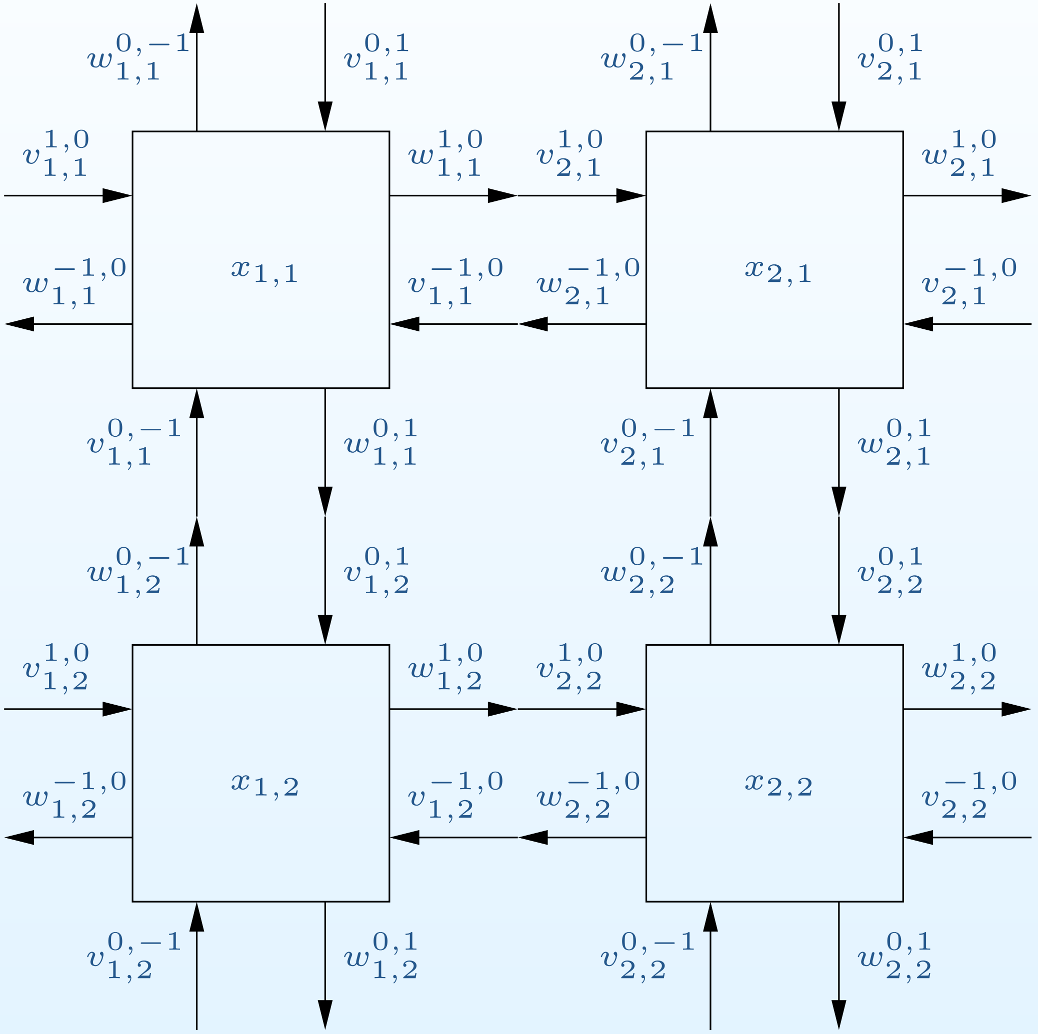
- x is the state vector;
- u is the vector of control inputs;
- $w^{\pm}$ are the *outputs*; and,
- $v^{\pm}$ are the coupling inputs.

Connecting the inputs to the outputs gives



Periodic Interconnections

Systems can be built with various topologies:



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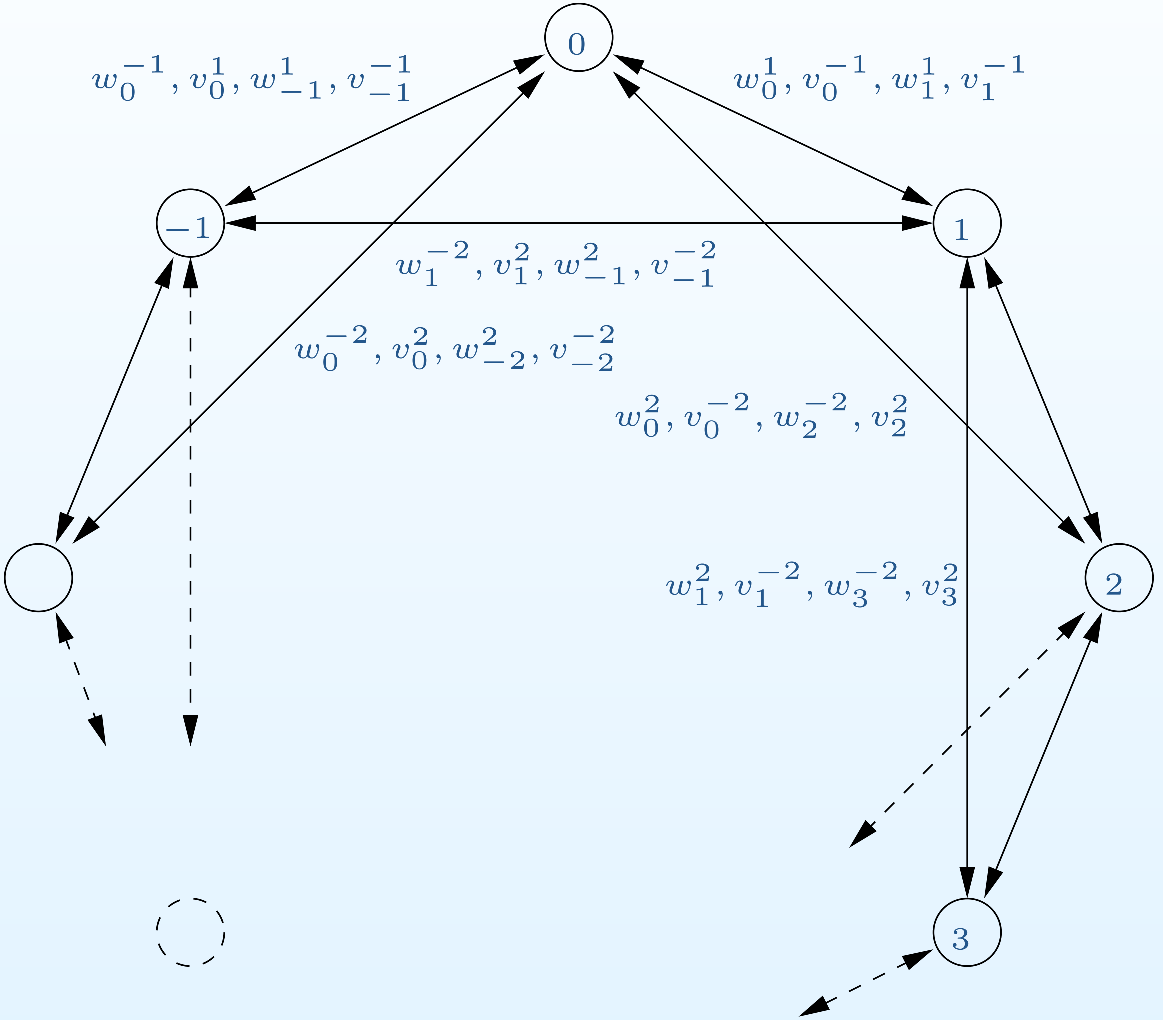
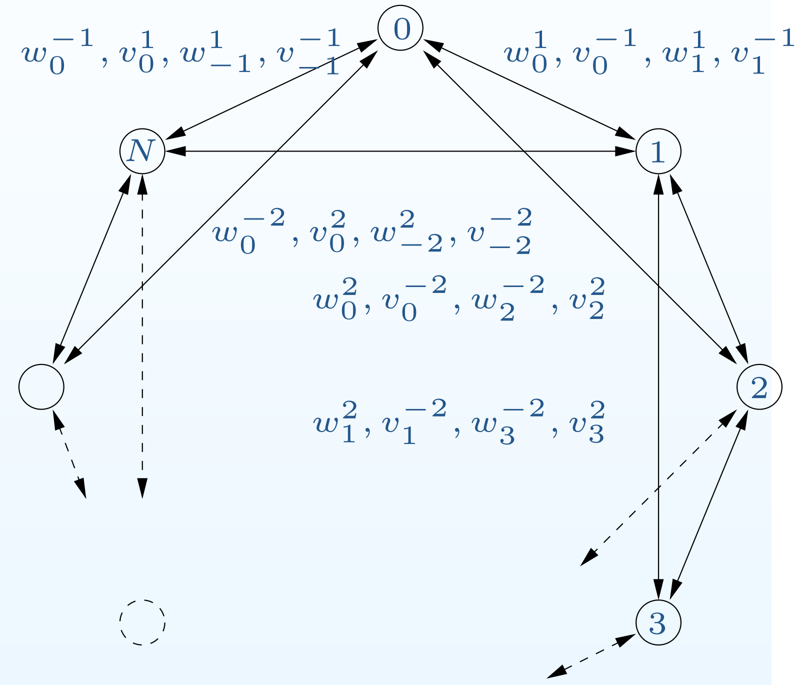


Group and Graph Theory Representation

- Represent connectedness with *generators*, $\{s_1, \ldots, s_n\}$ with $g_2 = s_i g_1$.
- *Cayley Graph*:
  - nodes = components
  - edges = communication
- *Equivalent connections* for two systems if they have the same generators.
- Example: $S = \{-2, -1, 1, 2\}$.
- Each component:

$$\dot{x}_i = f_i(x) + g_i(x)u$$
$$w_i^s(t) = w_i^s(x_i(t)).$$



- Periodic interconnections:

$$v_g^s(t) = w_{s^{-1}g}^s\left(x_{s^{-1}g}(t)\right)$$
$$w_g^s(t) = v_{sg}^s(x_g(t))$$

Symmetric and Equivalent Systems

A *symmetric system* has components with identical dynamics:
$f_{g_1}(x) = f_{g_2}(x)$, $g_{g_1,j}(x) = g_{g_2,j}(x)$, $w^s_{s^{-1}g_1}(x) = w^s_{s^{-1}g_2}(x)$
and identical control laws

$$u_{g_1,j}\left(x_1(t), w^{s_1}_{s_1^{-1}g_1}(x_2(t)), \ldots, w^{s_{|X|}}_{s_{|X|}^{-1}g_1}(x_{|X|+1}(t))\right) = u_{g_2,j}\left(x_1(t), w^{s_1}_{s_1^{-1}g_2}(x_2(t)), \ldots, w^{s_{|X|}}_{s_{|X|}^{-1}g_2}(x_{|X|+1}(t))\right)$$

for all $g_1 \in G_1$, $g_2 \in G_2$, $s \in X$, $x \in \mathbb{R}^n$,
$(x_1, x_2, \ldots, x_{|X|+1}) \in \mathbb{R}^n \times \mathbb{R}^n \times \cdots \times \mathbb{R}^n$ and $j \in \{1, \ldots, m\}$
where $m = m_{g_1} = m_{g_2}$.

Two systems are *equivalent* if they are both symmetric with equal components and the *same generators*.

Main Results

- **Stability:** if a symmetric distributed system is stable, so is any equivalent symmetric system, so can "grow" or "shrink."
  - Growing: useful for analysis/design on a small system with guaranteed invariance for larger equivalent systems
  - Shrinking: reconfigurable robustness
- **Robustness:** stability in the sense of Lyapunov is guaranteed even when components fail without any reconfiguration necessary.
- Requires a symmetric Lyapunov function: $V = \sum_{i \in G} V_i$ where

$$V_{g_1}\left(x_1, w^{s_1}_{s_1^{-1} g_1}(x_2), \ldots, w^{s_{|X|}}_{s_{|X|}^{-1} g_1}(x_{|X|+1})\right) = V_{g_2}\left(x_1, w^{s_1}_{s_1^{-1} g_2}(x_2), \ldots, w^{s_{|X|}}_{s_{|X|}^{-1} g_2}(x_{|X|+1})\right)$$

for all $g_1, g_2 \in G$ and $\left(x_1, x_2, \ldots, x_{|X|+1}\right) \in \mathbb{R}^n \times \cdots \times \mathbb{R}^n$.

Formation Control Examples

- From [18] second-order mechanical system agents:

$$\frac{d}{dt}\begin{bmatrix} x_i \\ \dot{x}_i \\ y_i \\ \dot{y}_i \end{bmatrix} = \begin{bmatrix} \dot{x}_i \\ 0 \\ \dot{y}_i \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} u_{i,1} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_{i,2}. \qquad (1)$$

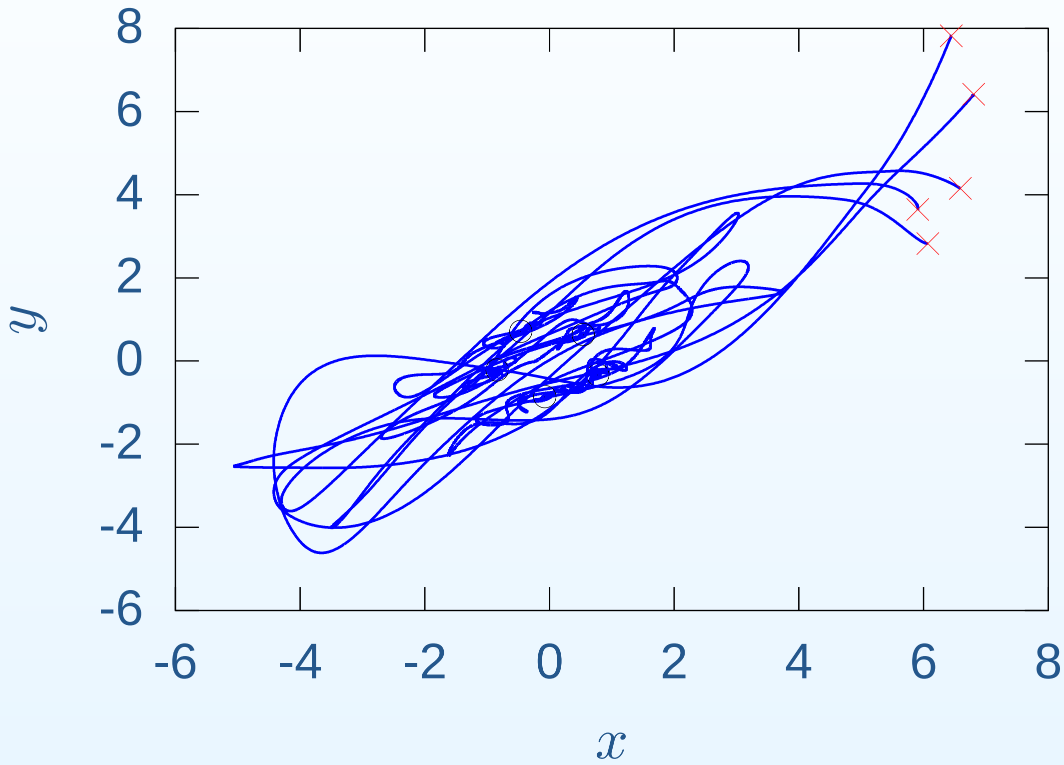
- Goal= regular $(N+1)$-polygon centered at the origin, hence

$$d_{ij} = \begin{cases} 1, & |i-j| = 1 \\ \frac{\sin\left(\frac{2\pi}{N+1}\right)}{\sin\left(\frac{\pi}{N+1}\right)}, & |i-j| = 2 \end{cases} \quad \text{and} \quad r_i = \frac{1}{2\sin\frac{\pi}{N}}.$$

- Take the control law to be $u =$

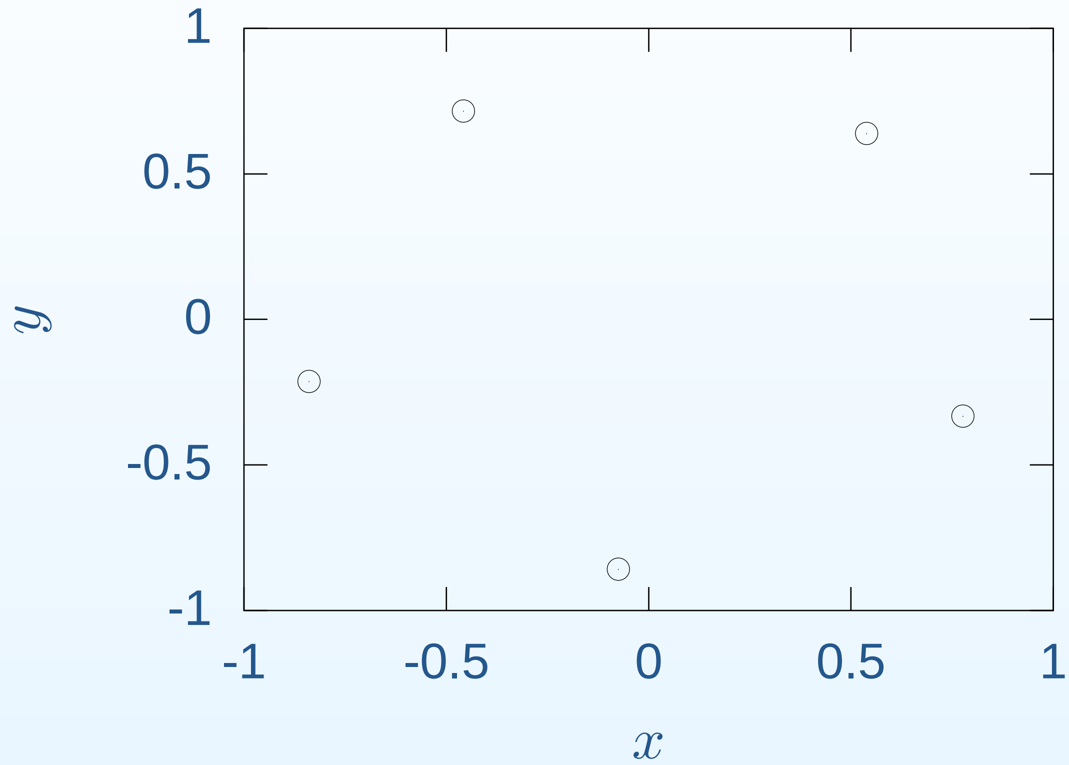
$$-\sum_j \begin{bmatrix} \frac{\left(\sqrt{(x_i-x_j)^2+(y_i-y_j)^2}-d_{ij}\right)}{\sqrt{(x_i-x_j)^2+(y_i-y_j)^2}}(x_i - x_j) \\ \frac{\left(\sqrt{(x_i-x_j)^2+(y_i-y_j)^2}-d_{ij}\right)}{\sqrt{(x_i-x_j)^2+(y_i-y_j)^2}}(y_i - y_j) \end{bmatrix} - k_d \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} - \begin{bmatrix} \frac{\sqrt{x_i^2+y_i^2}-r_i}{\sqrt{x_i^2+y_i^2}} x_i \\ \frac{\sqrt{x_i^2+y_i^2}-r_i}{\sqrt{x_i^2+y_i^2}} y_i \end{bmatrix}.$$

Formation Control Examples



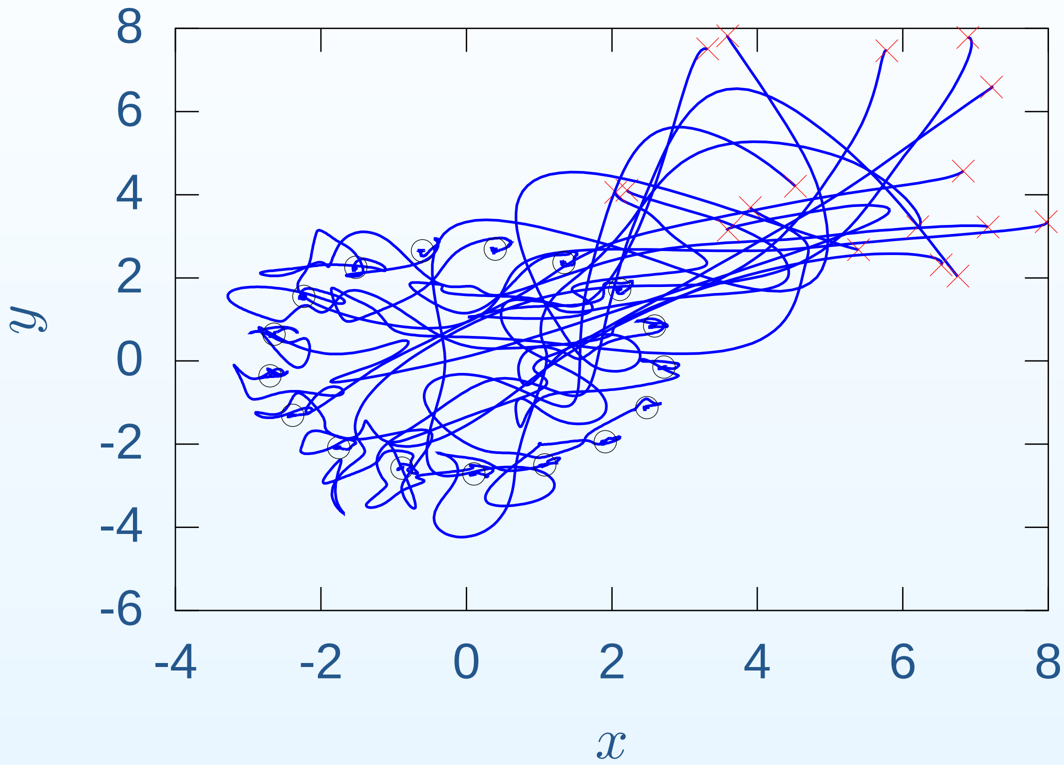
Five robot stable formation control.

Formation Control Examples



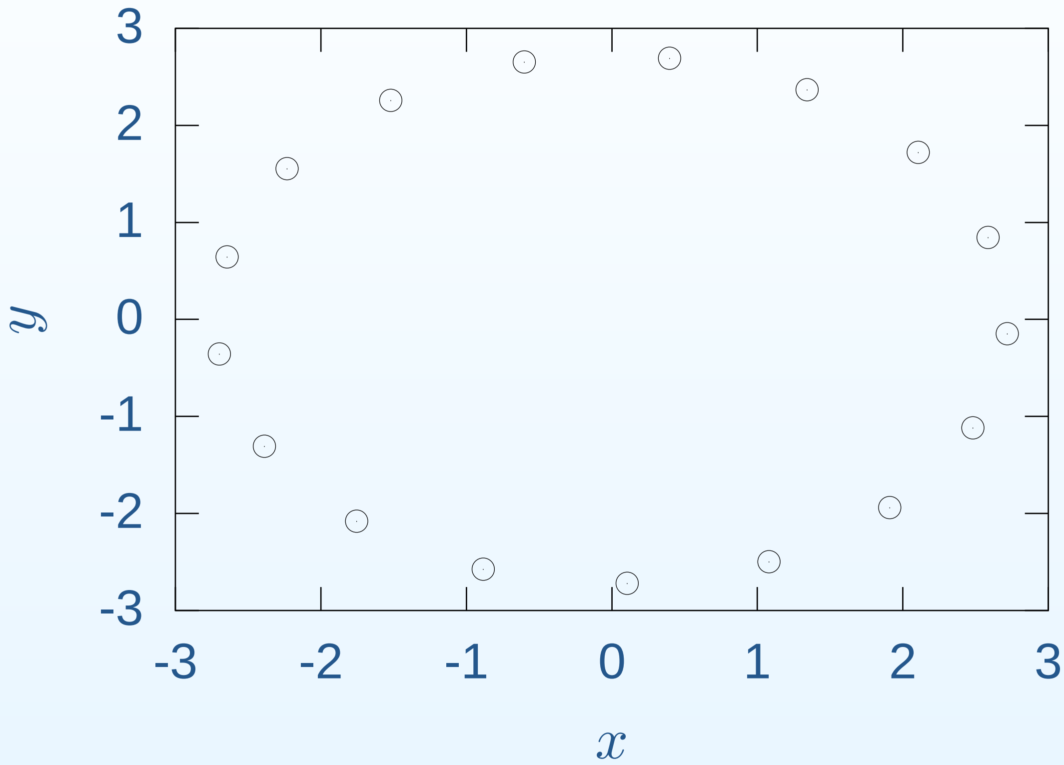
Initial and final configurations.

Formation Control Examples



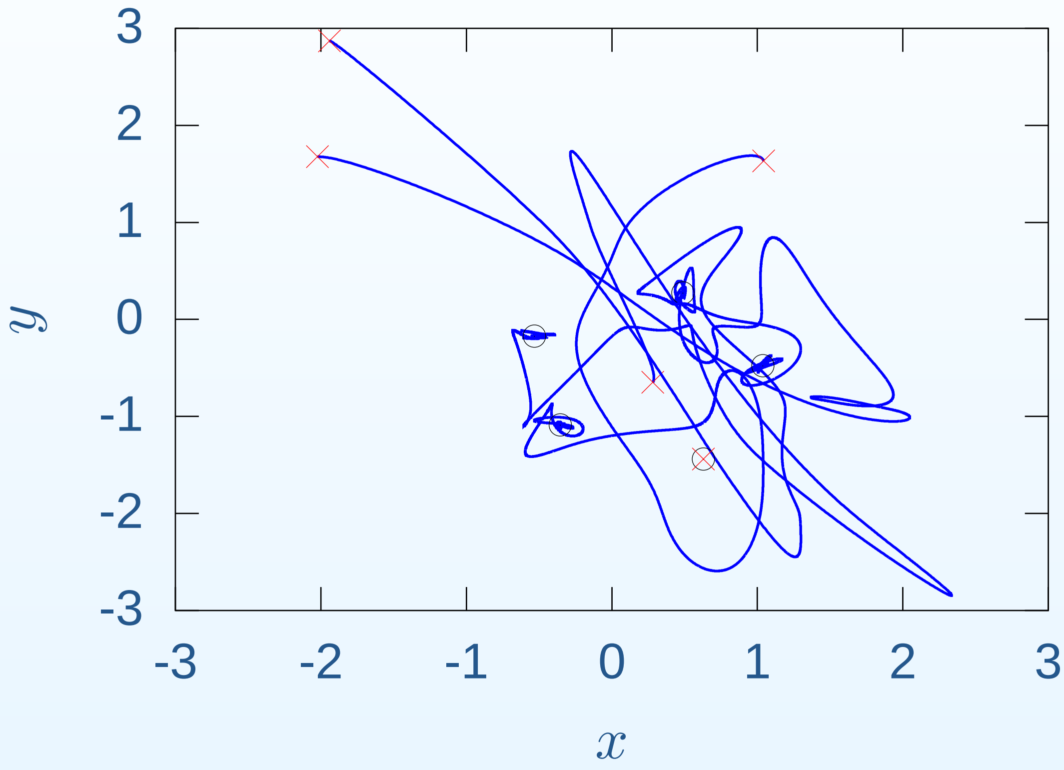
Seventeen-agent formation control.

Formation Control Examples



Starting and ending configurations.

Compositionality Robustness Examples



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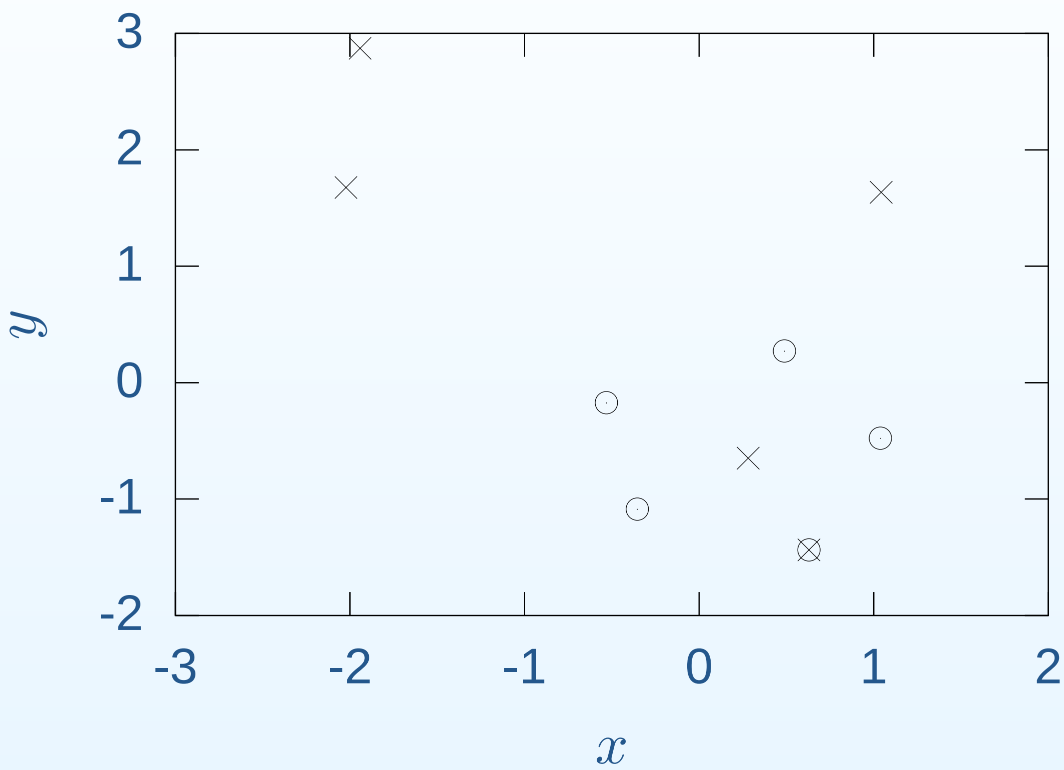
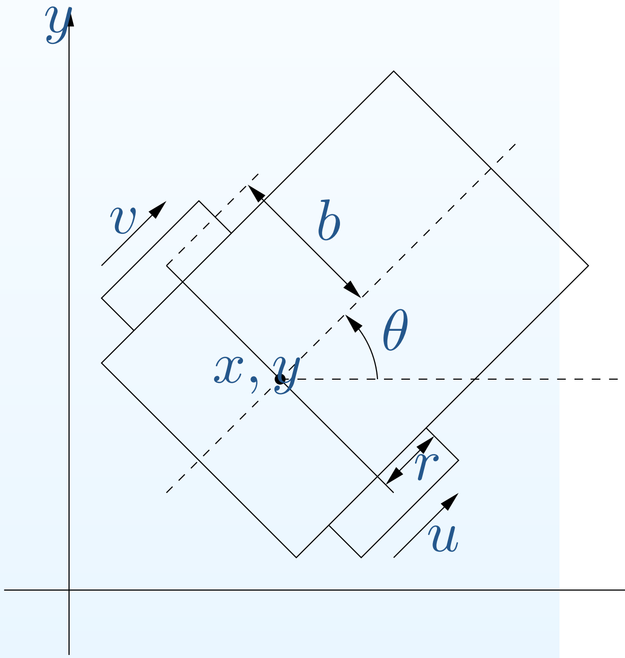


Optimal Control of Symmetric Systems

We are investigating the effect of approximate symmetries and symmetry-breaking by looking at optimal formation control.

Minimize control effort and deviation from desired formation



$$J = \int_0^{t_f} \Bigg[\frac{1}{2} \sum_{i=1}^{n} u_i^2 + v_i^2$$

$$+ \sum_{i=1}^{n} \Bigg[\lambda_{x_i} \left(\dot{x}_i - \frac{r}{2} \cos\theta_i (u_i + v_i) \right)$$

$$+ \lambda_{y_i} \left(\dot{y}_i - \frac{r}{2} \sin\theta_i (u_i + v_i) \right)$$

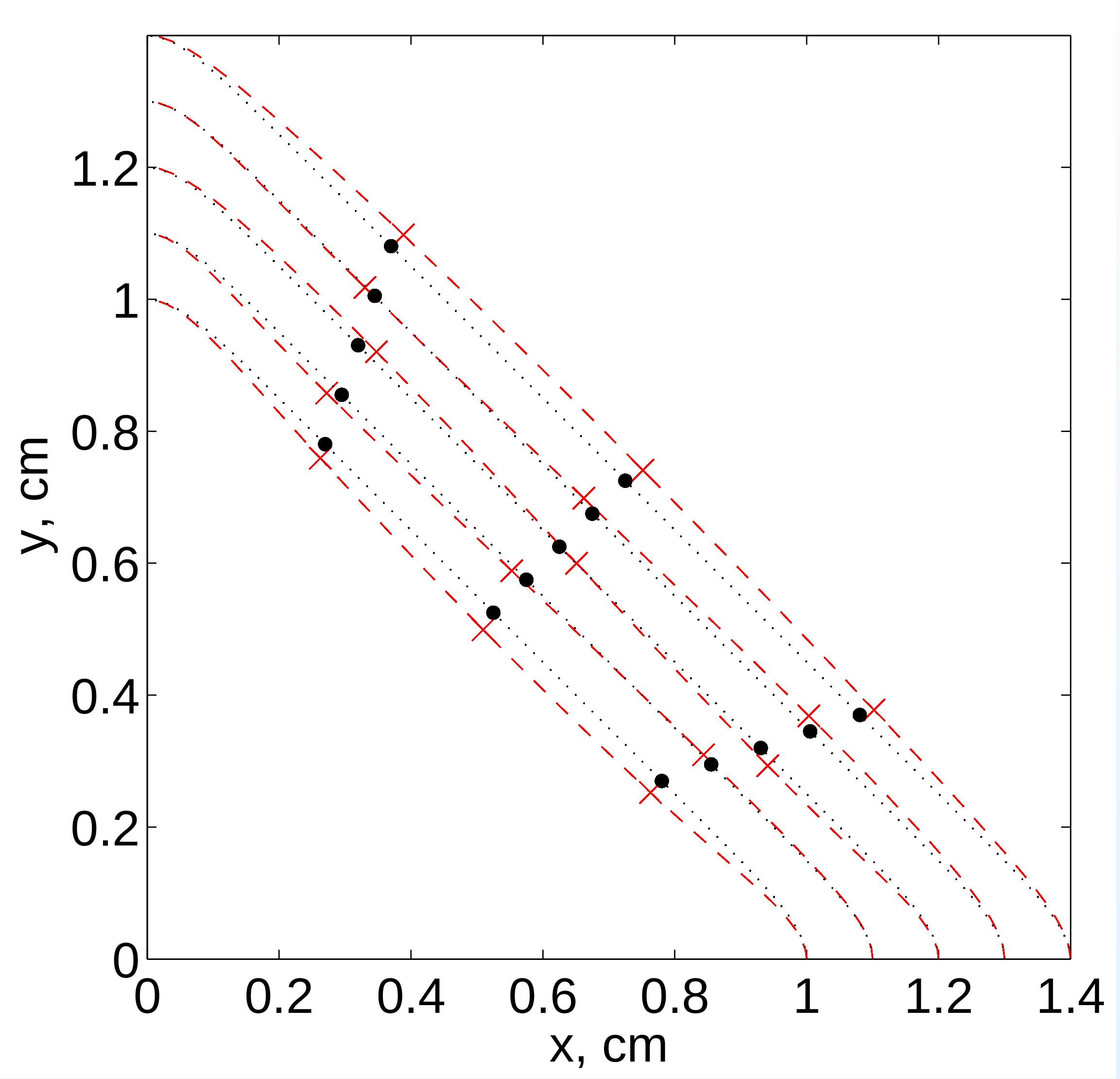
$$+ \lambda_{\theta_i} \left(\dot{\theta}_i - \frac{r}{2b} (u_i - v_i) \right) \Bigg]$$

$$+ k \sum_{i=1}^{n-1} (d_{i,i+1} - \tilde{d})^2 \Bigg] dt$$

Consider $k \in [0, +\infty)$.

Optimal Control of Symmetric Systems

Solutions:



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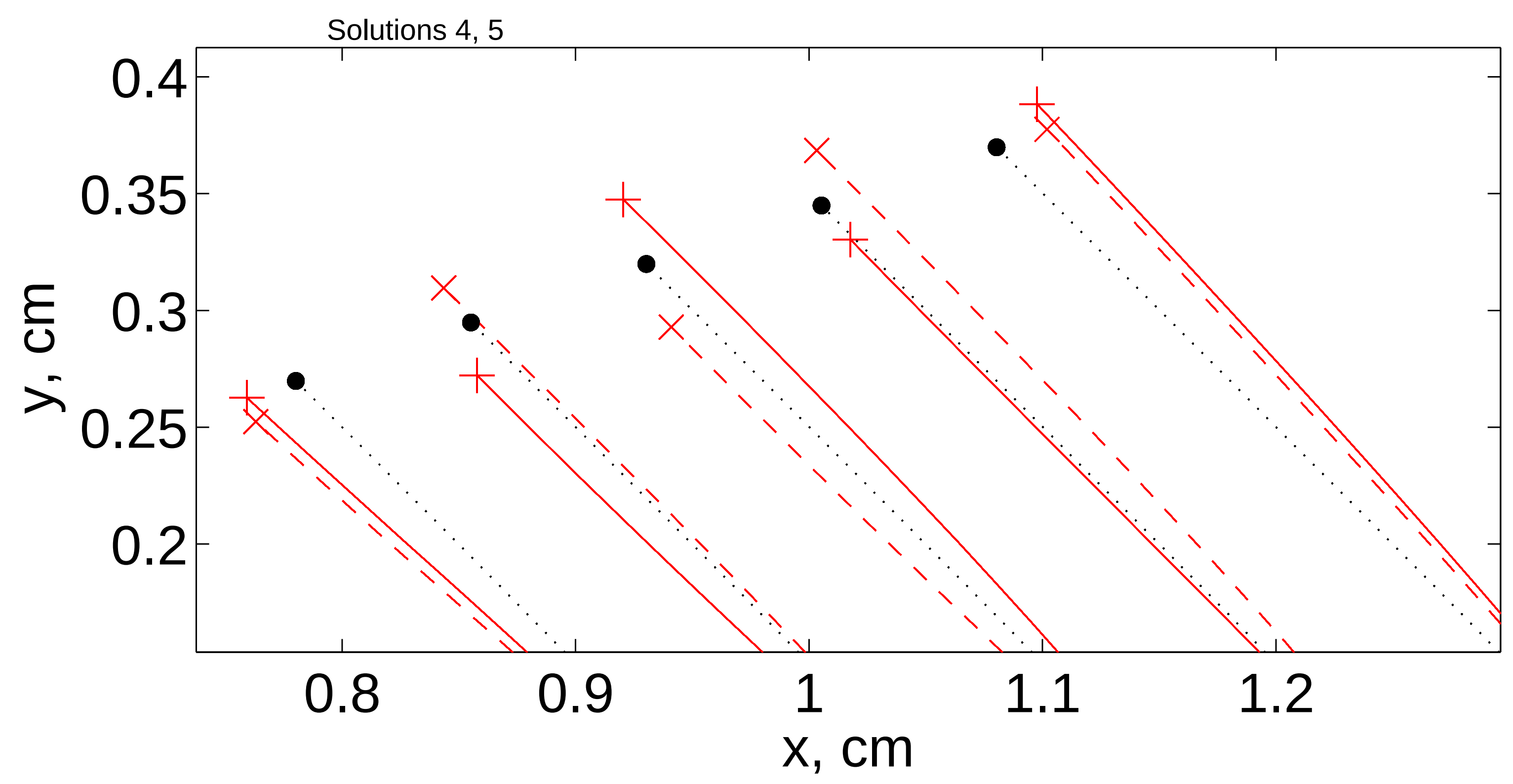
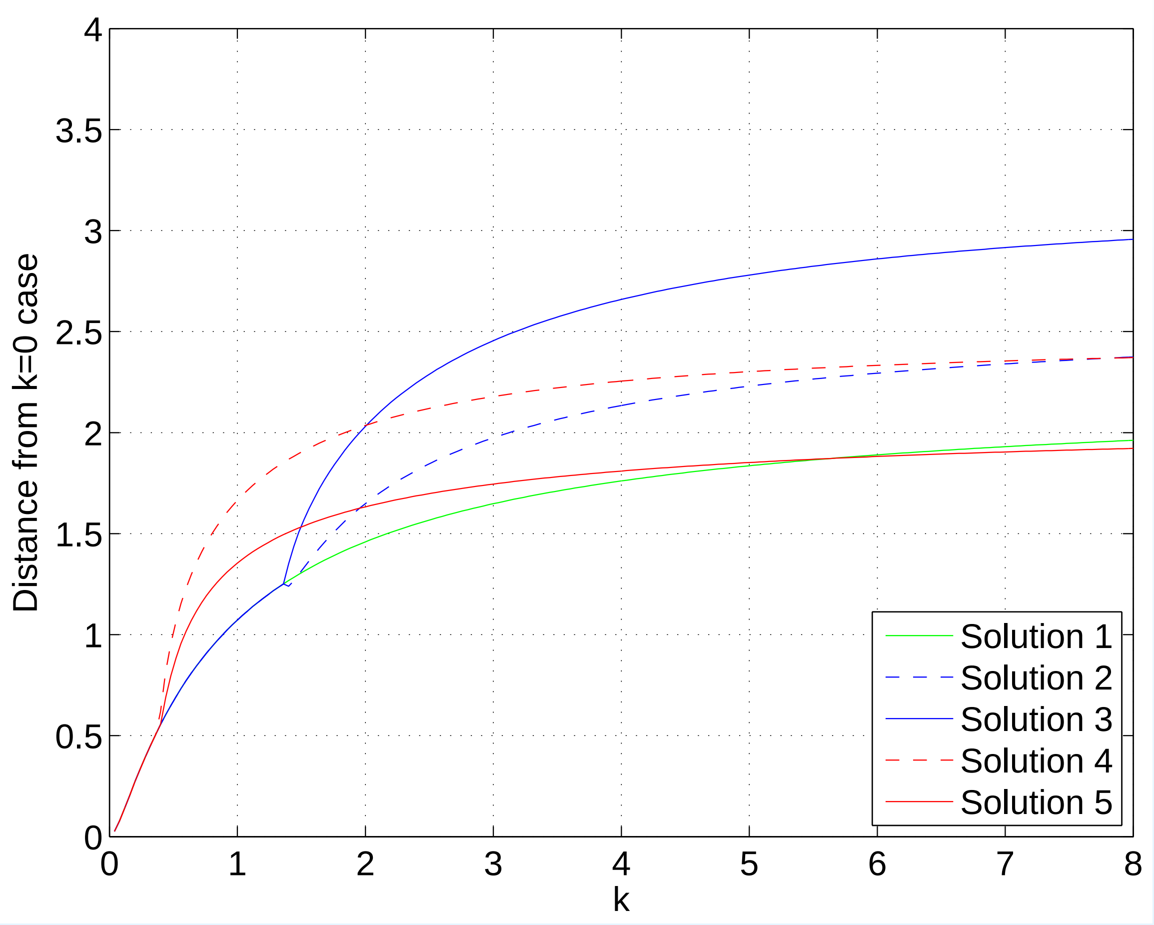


Optimal Control of Symmetric Systems

Bifurcating solutions:



Optimal Control of Symmetric Systems

Main Results:

- Existence of multiple solutions
  - Limiting cases: $k = 0$ and $k \to \infty$.
  - Characterization of bifurcations.
- For holonomic system: solutions *and* bifurcations must be symmetric [5, 6].
- For nonholonomic system: symmetry is broken by agent itself
  - Bifurcations have small deviations
  - On the order of the wheelbase vs. path length
  - Further investigation: to give insight into full range of approximate systems.

Control of *Mechanical* Systems

- Control + Mechanical = Cyber + Physical
- Instead of generic

$$\dot{x} = f(x) + g(x)u$$

  consider the case where the vector fields *came from* a first principle, i.e., Lagrange's equations.
- For $\Sigma = (M, \mathbb{G}, \{Y_1, \ldots, Y_m\}, U)$ the equations of motion are

$$\dot{x}^i = v^i$$
$$\dot{v}^i = -\Gamma^i_{jk} v^j v^k + u^a Y^i_a.$$

- Very difficult open questions concerning control away from equilibrium points. E.g., only sufficient conditions for controllability.
- Approach: decompose velocities into actuated and unactuated degrees of freedom and study the coupling between them.

Mechanical System Velocity Decomposition

- Decompose the velocity curve, $\dot{\gamma}(t)$ of Σ into

$$\dot{\gamma}(t) = w^a(t)X_a(\gamma(t)) + s^b X_b^{\perp}(\gamma(t)).$$

- From Lagrange's equations, it follows that

$$\dot{s}^b(t) = -\hat{\Gamma}^b_{ap} w^a w^p - 2\hat{\Gamma}^b_{ar} w^a s^r - \hat{\gamma}^b_{rk} s^r s^k.$$

- Direct control over the ws, therefore the vector-valued quadratic form $\hat{\Gamma}^b_{ap} w^a w^p$ plays a key role in control.
  - If it is indefinite, can both increase and decrease s, therefore can control all velocities.
  - If positive (negative) definite with other forms zero, can only increase (decrease) s (roller-racer).

Control of *Mechanical* Systems

Main results:

- Stopping (analysis at non-zero velocity) and stopping *algorithms*.
- Strongest algorithms for underactuated by one.
- Controllability computational simplicity.
- Focus is on *coupling dynamics* between actuated and unactuated degrees of freedom.
- Natural extension to system integration and coupling between components.
- References: [7, 15, 16, 17] and related work: [1, 3, 2, 20, 8, 21].

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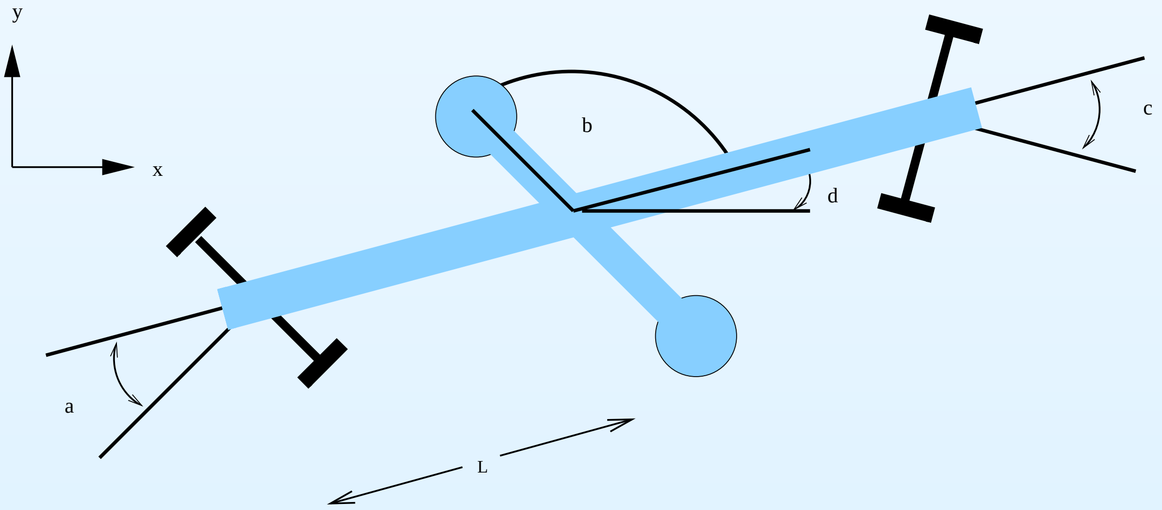

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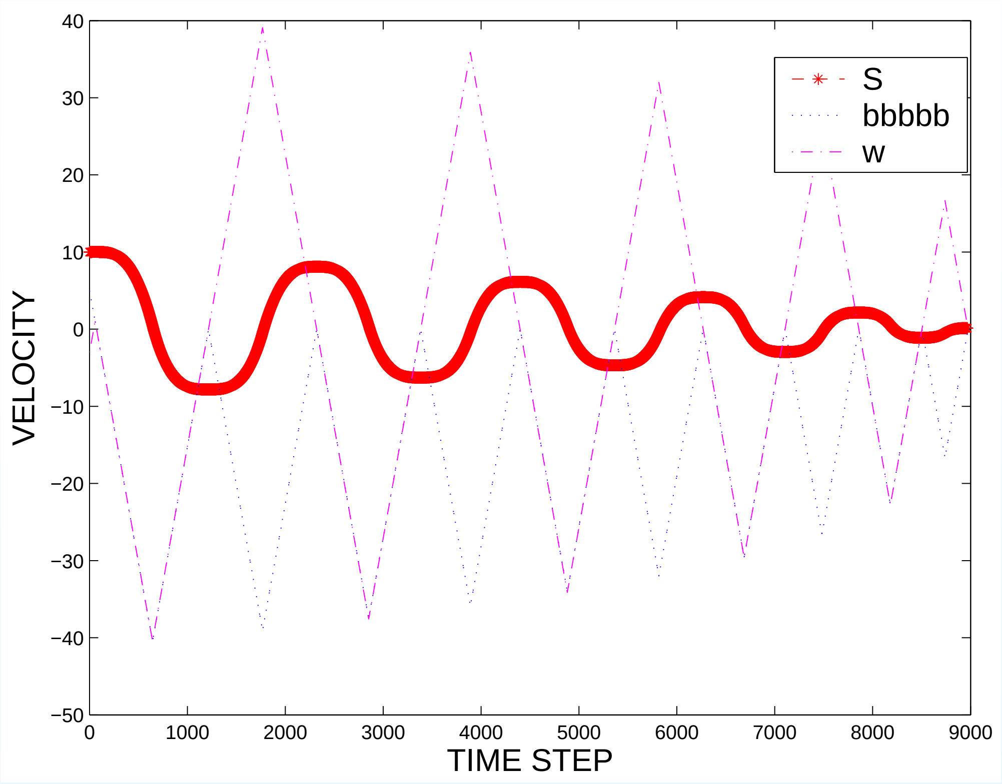

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