Computationally Aware Cyber-Physical Systems

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Overview

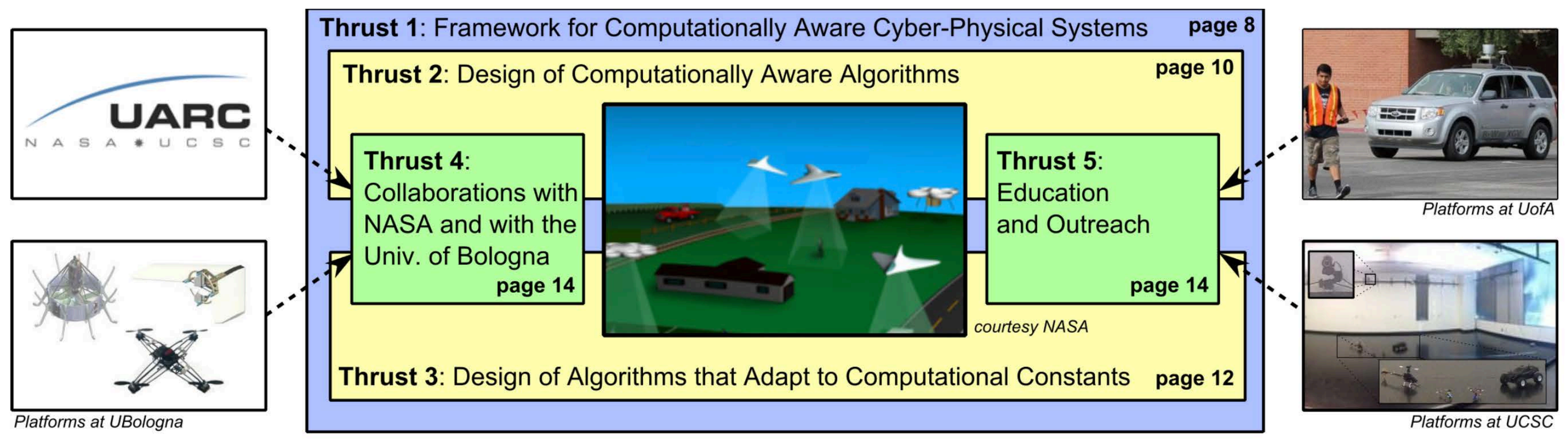
Project Goal
Generate tools for systematic analysis and design of computationally aware algorithms in cyber-physical systems.

Technical Objectives

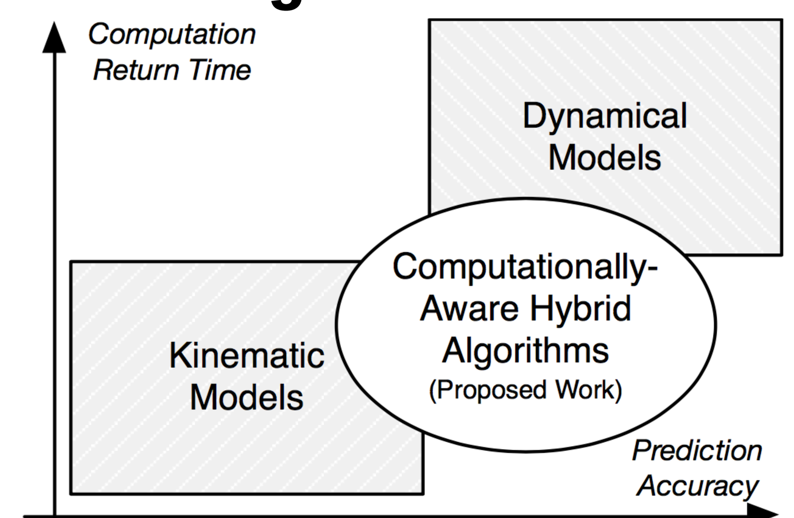
1. **Design tools** capable of accounting for **computational capabilities** in real-time
1. **Design hybrid feedback algorithms** that include more accurate prediction schemes **exploiting computational capabilities**, within the time constraints.

Design and Application of Cyber-physical systems

- Tight coupling between computation, communication, and control in the design and application of cyber-physical systems (CPSs)
- When system plants are complex, predictive strategies require the use of accurate models with higher computation times
- Timely decisions are required



Challenges



Only rarely the model of a CPS can be approximated such that the modeling error is negligible. We are faced with a complex tradeoff:

Should one select an accurate model for prediction, which will take longer to perform the optimization step?

Or should one choose a less accurate model for prediction, which will return an answer sooner, but one that is likely far from optimal?

Driving application — Safe and Robust Integration of UAS in the National Air Space

A key issue of scale in the NAS is the number of vehicles that can occupy a region. Approaches to Sense and Avoid (SAA) can improve the ability of individual aircraft to avoid violations of Minimum Separation Infringement (MSI) zone, and the Vertical Separation Minimum (VSM) standard. However, **what models are used to establish controllers for these zones, and an ability to predict the potential flight paths of other vehicles when navigating?**

Technical Approach

Thrust 1: Mathematical Framework for computationally aware CPS

Approach:
Employ hybrid dynamical models to capture the behavior of cyber-physical systems and their components

Physical Component of the CPS

$$\dot{z} \in F_P(z,u), \ \ y = h(z,u)$$
$$(z,u) \in C_P \subset \mathcal{Z} \times \mathcal{U}$$

where

- $z \in \mathbb{R}^{n_P}$ is the state variable
- $u \in \mathbb{R}^{m_P}$ is the input
- $y \in \mathbb{R}^{r_P}$ is the output
- $F_P : \mathbb{R}^{n_P} \times \mathbb{R}^{m_P} \rightrightarrows \mathbb{R}^{n_P}$ is a set-valued map
- $h : \mathbb{R}^{n_P} \times \mathbb{R}^{m_P} \rightrightarrows \mathbb{R}^{r_P}$ is a function
- $\mathcal{Z} \subset \mathbb{R}^{n}$ state constraint
- $\mathcal{U} \subset \mathbb{R}^{n}$ input constraint

Cyber Component of the CPS

$$\eta^+ \in G_C(\eta, v), \ \ \zeta = \kappa(\eta, v)$$
$$(\eta, v) \in D_C \subset \Upsilon \times \mathcal{V}$$

where

- $\eta \in \mathbb{R}^{n_C}$ is the state variable
- $v \in \mathbb{R}^{m_C}$ is the input signal
- $\zeta \in \mathbb{R}^{r_C}$ is the output
- $G_C : \mathbb{R}^{n_P} \times \mathbb{R}^{m_P} \rightrightarrows \mathbb{R}^{n_P}$ is a set-valued map
- $\kappa : \mathbb{R}^{n_P} \times \mathbb{R}^{m_P} \rightrightarrows \mathbb{R}^{r_P}$ is the output function
- $\Upsilon \subset \mathbb{R}^{n_C}$ is the state space
- $\mathcal{V} \subset \mathbb{R}^{n}$ input constraint

Continuous dynamics of the CPS have to be discretized, leading to

$$\hat{z}^+ \in \hat{F}^q_P(\hat{z},\hat{u}), \ \ \hat{y} = \hat{h}^q(\hat{z},\hat{u})$$
$$(\hat{z},\hat{u}) \in \hat{C}^q_P$$

where

- $\hat{z}$, $\hat{u}$, $\hat{y}$, $\hat{F}^q_P$, $\hat{h}^q$ and $\hat{C}^q_P$ are discretized versions of the variables
- $q \in \mathbb{Q}$ state variable that indicates the chosen approximation of the actual model

and algorithms need to compensate for discretization error.

Thrust 2: Generate synthesis methods for algorithms considering computational capabilities of CPS

Approach:
Consider multiple models of continuous dynamics of the CPS and, for a given computational model, design algorithms that incorporate computational constraints

Study the solution of the optimization problem

$$\mathbf{P}^q_{\text{CPS}}(\hat{z}_k, \hat{\eta}_k) : \underset{(\Xi^q_k, \hat{U}^q_k) \subset \hat{C}^q_P, (\Lambda^q_k, \hat{V}^q_k) \subset D_C}{\text{argmin}} J^q_N(\hat{z}_k, \hat{\eta}_k, \hat{U}^q_k, \hat{V}^q_k)$$

with

$$J^q_N(\hat{z}_k, \hat{\eta}_k, \hat{U}^q_k, \hat{V}^q_k) = \sum_{t=k}^{k+N-1} \ell^q(\hat{z}_{k,t}, \hat{\eta}_{k,t}, \hat{u}_{k,t}, \hat{v}_{k,t}) + \varphi^q(\hat{z}_{k,k+N}, \hat{\eta}_{k,k+N})$$

where

- ℓ^q is the stage cost function
- φ^q is the terminal cost function for the given value of $q \in \mathbb{Q}$

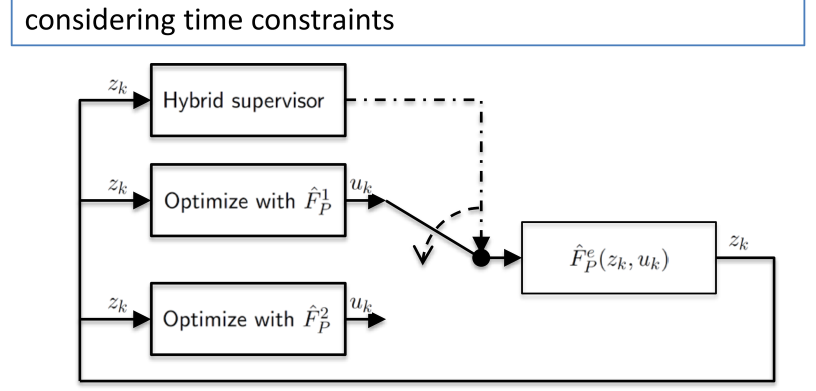
Measure of the mismatch between the physics of the system and the approximation provided

$$\Gamma^q(\hat{z},\hat{u}) = \hat{F}^q_P(\hat{z},\hat{u}) - \hat{F}^e_P(\hat{z},\hat{u})$$

where

- $\hat{F}^e_P$ is the exact discretization of $\hat{F}_P$
- Γ^1 provides measure of mismatch

Select approximations to achieve sufficient accuracy, considering time constraints



Thrust 3: Generate tools to design algorithms to adaptively select an appropriate CPS model

Approach:
Study the stability of the system under input perturbations, where the perturbation models the premature termination of the computations

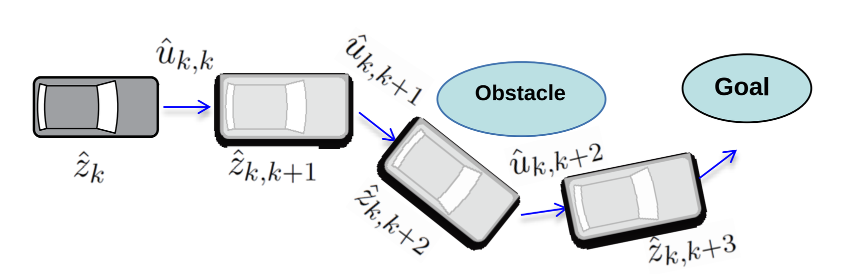
Results

Path Following for Autonomous vehicles

Goal: Achieve obstacle avoidance **with timely response for vehicle control.**

Approach

1. Vehicle control uses a **predictive strategy (MPC)** that creates a control input based on state observation (and prediction)

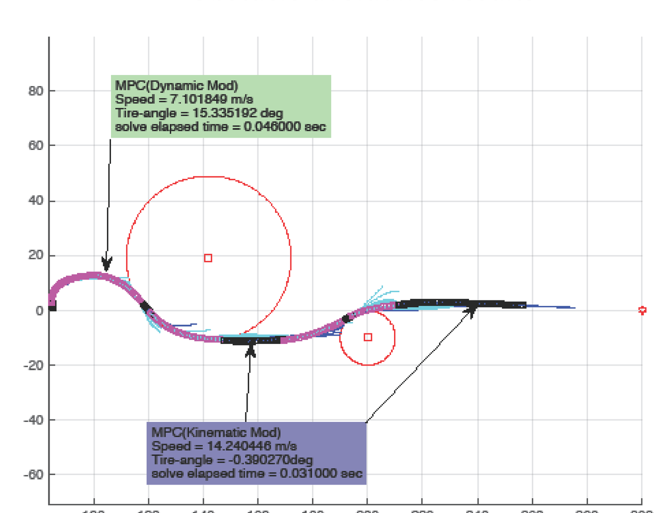


2. **Uncontrollable divergence (UD) metric,** quantifies the divergence between actual state and prediction

$$\Lambda^q(\hat{z}_k) := \hat{z}_{\text{next}(k)} - \hat{z}^{*q}_{k,\text{next}(k)} \qquad \text{UD}^q(\hat{z}_k) := \|\Lambda^q(\hat{z}_k)\|$$

3. A **hybrid strategy** switches between predictive models such that the UD is minimized

Obstacle avoidance results



Robustness of Model Predictive Control (MPC) to Computational Errors

Goal: Overcome bounded computation time in MPC using hybrid systems tools and their robustness properties

Consider the constrained continuous-time plant to be controlled

$$\dot{z} = F_P(z,u) \quad (z,u) \in \mathcal{Z} \times \mathcal{U}$$

- The measurement z_m is obtained at times $t_1, t_2 \ldots$, but due to computational limitations, the time elapsed between consecutive measurements, $t_{j+1} - t_j$, is not constant.
- Instead, it belongs to an interval $[T^1_m, T^2_m]$, describing the minimum and maximum time between samples.

Using a timer variable τ_m, this is described by the **hybrid system**

$$\begin{cases} (\dot{z}_m, \dot{\tau}_m) = (0, -1) & \tau_m \in [0, T^2_m] \\ (z^+_m, \tau^+_m) \in \{z\} \times [T^1_m, T^2_m] & \tau_m = 0, \end{cases}$$

whose jumps correspond to measurements.

Substituting the optimal control $u = \kappa(\tau_c, z_m)$ to the plant, where τ_c is a timer variable, and combining it with the measurement model results in the closed-loop hybrid system with state $x := (z, z_m, \tau_m, \tau_c)$

$$\begin{cases} \dot{x} = F_{cl}(x) & x \in C_{cl} \\ x^+ \in G_{cl}(x) & x \in D_{cl} \end{cases}$$

Approach:
1. Use hybrid systems tools along with conventional MPC methods to derive closed-loop stability.
2. Model computational errors (asynchronous timers, data dropouts) as perturbations to the closed-loop hybrid system
3. Invoke hybrid semiglobal practical stability results for nominal robustness, conclude that MPC can tolerate small computational errors.

Set-Based Predictive Control for Collision Detection and Evasion

Goal: Predict inbound dynamic obstacles and guide a vehicle towards a target while prioritizing safety

Approach

1. Consider a **set-valued predictive control strategy** where sets are used to represent uncertainty effects on the system.
   The new state is $Z := \{z + \delta\mathbb{B}\}$ with δ disturbance parameter and $\mathbb{B}$ is the unit ball. The set-valued system is $Z^+ = \hat{F}_P(Z,U)$ for $Z = (Z_O, Z_V)$ with Z_O: obstacle state, Z_V: vehicle state
2. Find a **sequence of input sets,** such that the solution satisfies the constraints, which include **avoiding the obstacle,** and minimizing a cost to **reach the target.**

Set-valued MPC Problem
Given: prediction horizon N, control horizon M, stage cost L, terminal cost V, initial condition set Z, and constraint sets $\mathcal{S}$ and Z_V, find a pair $Z := Z_0 \times Z_1 \cdots \times Z_N$, $U := U_0 \times U_1 \cdots \times U_N$, minimizing the cost

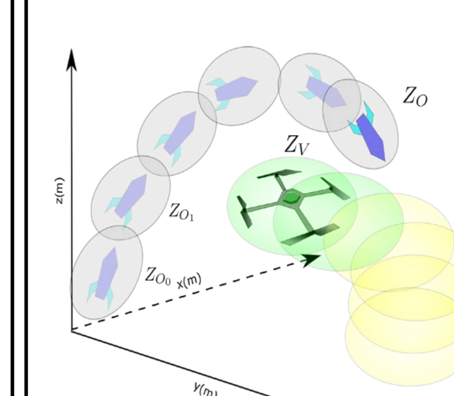
$$J_N(Z,U) = \sum_{j=0}^{N-1} L(Z_j, U_j) + V(Z_N)$$

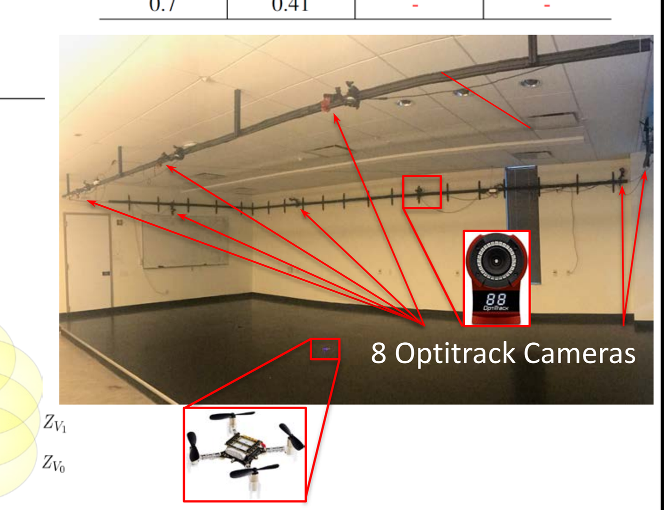
subject to the constraints
$Z_{j+1} = \hat{F}_P(Z_j, U_j)$, $Z_j \times U_j \subset \mathcal{S}$, $Z_0 = Z$, and $Z_N \subset Z_V$

Algorithm for set-based MPC

1: Obtain the initial system state Z
2: Solve set-valued MPC Problem
3: $j = 0$
4: **for** $j \leq M - 1$
5: $\quad Z_{j+1} = \hat{F}_P(Z_j, U_j)$
6: **end**
7: Set $Z = Z_M$

Obstacle distance [m]	Evasion time [s] ($\varsigma = 0.3$m)	Evasion time [s] ($\varsigma = 0.4$m)	Evasion time [s] ($\varsigma = 0.5$m)
1.5497	1.14	1.26	1.39
1.4579	1.04	1.22	1.38
1.318	0.62	0.655	1
0.945	.42	0.62	0.97
0.7	0.41	-	-





Crazyflie 2.0 quadrotor

Selected Products

1) K. Zhang, J. Sprinkle, and R. G. Sanfelice *Computationally-Aware Control of Autonomous Vehicles: A Hybrid Model Predictive Control Approach*, Autonomous Robots, vol. 39, pp. 503-517, 2015
2) K. Zhang, J. Sprinkle, and R. G. Sanfelice *Computationally-Aware Switching Criteria for Hybrid Model Predictive Control Of Cyber-Physical Systems*, IEEE Transactions on Automation Science and Engineering, vol. 13, pp. 479--490, 2016
3) B. Altın, and R. G. Sanfelice, *Model Predictive Control under Intermittent Measurements due to Computational Constraints: Feasibility, Stability, and Robustness*. Submitted to American Control Conference 2018
4) J. Crowley, Y. Zeleke, B. Altın, and R. G. Sanfelice, *Set-Based Predictive Control for Collision Detection and Evasion*. Submitted to International Conference on Robotics and Automation 2018
5) J. Crowley, Y. Zeleke, B. Altın, and R. G. Sanfelice, Video accompanying *Set-Based Predictive Control for Collision Detection and Evasion*. https://youtu.be/CuwnI2us8V8

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Intellectual merit

- *Mathematical framework for* computational limitations of CPSs
- *Novel architectures* that consider computational limitations to switch between controllers
- *Deep understanding* of the conditions for stability for computationally aware controllers
- *Tools and design techniques* that permit engineers to deploy computationally aware controllers

Broader impacts

- **Computational-aware control design tools**
- **Collaborations with the University of Bologna and NASA**

Outreach

- **Science and Internship Program for high school students**
- **REU program for undergraduate students**