The KeYmaera X Theorem Prover for Hybrid Systems

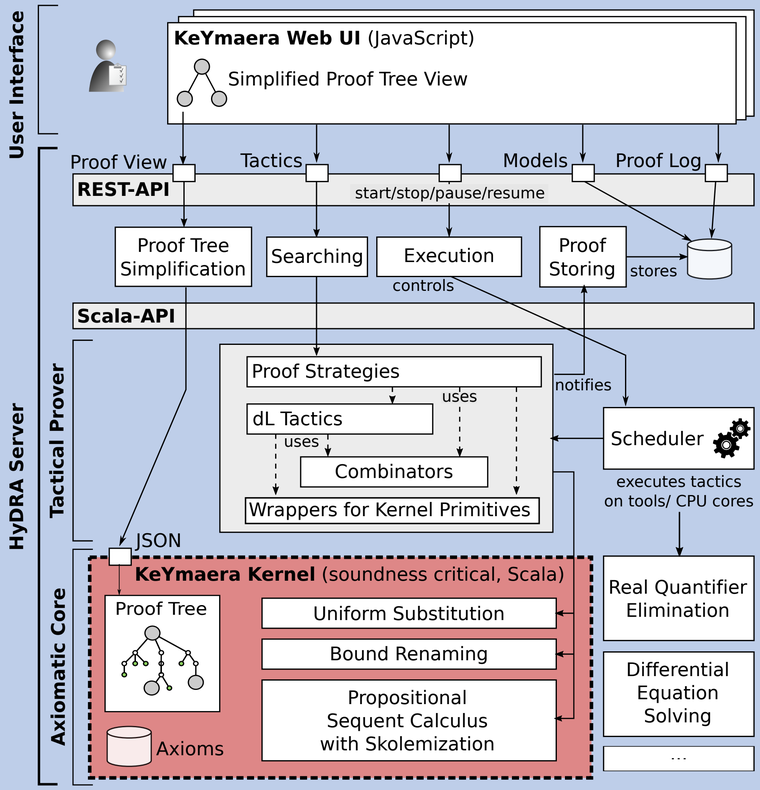
Logical Systems Lab, Carnegie Mellon University

Abstract

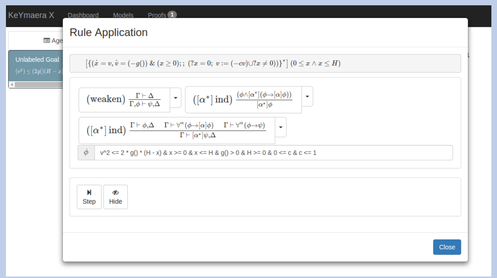
KeYmaera X is a theorem prover for specifying and verifying correctness properties of *hybrid systems* (systems that mix discrete and continuous dynamics). KeYmaera X implements *differential dynamic logic* (d$\mathcal{L}$) and provides a high degree of control over automated proof search.

Architectural Overview

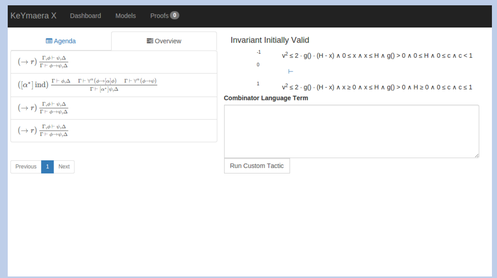
KeYmaera X features a minimal core that isolates soundness-critical axiomatic reasoning. Tactics built on top of this core drive automated proof search, and a modern web-based front-end provides a clean interface for both interactive and automated proving.



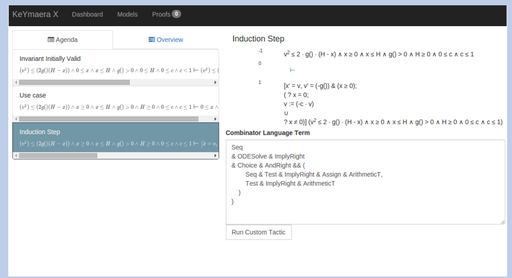
Core Features of KeYmaera X



(a) KeYmaera X provides a list of applicable tactics when a goal is selected.



(b) A list of previously executed tactics provides an overview of the proof history.



(c) User-written tactics may be applied to both entire problems and subproblems.

KeYmaera X supports both interactive and automated proof search for hybrid systems models.

- The web-based user interface (pictured above) supports interactive proving and exposes built-in general-purpose proof search tactics that suffice for many models.
- Domain or problem-specific proof search techniques are implemented using a tactic combinator library.
- An isolated soundness-critical core ensures that bugs in custom tactics cannot introduce unsoundness.

Tactical Theorem Proving for Hybrid Systems

The following d$\mathcal{L}$ formula describes a safety property for a car model.

$$\underbrace{v \geq 0 \land A > 0}_{\textit{precondition}} \rightarrow [(\underbrace{a := A \cup a := 0}_{\textit{ctrl}} ; \underbrace{\{p' = v,\ v' = a\}}_{\textit{plant}})^*] \underbrace{v \geq 0}_{\textit{postcondition}}$$

The general-purpose tactics shipped with KeYmaera X will discover a proof for this model automatically. An efficient tactic specialized to this problem can be implemented using the tactic combinator library:

```
ImplyRight & Loop("v>=0".asFormula) & onLabel(
  ("base case", Master),
  ("induction step", ImplyRight & Seq & Choice & AndRight &&
      (Assign & ODESolve & Master,
       Assign & ODESolve & Master) ),
  ("use case", Master)
)
```

Try KeYmaera X!

KeYmaera X is available for download at keymaerax.org

Carnegie Mellon University

keymaerax@keymaerax.org · keymaerax.org