Time and controls
(When to exercise control?)

Paulo Tabuada

Cyber-Physical Systems Laboratory
Department of Electrical Engineering
University of California at Los Angeles

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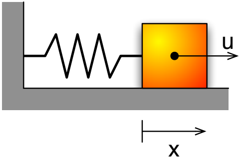
Let me argue as we sip different cocktails of time and control.

We start with a textbook example to provide a concrete context for our cocktails.



$$
\begin{aligned}
\dot{x} &= v \\
\dot{v} &= -\frac{k}{m}x + \frac{1}{m}u
\end{aligned}
$$

The objective is to design a control law $u = Kx$ to stabilize the mass at $x = 0$. For simplicity we take $m = 1$ and $k = 10$.

Several alternatives are possible:

1. Periodic control (classical approach, first part of my talk);
2. Event-triggered, self-triggered control and beyond (second part of my talk).

Classical approach:

1. Design a controller ($u = -4v$) resulting in an asymptotically stable[1] closed-loop system assuming ideal sensors, ideal actuators, and instantaneous infinite precision computation.

[1] With eigenvalues at $\lambda = -2 \pm i\sqrt{6}$.

Cocktails of time and control

The first cocktail

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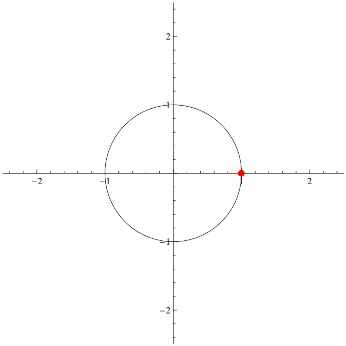
Perhaps due to this result, it is widely believed that control requires short periods (high sampling rates) and that stability is lost if deadlines are missed or periods are increased.

[1] With eigenvalues at $\lambda = -2 \pm i\sqrt{6}$.

For a periodic implementation asymptotic stability is equivalent to the closed-loop eigenvalues being strictly inside the unit disk.

Let us plot the location of the eigenvalues as a function of the chosen period.



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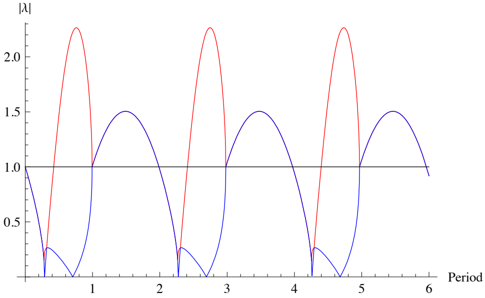

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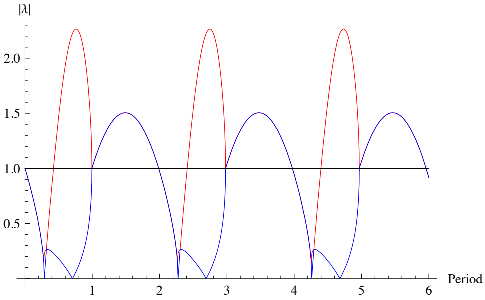
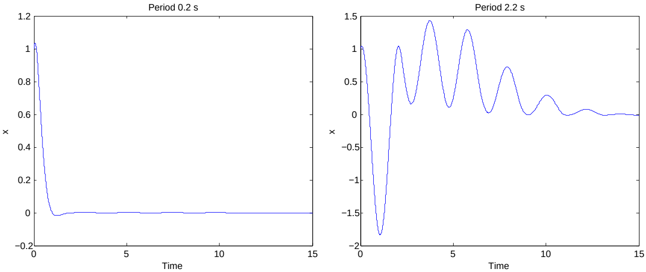


Stability is achieved with small periods as well as with (arbitrarily) large periods!

Stability, however, is not the full story.



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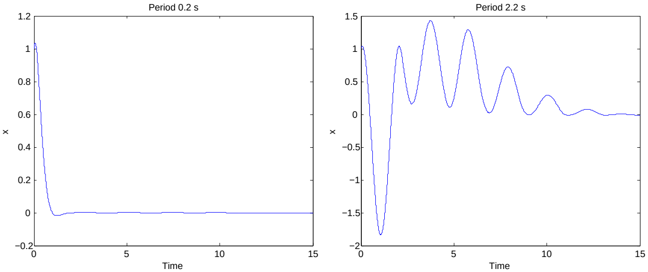

Is performance monotonically increasing with decreasing periods?

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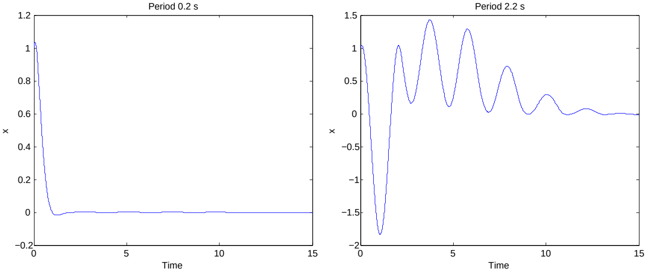
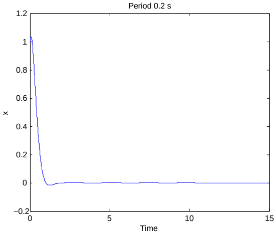


Is performance monotonically increasing with decreasing periods?

No! Shorter periods magnify the undesired effects of sensor noise and quantization[2].

[2] B. Bamieh. *Intersample and finite wordlength effects in sampled-data problems*. IEEE Transactions on Automatic Control, 48(4), 639–643, 2003.

Periods and performance, different cocktails for different tastes.



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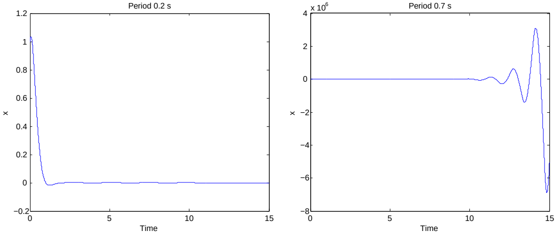

Cocktails of time and control

The third cocktail: mixing periods

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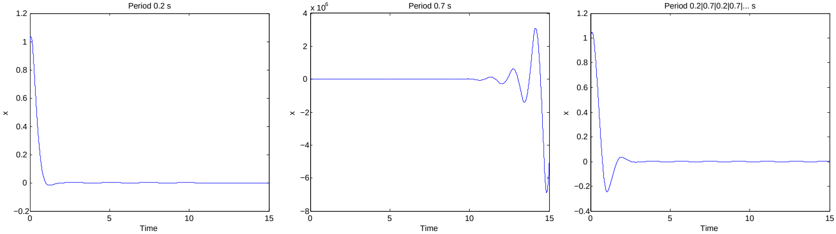

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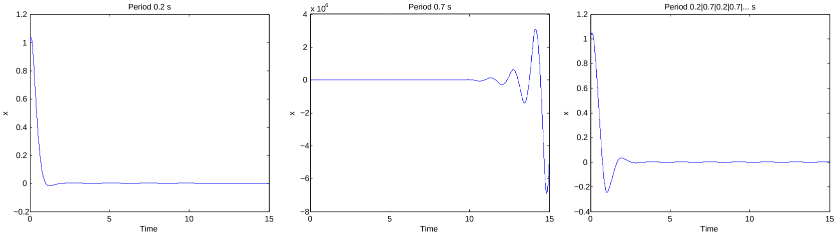

$$J(x(0), v(0)) = \int_0^\infty x^2(\tau) + v^2(\tau) + u^2(\tau) d\tau$$

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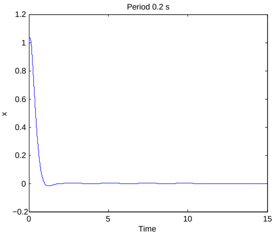

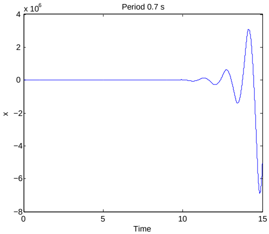

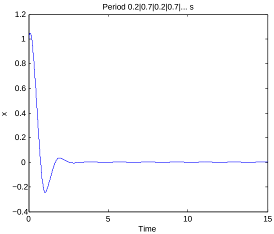

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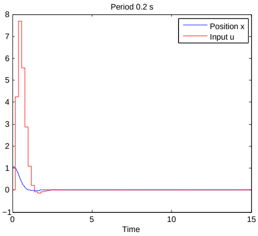

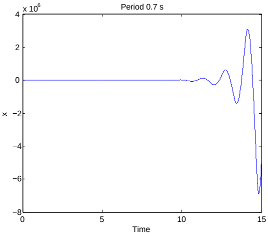

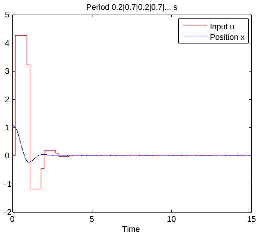

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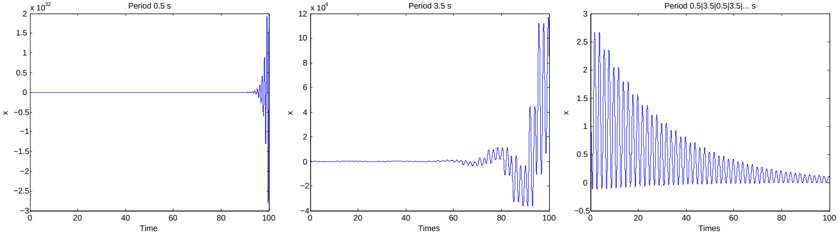
We can even stabilize this system using only periods for which the system is unstable!



Cocktails of time and control

Reflections of an inebriated man

- The relation between time, stability, and performance is quite complex and not well understood[3];

[3] Beware of good looking (convex) Quality of Control or utility functions.

Cocktails of time and control

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- The periodic paradigm results in a conservative usage of resources since the period is chosen for worst case operating conditions that rarely happen;
- The periodic paradigm can be seen as open-loop sampling;
- Recently, event-triggered and self-triggered control have been proposed as a way of introducing *feedback* in the sampling process.

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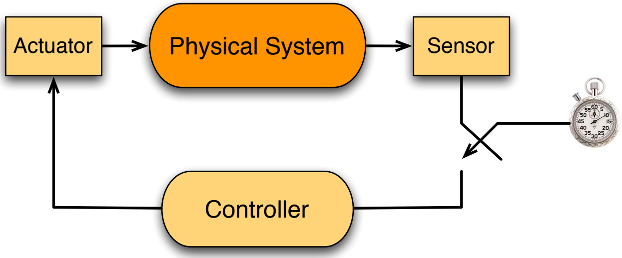
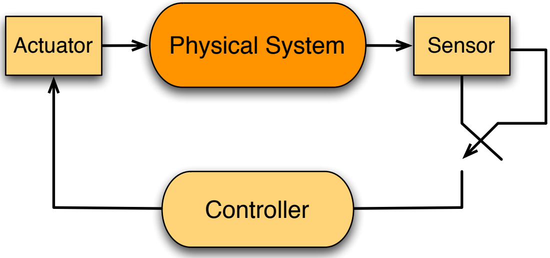
In time-triggered control the sensing, control, and actuation are driven by a clock.



Event-triggered and self-triggered control

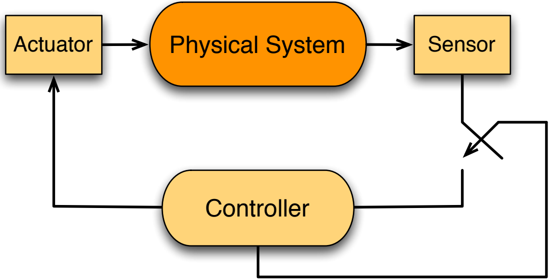
A new look at old ideas

In event-triggered control the input is held constant, not periodically, but while performance is satisfactory.



This requires the constant monitoring of the state to determine the current performance.

In self-triggered control, the current state is used not only to compute the input to the system, but also the next time the control law should be recomputed;



Constant monitoring of the state is no longer needed although the loop is still closed based on the current performance.

As an example we consider the control of a jet engine compressor:

$$
\begin{aligned}
\dot{x}_1 &= -x_2 - \frac{3}{2}x_1^2 - \frac{1}{2}x_1^3 \\
\dot{x}_2 &= \frac{1}{\beta^2}(x_1 - u) \\
u &= x_1 - \frac{\beta^2}{2}(x_1^2+1)(y + x_1^2 y + x_1 y^2) + 2\beta^2 x_1 \\
y &= 2\frac{x_1^2 + x_2}{x_1^2+1}
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$$

New deadline is a function of the current state of the physical system:

$$\tau(x_1, y) = \frac{29x_1 + x_1^2 + y^2}{5.36x_1^2\sqrt{x_1^2 + y^2} + x_1^2 + y^2} \cdot \tau^*.$$

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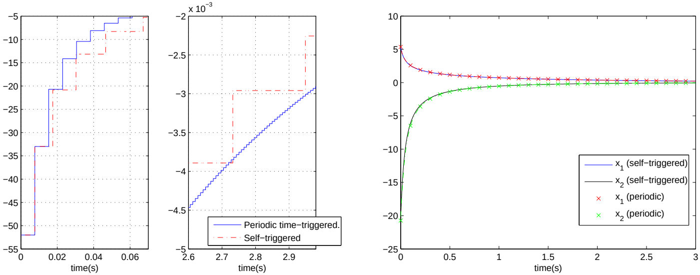

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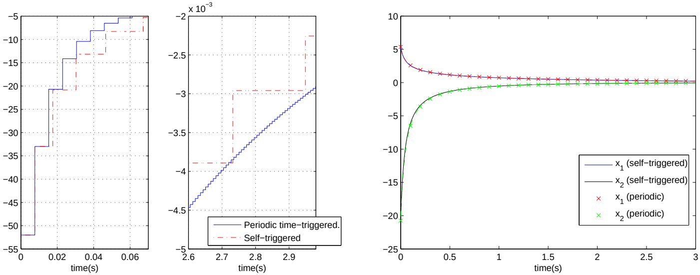

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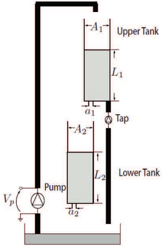
It works in theory... How about in practice?

Event-triggered and self-triggered control

Welcome to the real world!

In collaboration with Kalle Johansson at KTH we tested these ideas in a real deployment.



- Two independent water tanks controlled over the same wireless network.
- 1 sensing, 1 controller, and 1 actuator node per water tank.
- Additional temperature and humidity sensors on the wall to generate low priority traffic.

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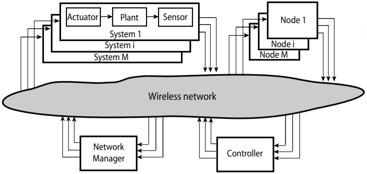

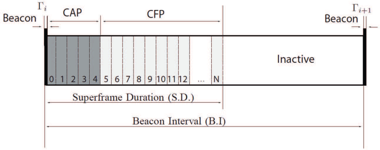

- Slight modification of the IEEE 802.15.4 MAC protocol (used in the proposed WirelessHART);
- The IEEE 802.15.4 MAC protocol offers a Contention Access Period (CAP) using CSMA/CA and a Contention Free Period (CFP) using TDMA;
- Telos motes with 250kbps 2.4 GHz Chipcon CC2420 IEEE 802.15.4 compliant radio.

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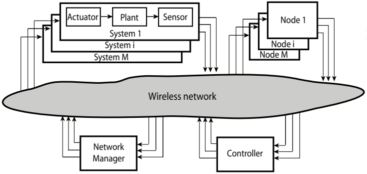

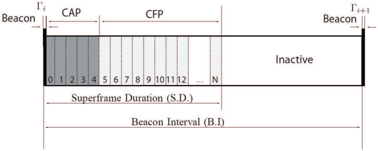

Scheme	Integral of Absolute Error	#updates in 220 s.	Battery life (days)
Event-triggered	88.26	49	879.84
Self-triggered	101.65	34	1010.18
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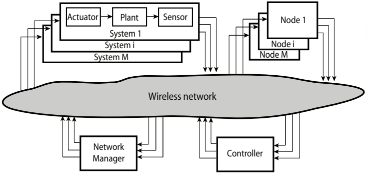

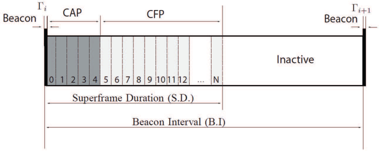

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  - Anytime attention control: based on a deadline provided by the real-time scheduler, compute an input that does not need to be changed before a given deadline while maximizing control performance;
  - Asynchronous control: design an observer/controller that works even when receiving out-of-order sensor measurements.

Final thoughts

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  - How to provide real-time guarantees when events are only known at execution time?
  - How to dynamically trade computation and communication resources for performance?
  - How to mix time-triggered with event-triggered controllers?
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Does control need real-time?

Yes, but not the *old clockwork* based on periods and deadlines!

For papers and more information:
`http://www.cyphylab.ee.ucla.edu`