Topological Abstraction for Robot Path Planning

Subhrajit Bhattacharya
Lehigh University

Background: Topological Path Planning (TPP)

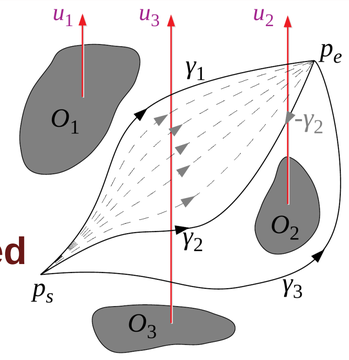
Homotopy Classes:

Homotopy invariant:

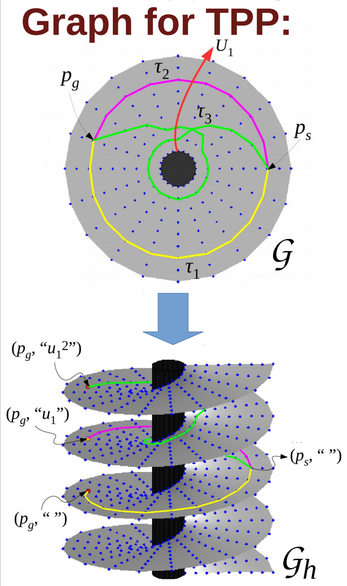
$h(\gamma_1) = h(\gamma_2) = \text{“}u_3\, u_2\text{”}$

$h(\gamma_3) = \text{“}u_3\text{”}$

(presentation of the freely-generated fundamental group)

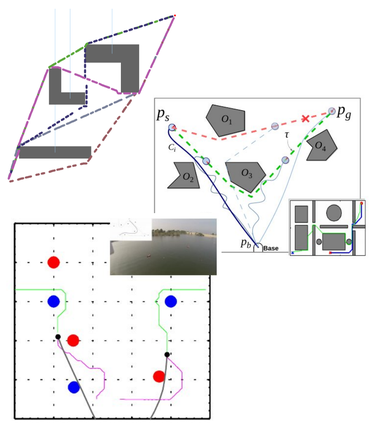


Homotopy-augmented Graph for TPP:



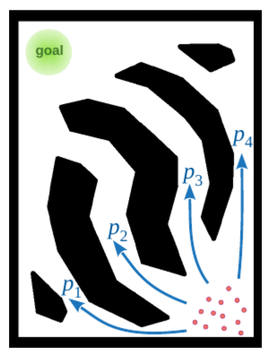
(universal covering space)

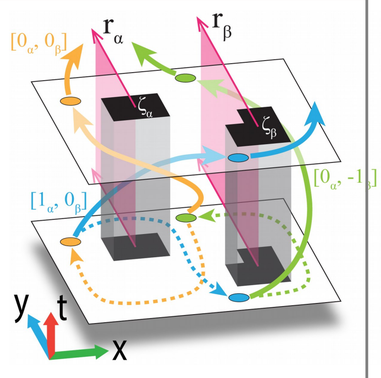
TPP Basic Results:



Coordination-free Multi-robot Path Planning for Congestion Reduction Using Topological Reasoning

Problem: Multi-robot path planning in a complex, cluttered environment with the aim of reducing overall congestion, while avoiding any inter-robot communication or coordination (due to privacy restrictions or lack of communication).





Solution Approach (for each robot):

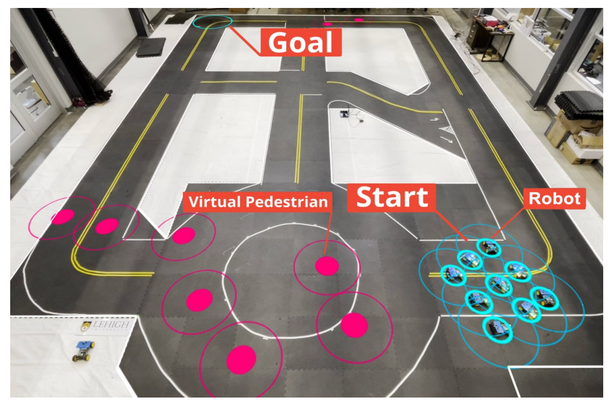
- Compute topologically distinct paths
- Compute *path choice probabilities* based on estimated traffic density

$$\min_{P_1,P_2,\cdots,P_m} \sum_{\sigma\in S^n} C(\sigma) \prod_{i=1}^{n} P_{\sigma_i}$$

$$\text{s.t.} \quad \sum_{j=1}^{m} P_j = 1\,, \qquad 0 \le P_j \le 1,\ \forall j \in S$$

Simplifications:
- 2-robot model (QP)
- Ensemble model (convex optimizarion)

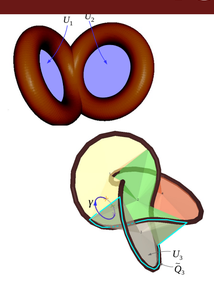
- Stochastically assign robot to a topological class based on computed probabilities,
- Stay committed to the assigned class while locally avoiding collisions and replanning in spatio-temporal domain using a topology-informed heuristic function.



Beyond Topology: Geometric Classes of Paths – Topo-geometric Planning in 3D

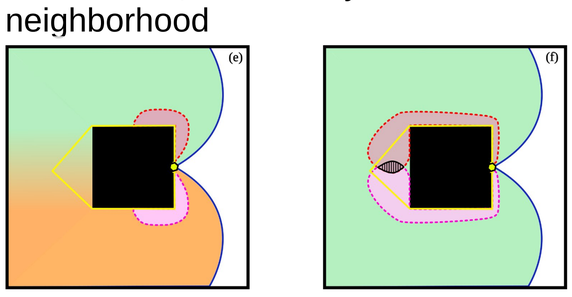
Challenge:

Homotopy invariants are difficult to compute in 3D and requires non-trivial constructions.



Solution Approach:

Neighborhood-augmented graph:
Vertices not only distinguished by coordinates, but also by their immediate neighborhood



Same vertex, different neighborhoods.

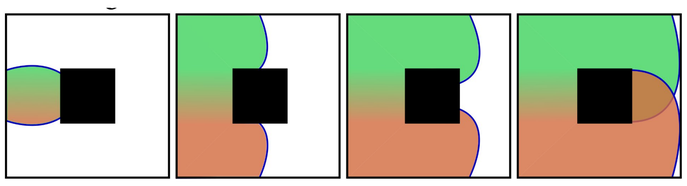
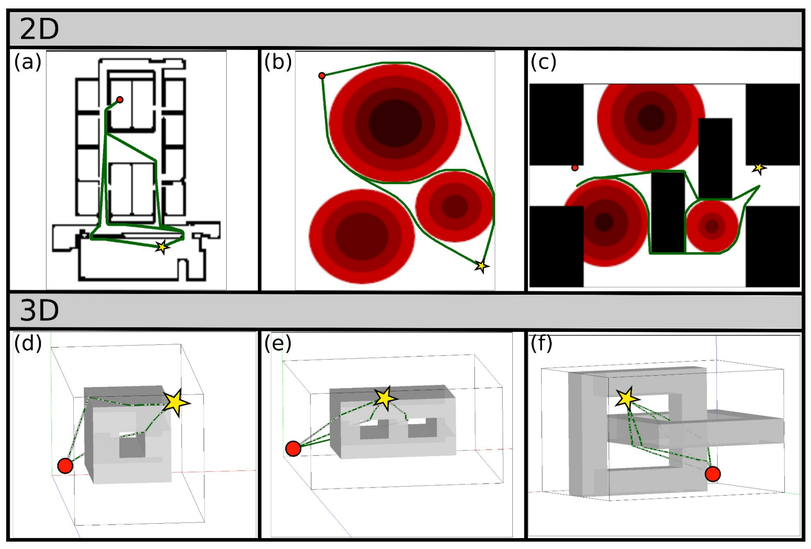
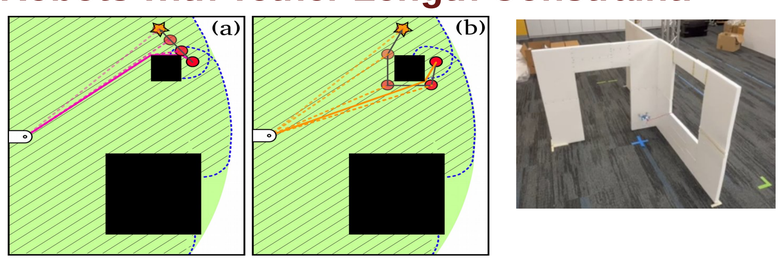


Illustration of progress of search in neighborhood-augmented graph

Initial Results 2D/3D:

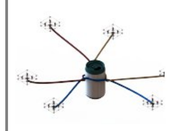


Application to Planning for Aerial Tethered Robots with Tether Length Constraint:



Hitch Planning for Object Transportation

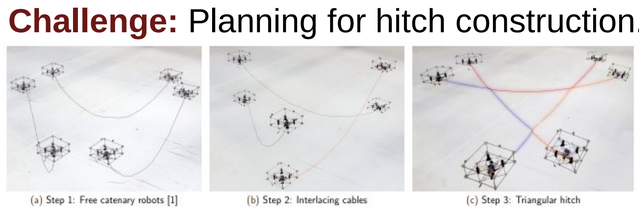
[In collaboration with Davis Saldana]



Objective: Object transportation using multiple cables carried by UAVs.

Hitch Topologies:

Challenge: Planning for hitch construction.



Outreach

- **Summer 2022 Lehigh CHOICES outreach program for middle-school girls:**
  - A “Capture-the-flag” Activity using a Team of Ground Robots for Teaching Fundamentals of Robot Motion Planning and Control.
  - “Lesson” on Robot Autonomy, with introduction to concepts in perception, planning and control. Introduces concepts from topology in context of robot configuration spaces.
- Co-organized presentations and demonstrations at the AIR lab for the LWE PreLUsion program. PreLUsion is a summer pre-orientation experience offered for female students starting at Lehigh in the Fall.
- Building a mentoring ecosystem around these events involving graduate students as well as undergraduate students.

Future Directions

Fundamental:
Topological abstractions for configuration space of articulated systems (robot arms, snake-like robots) and soft robotic manipulators.

Applications:
Systems of tethered robots, teams of robots carrying cables, multi-robot planning with topological reasoning in spatial domains.

Education & Outreach:
Refine educational materials and make them publicly available through project website.

2023 FRR & NRI Principal Investigators' Meeting
May 2-3, 2023

Award ID#: 2144246