Towards Passivity based software synthesis

Shravan Sajja

University of Notre Dame,
In collaboration with Michael Mccourt, Vijay Gupta, Bill Goodwine, Panos Antsaklis

April 22, 2013

Objective

Cyber Physical System: Large scale interconnected systems with tight coupling between physical dynamics, computational dynamics, and communication networks.

Highly complex models at different levels and with complex relationships between them.

Proposed solution: Finite state approximations for continuous models.

- Automated analysis and design (**Verification** and **Synthesis**),
- common language for continuous dynamics and software implementation of control algorithms (**Hybrid Systems**),
- framework for control over finite actuation and coarse sensing due to network limitations.

Passivity-Based Control Design for Cyber-Physical Systems

Advantages:

- Robustness w.r.t. structural uncertainty,
- Compositional property,
- Orthogonality w.r.t. network effects,
- Simplicity,...

Roadblocks:

- Notion of Passivity for Finite State Approximations,
- Finite state approximation methods that preserve passivity.
- Compositionality for passive finite state approximations.

Outline

1. Hybrid Input/Output Automaton
2. Dissipativity for Finite State Automata
3. Finite State Approximations and Passivity preserving

Other works...

- C. M. Özveren, A. S. Willsky and P. J. Antsaklis, Stability and stabilizability of discrete event dynamic systems, Journal of the ACM, Vol. 38, No. 3, pp. 729–751, 1991.
- K. Passino, A. N. Michel, P. J. Antsaklis, Lyapunov stability of a class of discrete event systems, IEEE Transactions on Automatic Control, Vol. 39, No. 2, pp. 269 – 279, 1994.
- N. Lynch, R. Segala, and F. Vaandrager, Hybrid I/O automata, Inf. Comput., Vol. 185, No. 1, pp. 105 – 157, 2003.
- D. C. Tarraf, A. Megretski and M. A. Dahleh, A framework for robust stability of systems over finite alphabets, IEEE Transactions on Automatic Control, Vol. 53, No. 5, pp. 1133 –1146, 2008.
- P. Tabuada, Verification and Control of Hybrid Systems: A Symbolic Approach, Springer, 2009.
- E. Feron, From Control Systems to Control Software, Control Systems, IEEE, Vol. 30, No. 6, pp. 50 – 71, 2010.
- M. Zamani, G. Pola, and P. Tabuada, Symbolic Models for Nonlinear Control Systems Without Stability Assumptions, IEEE Transactions on Automatic Control, Vol. 57, No. 7, pp. 1804 – 1809, 2012.
- Y. Oishi, Passivity degradation under the discretization with the zero-order hold and the ideal sampler, 49th IEEE Conference on Decision and Control, pp. 7613–7617, 2010.

Hybrid Input/Output Automaton

A hybrid input-output automaton Σ_H is comprised of the following components

$$\Sigma_H = \{Q, Z, F, H, Init, Inv, E, G, R\},$$

where

- Q is the set of discrete states,
- $Z = \{U, Y, X\}$ is the set of continuous variables,
- $F = \{f_i\}$ is the set of vector fields $f_i(\cdot, \cdot) : X \times U \rightarrow \mathbb{R}^n$,
- $H = \{h_i\}$ is the set of output equations $h_i(\cdot, \cdot) : X \times U \rightarrow \mathbb{R}^p$,
- $Init \subset Q \times X$ denotes the valid set of initial conditions,
- $Inv : Q \rightarrow 2^X$ denotes the portion of X where each $q \in Q$ may be active,
- $E \subset Q \times Q$ is the set of all edges,
- $G : E \rightarrow 2^X$ is the guard set, and
- $R : E \times X \rightarrow 2^X$ is the reset map for continuous state $x \in X$.

Dissipative Hybrid I/O Automata

Σ_H is **dissipative** if for each mode $q_i \in Q$ there exists a storage function $V_i(x)$ such that

$$\underline{\alpha}(\|x\|) \leq V_i(x) \leq \overline{\alpha}(\|x\|)$$

for class-$\mathcal{K}$ functions $\underline{\alpha}$ and $\overline{\alpha}$ to satisfy the following conditions.

1. For all $q_i \in Q$ there exists a continuous energy supply rate $\omega_c^i : U \times Y \to \mathbb{R}$ such that when q_i is active between switching instants t_k and t_{k+1} ,

$$V_i(x(t_2)) \leq V_i(x(t_1)) + \int_{t_1}^{t_2} \omega_c^i(u, y)dt, \quad \text{for } t_k \leq t_1 \leq t_2 \leq t_{k+1}.$$

2. There exists a discrete energy supply rate $\omega_d : X \times E \to \mathbb{R}$ such that for each switching instant t_k, where the transition can be denoted $e = (q_i, q_j)$,

$$V_j(x(t_k^+)) \leq V_i(x(t_k^-)) + \omega_d(x, e),$$

and the rate ω_d is bounded by a class-$\mathcal{K}$ function $W(x)$, $\omega_d \leq W(x)$.

Passive Hybrid I/O Automata

Σ_H is **passive** if it is dissipative with

$$\omega_c^i(u, y) = u^T y, \quad \forall q_i \in Q$$

and if for all discrete mode switching times t_k the discrete supply rate satisfies

$$\sum_{k=1}^{T_n} \omega_d(x_k^-, e) \leq \phi_d(t)$$

where $\phi_d(t)$ is absolutely integrable .

Stability Results for Dissipative Hybrid I/O Automata

Theorem

Consider an unforced ($u(t) = 0, \forall t$) hybrid input output automaton Σ_H such that

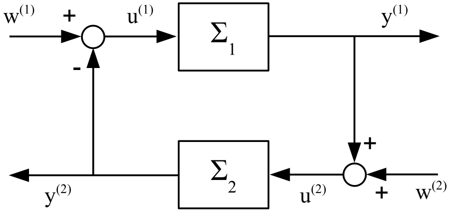
- $\forall q_i \in Q$, $\dim(x_i) = n$ and
- $\exists x_e$ such that $f_i(x_e, 0) = 0, \forall i$.

If the following conditions hold for all executions and all times T where a maximum of T_n switches occur in the time interval $[t_0, T]$:

- for all modes q_i,
$$\omega_c^i(0, y) \leq 0$$
- and for each switching instant t_k, $\sum_{k=1}^{T_n} \omega_d(x_k, e) \leq \phi_d(t)$ where $\phi_d(t)$ is absolutely integrable,

then Σ_H is Lyapunov stable.

The feedback interconnection of two hybrid automata



Theorem

The feedback interconnection of two dissipative (passive) hybrid I/O automata Σ_{H1} and Σ_{H2} where

$$\Sigma_{H1} = \{Q^{(1)}, Z^{(1)}, F^{(1)}, H^{(1)}, Init^{(1)}, Inv^{(1)}, E^{(1)}, G^{(1)}, R^{(1)}\} \quad and$$
$$\Sigma_{H2} = \{Q^{(2)}, Z^{(2)}, F^{(2)}, H^{(2)}, Init^{(2)}, Inv^{(2)}, E^{(2)}, G^{(2)}, R^{(2)}\},$$

forms a dissipative (passive) hybrid I/O automaton Σ.

Finite automata

A **finite automaton** is defined by the five-tuple:
$A = \{Q, \Sigma, \alpha, q_0, F\}$ where

- Q is the set of discrete states,
- $\mathcal{E}$ is the set of discrete events,
- $\alpha : Q \times \Sigma \rightarrow Q$ is the set of possible state transitions,
- $Q_0 \subset Q$ is the set of initial states, and
- $Q_e = F \subseteq Q$ is the set of final states.

Definition

An **energy storage function** $V : Q \rightarrow \mathbb{R}^+$ for a finite automaton $A = \{Q, \mathcal{E}, \alpha, q_0, F\}$ must satisfy

- $V(q) = 0$ for all $q \in Q_e$ and
- $V(q) > 0, \forall q \in Q \backslash Q_e$.

Dissipativity and Stability of finite automata

Definition

A finite automaton $A = \{Q, \mathcal{E}, \alpha, Q_0, F\}$ is **dissipative** with respect to supply rate $\omega(u, q)$ if there exists a energy storage function V such that the following inequality holds $\forall K \geq 0$,

$$\sum_{k=0}^{K-1} \omega(u(k), q(k)) \geq V(q(K)) - V(q(0)).$$

Theorem

Consider a dissipative finite automaton with a desired equilibrium set Q_e. This finite automaton has energy storage function $V(q)$ and a supply rate $\omega(q, e)$. The equilibrium of the finite automaton is an invariant set if $\omega(q, e) = 0$ for all $q \in Q_e$ and all $e \in \mathcal{E}$.

Finite State Approximations [Tabuada 2012]

Definition (Control Systems)

A control system is a quadruple $\Sigma = (\mathbb{R}^n, U, \mathcal{U}, f)$ consisting of:

- $\mathbb{R}^n$ is the state space;
- $U \subseteq \mathbb{R}^m$ is the input space;
- $\mathcal{U} : \mathbb{R} \to U$ is a subset of the set of all locally essentially bounded functions of time;
- $f : \mathbb{R}^n \times U \to \mathbb{R}^n$ is a continuous map satisfying the following Lipschitz assumption.

Transition systems

Definition (Pappas, Tabuada)

A system S is a quintuple $S = (X, U, \longrightarrow, Y, H)$ consisting of:

- A set of states X;
- A set of inputs U;
- A transition relation $\longrightarrow \subseteq X \times U \times X$;
- An output set Y;
- An output function $H : X \to Y$.

The state set X is equipped with a metric $\mathbf{d} : X \times X \to \mathbb{R}_0^+$.

$\|x\|$ represents the $\|x\|_\infty$ unless specified otherwise.

Objective: $\Sigma \xrightarrow[preserving\ dissipativity]{preserving\ stability} S$ with a countably finite set of states

Digital Control Systems as Transition Systems

$\Sigma = (X, U, \mathcal{U}, f)$ where

$X := [c_1, d_1] \times \cdots \times [c_n, d_n] \subseteq \mathbb{R}^n$ for some $c_i < d_i$, $i = 1, 2, \ldots, n$.

$U := [a_1, b_1] \times \cdots \times [a_m, b_m] \subseteq \mathbb{R}^m$ for some $a_i < b_i$, $i = 1, 2, \ldots, m$.

Let $\hat{\mu} = \min_{i=1,2,\ldots,m} |b_i - a_i|$ and $\hat{\eta} = \min_{i=1,2,\ldots,n} |d_i - c_i|$

Given $\tau > 0$, we consider is: $\mathcal{U}_\tau := \{\mathbf{u} \in \mathcal{U} \mid \mathbf{u}(t) = \mathbf{u}(0),\ t \in [0, \tau]\}$.

Sub-transition system $S_\tau(\Sigma) := (Q_1, L_1, \xrightarrow[1]{}, O, H)$, where

- $Q_1 = X$;
- $L_1 = \{l_1 \in \mathcal{U} | \ \mathbf{x}(\tau, x_0, l_1)$ is defined for all $x_0 \in X\}$;
- $q_1 \xrightarrow[1]{l_1} p_1$, if $\mathbf{x}(\tau, q_1, l_1) = p_1$;
- $O_1 = X$ and
- $H_1 = 1_X$.

$S_\tau(\Sigma)$ is countable but not finite

Digital Control Systems as Transition Systems

$\Sigma = (X, U, \mathcal{U}, f)$ where

$X := [c_1, d_1] \times \cdots \times [c_n, d_n] \subseteq \mathbb{R}^n$ for some $c_i < d_i$, $i = 1, 2, \ldots, n$.

$U := [a_1, b_1] \times \cdots \times [a_m, b_m] \subseteq \mathbb{R}^m$ for some $a_i < b_i$, $i = 1, 2, \ldots, m$.

Let $\hat{\mu} = \min_{i=1,2,\ldots,m} |b_i - a_i|$ and $\hat{\eta} = \min_{i=1,2,\ldots,n} |d_i - c_i|$

Given $\tau > 0$, we consider is: $\mathcal{U}_\tau := \{\mathbf{u} \in \mathcal{U} \mid \mathbf{u}(t) = \mathbf{u}(0),\ t \in [0, \tau]\}$.

Sub-transition system $S_\tau(\Sigma) := (Q_1, L_1, \xrightarrow[1]{}, O, H)$, where

- $Q_1 = X$;
- $L_1 = \{l_1 \in \mathcal{U} |\ \mathbf{x}(\tau, x_0, l_1)$ is defined for all $x_0 \in X\}$;
- $q_1 \xrightarrow[1]{l_1} p_1$, if $\mathbf{x}(\tau, q_1, l_1) = p_1$;
- $O_1 = X$ and
- $H_1 = 1_X$.

$S_\tau(\Sigma)$ is countable but not finite

Finite, Countable Transition Systems

Given a control system Σ, any $\tau > 0$, $\eta > 0$ and $\mu > 0$, a finite, countable transition system can be defined as: $S_{\tau,\eta,\mu}(\Sigma) := (Q_2, L_2, \xrightarrow[2]{}, O_2, H_2)$ such that

- $Q_2 = [X]_\eta$;
- $L_2 = [L_1]_\mu$;
- $q_2 \xrightarrow[2]{l_2} p_2$, if $l_2 \in L_2(q_2)$ and $\|p_2 - \mathbf{x}(\tau, q_2, l_2)\| \leq \eta/2$;
- $O_2 = [X]_\eta$
- $H_2 = i : Q_2 \to O_2$

where $A \subseteq \mathbb{R}^n$ and $\mu \in \mathbb{R}^+$,

$$[A]_\mu := \{a \in A | a_i = k_i \mu, \ k_i \in \mathbb{Z}, i = 1, \ldots, n\}.$$

If we define $\mathcal{B}_\epsilon(x) = \{y \in \mathbb{R}^n | \ \|x - y\| \leq \epsilon\}$, then $\mathbb{R}^n \subseteq \bigcup_{q \in [\mathbb{R}^n]_\mu} \mathcal{B}_{\mu/2}(q)$

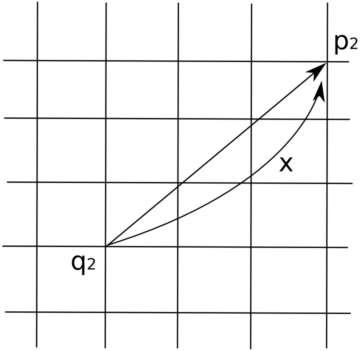


Figure : Principle for computation of Finite approximation

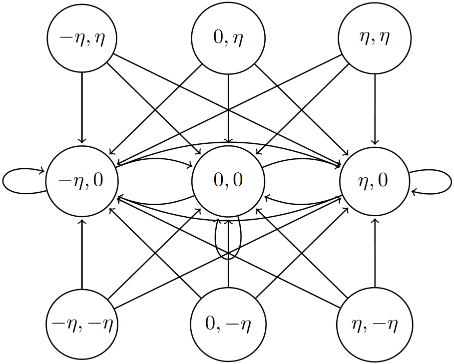


Figure : Example of a finite state approximation for a 2-D system [Tabuada 2012]

Approximate Simulation

Definition (ϵ-Approximate Simulation)

Let $S_1 := (Q_1, L_1, \xrightarrow[1]{}, O_1, H_1)$, $S_2 := (Q_2, L_2, \xrightarrow[2]{}, O_2, H_2)$ be deterministic metric transition systems with the same sets of labels L and outputs O equipped with the metric $\mathbf{d}$. Let $\epsilon \in \mathbb{R}_0^+$ be a given precision, a relation $R \subseteq Q_1 \times Q_2$ is said to be an ϵ-approximate simulation relation between S_1 and S_2 if the following three conditions are satisfied:

(i) for every $q_1 \in Q_1$, there exists $q_2 \in Q_2$ with $(q_1, q_2) \in R$;

(ii) for every $(q_1, q_2) \in R$ we have $\mathbf{d}(H_1(q_1), H_2(q_2)) \leq \epsilon$;

(iii) for every $(q_1, q_2) \in R$ we have that $q_1 \xrightarrow[1]{l_1} p_1$ in S_1 implies the existence of $q_2 \xrightarrow[2]{l_2} p_2$ in S_2 satisfying $(p_1, p_2) \in R$.

Approximate Bisimulation

Definition (ϵ-Approximate Bisimulation)

Let $S_1 := (Q_1, L_1, \xrightarrow[1]{}, O_1, H_1)$, $S_2 := (Q_2, L_2, \xrightarrow[2]{}, O_2, H_2)$ be deterministic metric transition systems with the same sets of labels L and outputs O equipped with the metric $\mathbf{d}$. Let $\epsilon \in \mathbb{R}_0^+$ be a given precision, a relation $R \subseteq Q_1 \times Q_2$ is said to be an ϵ-approximate bisimulation relation between S_1 and S_2 if the following three conditions are satisfied:

(i) for every $(q_1, q_2) \in R$ we have $\mathbf{d}(H_1(q_1), H_2(q_2)) \leq \epsilon$;

(ii) for every $(q_1, q_2) \in R$ we have that $q_1 \xrightarrow[1]{l_1} p_1$ in S_1 implies the existence of $q_2 \xrightarrow[2]{l_2} p_2$ in S_2 satisfying $(p_1, p_2) \in R$.

(iii) for every $(q_1, q_2) \in R$ we have that $q_2 \xrightarrow[2]{l_2} p_2$ in S_2 implies the existence of $q_1 \xrightarrow[1]{l_1} p_1$ in S_1 satisfying $(p_1, p_2) \in R$.

Incrementally Input to State Stable

Definition

A control system Σ is incrementally input to state stable (δ- ISS) if it is forward complete and there exist a $\mathcal{KL}$ function β and a $\mathcal{K}_\infty$ function γ such that for any $t \in \mathbb{R}_0^+$, any $x, x' \in \mathbb{R}^n$ and any $v, v' \in \mathcal{U}$ the following condition is satisfied:

$$\|\zeta_{x,v}(t) - \zeta_{x',v'}(t)\| \leq \beta(\|x - x'\|, t) + \gamma(\|v - v'\|) \tag{1}$$

Theorem (Tabuada 2008)

Consider a control system Σ and any desired precision $\epsilon > 0$. If Σ is δ-ISS then for any $\tau > 0$, $\eta > 0$ and $\mu > 0$ satisfying the following inequality:

$$\beta(\epsilon, \tau) + \gamma(\mu) + \eta/2 \leq \epsilon, \tag{2}$$

the transition system $S_{\mathcal{U}_\tau}(\Sigma)$ is ϵ-bisimilar to $S_{\tau,\eta,\mu}(\Sigma)$.

Incrementally Forward Complete Systems (δ-FC)

Definition

A control system Σ is δ-FC if there exist continuous functions $\beta : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \to \mathbb{R}_0^+$ and $\gamma : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \to \mathbb{R}_0^+$ such that for every $s \in \mathbb{R}^+$, the functions $\beta(\cdot, s)$ and $\gamma(\cdot, s)$ belong to class $\mathcal{K}_\infty$, and for any $x, x' \in \mathbb{R}^n$, any $\tau \in \mathbb{R}^+$, and any $v, v' \in \mathcal{U}$, where $v, v' : [0, \tau) \to \mathbf{U}$, the following condition is satisfied for all $t \in [0, \tau]$:

$$\|\zeta_{x,v}(t) - \zeta_{x',v'}(t)\| \leq \beta(\|x - x'\|, t) + \gamma(\|v - v'\|, t).$$

Theorem (Tabuada 2012)

If Σ is δ-FC with desired precision $\epsilon > 0$ then for any $\tau > 0$, $\theta > 0$, $\eta > 0$ and $\mu > 0$ satisfying the following inequality:

$$\beta(\theta, \tau) + \gamma(\mu, \tau) + \eta \leq \epsilon,$$

such that $\mu \leq \hat{\mu}$ and $\eta \leq \hat{\eta} \leq \epsilon \leq \theta$, then the transition system $S_\tau(\Sigma)$ is ϵ-approximately similar to $S_{\tau,\theta,\mu}(\Sigma)$.

Passivity

A nonlinear system Σ with an output $y(t) = h(x(t), u(t))$ is input output strictly passive (IOSP) with a storage function V if

$$\dot{V}(x(t)) \leq u^T(t)y(t) - \rho y^T(t)y(t) - \nu u^T(t)u(t) \qquad \forall t \geq 0$$

where $\nu > 0$, $\rho > 0$.

We assume that V is Lipschitz continuous, i.e.,

$$|V(x_1) - V(x_2)| \leq K(\|x_1 - x_2\|)$$

where K is a Lipschitz constant.

Let $\mathbf{x}(t, q, I)$ denote the point reached at time $t \in [0, \tau]$, under the input I and initial condition q. Also let

$$\mathbf{y}(t, q, I) = h(\mathbf{x}(t, q, I), I).$$

Passivity for a transition system

For the transition system $S_\tau(\Sigma) := (Q_1, L_1, \underset{1}{\longrightarrow}, O, H)$ consider a transition $q_1 \xrightarrow[1]{l_1} p_1$ where $\mathbf{x}(\tau, q_1, l_1) = p_1$.

$S_\tau(\Sigma)$ is $(\nu,\ \rho)$ - IOSP if all the transitions satisfy inequalities like

$$V(\mathbf{x}(t, q_1, l_1)) - V(q_1) \leq \langle l_1, \mathbf{y}(t, q_1, l_1)\rangle - \rho\langle \mathbf{y}(t, q_1, l_1), \mathbf{y}(t, q_1, l_1)\rangle \\ -\nu\langle l_1, l_1\rangle \qquad 0 \leq t \leq \tau.$$

Where V is the storage function and

$V(\mathbf{x}(t, q_1, l_1)) - V(q_1)$ is the increase in stored energy.

$\langle l_1, \mathbf{y}(t, q_1, l_1)\rangle - \rho\langle \mathbf{y}(t, q_1, l_1), \mathbf{y}(t, q_1, l_1)\rangle - \nu\langle l_1, l_1\rangle$ is the energy supplied during the transition.

Passivity for a symbolic model

For the symbolic system $S_{\tau,\eta,\mu}(\Sigma) := (Q_2, L_2, \xrightarrow[2]{}, O_2, H_2)$ consider a transition $q_2 \xrightarrow[2]{l_2} p_2$, where $l_2 \in L_2(q_2)$ and

$$\|p_2 - \mathbf{z}(t, q_2, l_2)\| \leq \eta/2 \leq \epsilon \text{ for } \delta \text{ - ISS systems}$$
$$\|p_2 - \mathbf{z}(t, q_2, l_2)\| \leq \beta(\theta, \tau) + \gamma(\mu, \tau) + \eta/2 \leq \epsilon \text{ for } \delta \text{ - FC systems}$$

$S_{\tau,\eta,\mu}(\Sigma)$ is (ν_F, ρ_F) - IOSP if all the transitions satisfy

$$V(p_2) - V(q_2) \leq l_2^T h(q_2, l_2)\tau - \rho_F h^T(q_2, l_2)h(q_2, l_2)\tau - \nu_F l_2^T l_2\tau$$

Proposition: If there exists $\underline{\alpha}, \overline{\alpha} \in \mathcal{K}_\infty$ such that

$$\underline{\alpha}(\|x\|) \leq V(x) \leq \overline{\alpha}(\|x\|)$$

then (ν_F, ρ_F) - IOSP Passivity for a symbolic system $S_{\tau,\eta,\mu}(\Sigma)$ leads to 0-input Lyapunov stability [Passino 1991].

Practical Passivity

$S_{\tau,\eta,\mu}(\Sigma)$ is $(\varepsilon, \rho_F, \nu_F)$ - practically IOSP if all the transitions satisfy

$$V(p_2) - V(q_2) \leq l_2^T h(q_2, l_2)\tau - \rho_F h^T(q_2, l_2)h(q_2, l_2)\tau - \nu_F l_2^T l_2 \tau + \varepsilon$$

where $\varepsilon \in \mathbb{R}_0^+$.

Proposition: This definition of $(\varepsilon, \rho_F, \nu_F)$ - practically IOSP leads to

- 0-input Lyapunov stability if $\|h(q_2, 0)\|_2^2 \geq \frac{\varepsilon}{\rho_F}$.
- 0-input practical asymptotic stability, i.e.,

$$\|x_{i+1}\| \leq \beta(\|x_i\|, \tau) + \delta$$

if there exist $\alpha \in K_\infty$ such that $h^T(x, 0)h(x, 0) \geq \alpha(\|x\|)$

Preserving Passivity

Assumption 1 [Oishi 2010]

Assume that the operator from $u(t)$ to $\dot{y}(t)$ has the finite L_2 gain, γ, that is

$$\int_0^T \|\dot{y}(t)\|_2^2 \mathrm{d}t \leq \gamma^2 \int_0^T \|u(t)\|_2^2 \mathrm{d}t$$

for any $T \geq 0$ and admissible $u(t)$.

Theorem

Suppose that the original continuous-time system Σ is $(\nu,\, \rho)$ - IOSP and Assumption 1 is satisfied. Let $S_\tau(\Sigma)$ be the transition system defined by Σ. If $S_{\tau,\eta,\mu}(\Sigma)$ is ϵ - approximately bisimilar (or similar) to $S_\tau(\Sigma)$, then $S_{\tau,\eta,\mu}(\Sigma)$ is $(K(\epsilon),\, \nu_F,\, \rho_F)$ - practically IOSP where

$$\begin{aligned} \nu_F &= \nu - \tau\gamma - \tau\gamma|\rho| - \tau^2\gamma^2|\rho| \\ \rho_F &= \rho - \tau\gamma|\rho|. \end{aligned}$$

Outline of the proof

IOSP passivity of Σ leads to

$$\begin{aligned}\langle l_2, \mathbf{y}(t, q_2, l_2)\rangle - \rho\langle \mathbf{y}(t, q_2, l_2), \mathbf{y}(t, q_2, l_2)\rangle - \nu\langle l_2, l_2\rangle \\ + V(q_2) - V(\mathbf{z}(t, q_2, l_2)) \geq 0,\end{aligned}$$

it is required to prove that

$$l_2^T h(q_2, l_2)\tau - \rho_F h^T(q_2, l_2) h(q_2, l_2)\tau - \nu_F(l_2^T l_2)\tau + V(q_2) - V(p_2) \geq 0,$$

Outline of the proof

For $0 \leq t \leq \tau$ we compare $\langle l_2, \mathbf{y}(t, q_2, l_2)\rangle$ and $\tau l_2^T h(q_2, l_2)$

$$\begin{aligned} & |\langle l_2, \mathbf{y}(t, q_2, l_2)\rangle - \tau l_2^T h(q_2, l_2)| \\ \leq \quad & \tau.\gamma.\tau(l_2^T l_2) \\ \Rightarrow \quad & \langle l_2, h(\mathbf{z}(t, q_2, l_2), l_2)\rangle \leq \tau^2\gamma(l_2^T l_2) + \tau l_2^T h(q_2, l_2) \end{aligned}$$

Other comparisons:

$$|\langle \mathbf{y}(t, q_2, l_2), \mathbf{y}(t, q_2, l_2)\rangle - \tau h^T(q_2, l_2) h(q_2, l_2)|$$

$$\leq (\tau\gamma + \tau^2\gamma^2)\tau(l_2^T l_2) + \tau^2\gamma.h^T(q_2, l_2)h(q_2, l_2),$$

$$-\nu\langle l_2, l_2\rangle = -\nu.\tau(l_2^T l_2)$$

Outline of the proof

For Lipschitz continuous storage functions

$$\begin{aligned} V(p_2) &\leq V(\mathbf{z}(t, q_2, l_2)) + K(\|p_2 - \mathbf{z}(t, q_2, l_2)\|) \\ &= V(\mathbf{z}(t, q_2, l_2)) + K(\|p_2 - \mathbf{z}(t, q_2, l_2)\|) \end{aligned}$$

For δ - ISS systems

$$\|p_2 - \mathbf{z}(t, q_2, l_2)\| \leq \eta/2 \leq \epsilon$$

For δ - FC systems

$$\|p_2 - \mathbf{z}(t, q_2, l_2)\| \leq \beta(\theta, \tau) + \gamma(\mu, \tau) + \eta/2 \leq \epsilon$$

$$\Rightarrow V(p_2) \leq V(\mathbf{z}(t, q_2, l_2)) + K(\epsilon)$$

Thus, we have practical IOSP of the form

$$l_2^T h(q_2, l_2)\tau - \rho_F h^T(q_2, l_2) h(q_2, l_2)\tau - \nu_F(l_2^T l_2)\tau + K(\epsilon) \geq V(p_2) - V(q_2)$$

Future work...

- Consequences of passivity for a symbolic system for Σ [Xia 2012].
- Compositionality property for Parallel and Feedback compositions of symbolic models.
- Robustness for symbolic models.
- Verification of passivity of symbolic models.