Traveling-Salesman And Related Scheduling Problems

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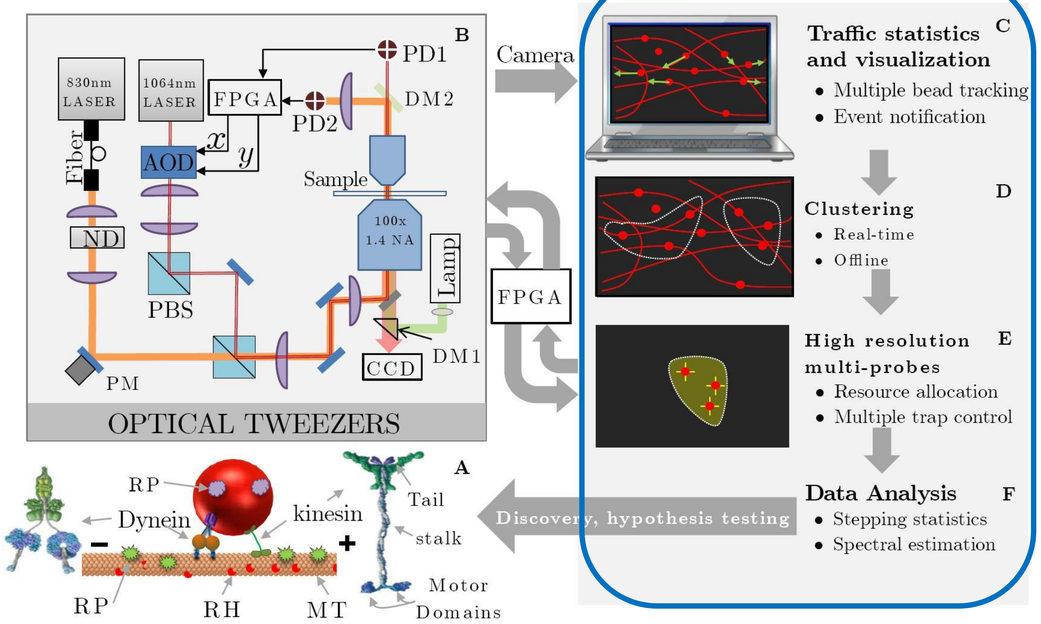
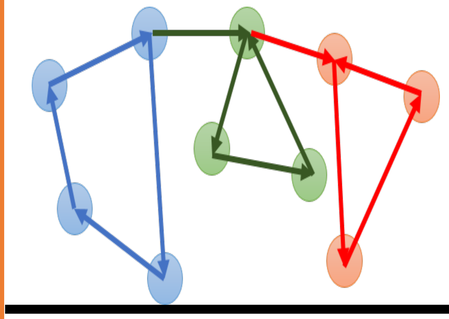
Overview



INFERENCE AND CONTROL ENGINE



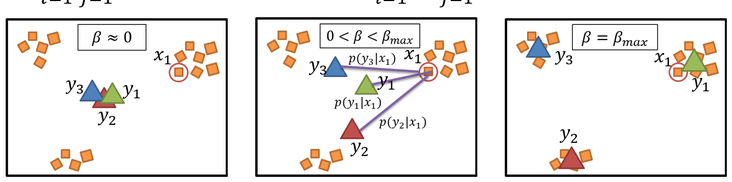
- Clustering of cargo based on measurements
- Traveling-salesman problem (TSP) with scheduling
  - High resolution sensing and manipulation resources are scarce
  - Optimal allocation needed

Deterministic Annealing (DA) algorithm

- Given N sities located at $\{x_i : 1 \le i \le N\}$, find K facility locations $\{y_j : 1 \le j \le K\}$ such that distance of a site i to its nearest facility j is minimized
- DA addresses optimal resource allocation as

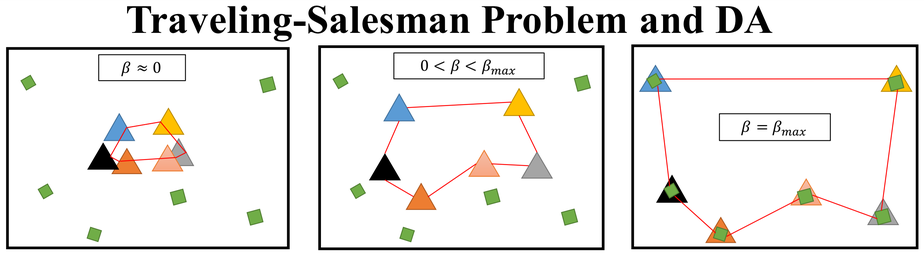
$$\min_{\{y_j, p_{j|i}\}} D - \frac{1}{\beta} H = \min_{\{y_j\}} \; -\frac{1}{\beta} \sum_{i=1}^{N} p_i \log \left(\sum_{j=1}^{K} e^{-\beta d(x_i, y_j)} \right), \text{ where,}$$

$$D = \sum_{i=1}^{N} \sum_{j=1}^{K} p_i p_{j|i} d(x_i, y_j); \quad H = -\sum_{i=1}^{N} p_i \sum_{j=1}^{K} p_{j|i} \log p_{j|i}; p_{j|i} = \frac{e^{-\beta d(x_i, y_j)}}{\sum_k e^{-\beta d(x_i, y_k)}}; p_i = \frac{1}{N}$$



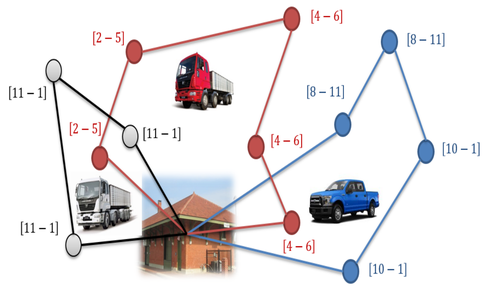
Working methodology of DA

Traveling-Salesman Problem and DA



$$\min_{\{y_j\}} \; -\frac{1}{\beta} \sum_{i=1}^{N} \log \left(\sum_{j=1}^{N} e^{-\beta d(x_i, y_j)} \right) + \left(\sum_{j=1}^{N} d(y_j, y_{j+1}) \right)$$

TSP with Scheduling



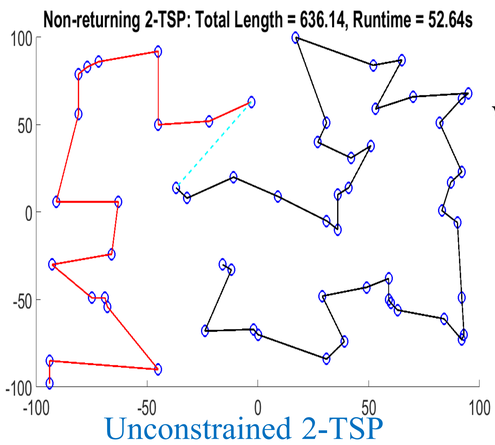
- Each customer is equipped with a service time-window $[t_i^s - t_i^f]$
- $d(x_i, y_j)$ in DA is as

$$d(x_i, y_j) = \left\| \begin{matrix} x_i^{(1)} - y_j^{(1)} \\ x_i^{(2)} - y_j^{(2)} \\ \lambda\left(0.5\left(t_i^s + t_i^f\right) - y_j^3\right) \end{matrix} \right\|^2$$

Implementation on Real and Benchmark Datasets

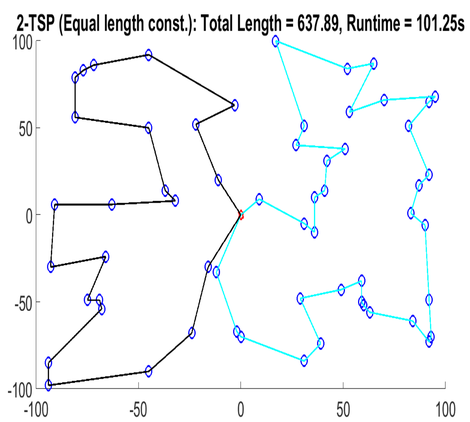
Results and Comparisons

- *Flexibility* to include multiple type constraints, such as, capacity constraints, same/different starting or end-point constraints, close-enough type constraints
- *Simultaneous* resource allocation and route-optimization (RARO)
- *Extremely fast* approach: Generates high quality solutions (within 10% of the best-known) for a 1000 cities Gehring & Homberger benchmark data within < 25 min. (as opposed to few hours to even a day for other approaches)



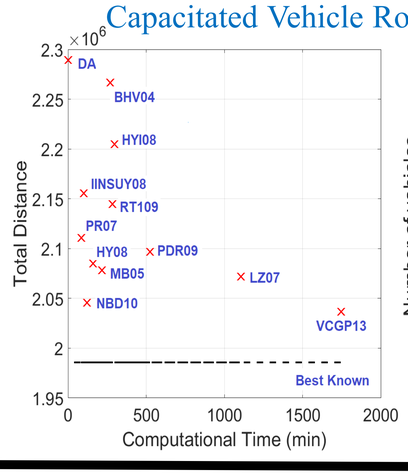
Unconstrained 2-TSP

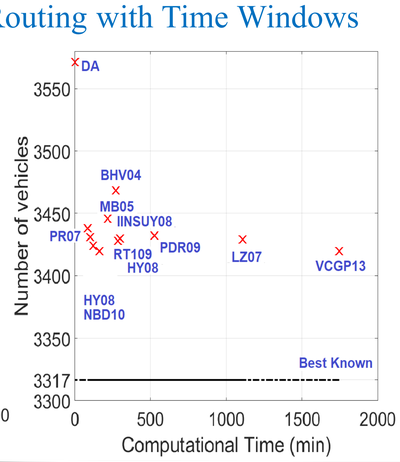
Constraint met without significant increase in tour length; simultaneous optimization & constraint satisfaction

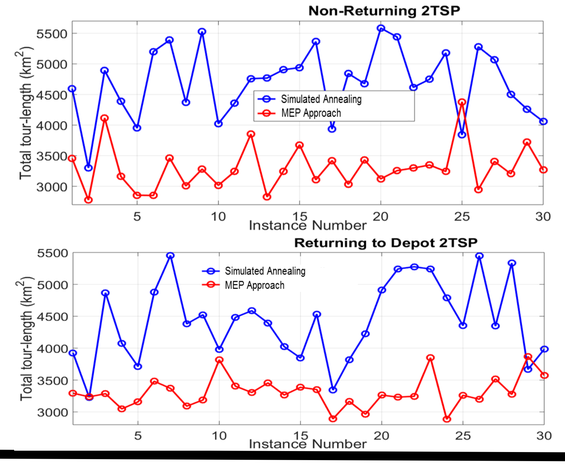


Equality Constrained 2-TSP

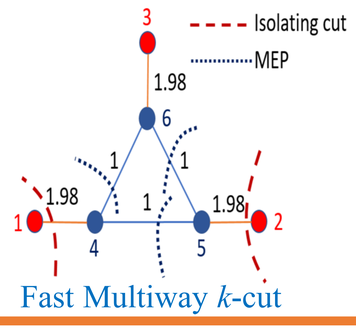
Capacitated Vehicle Routing with Time Windows



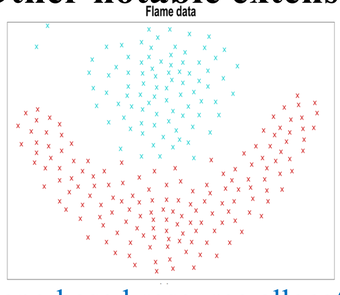




Other notable extensions



Fast Multiway k-cut



Shape-based resource allocation

$$\min_{x \in \mathbb{R}^N, z \in \{0,1\}^M} \; a^T x + b^T z$$
$$\text{s. t.,} \quad Ax + Bz \le c$$
$$x_i \ge 0 \; \forall \, i \in \{1, \ldots, N\}$$

Mixed-Integer Programs