Triggered Control of Cyber Physical Systems with Communication Channel Constraints

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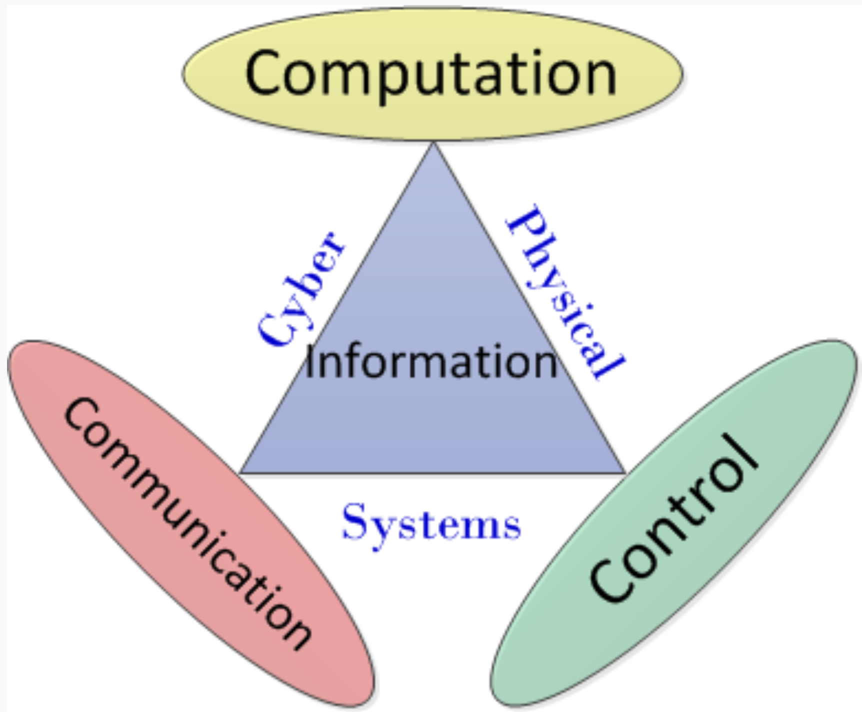
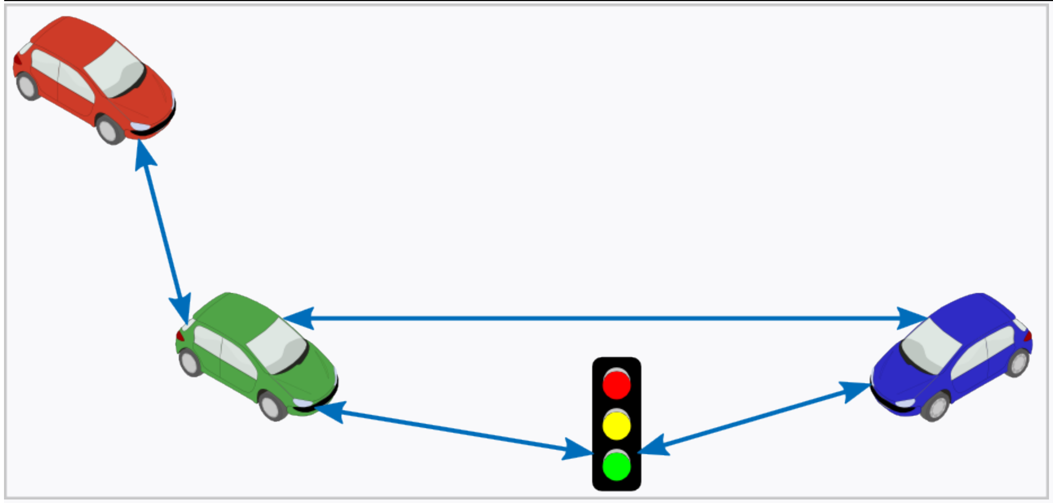
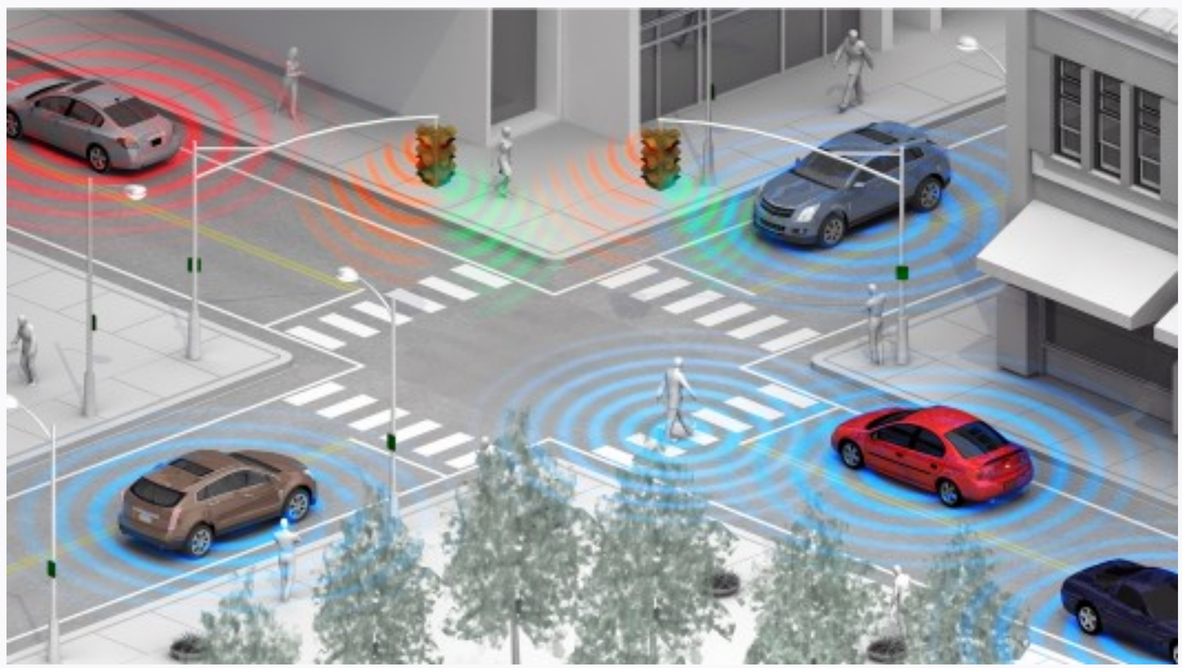
Coll. P. Tallapragada, P. Minero

Cyber Physical Systems



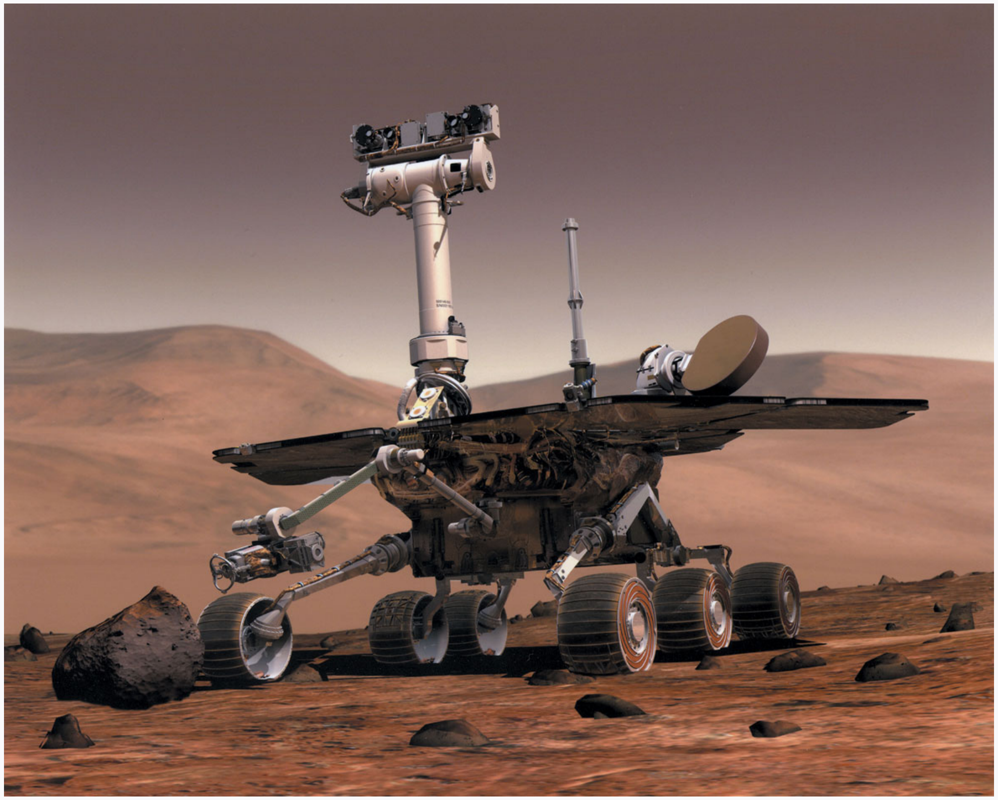
V2V and V2I Aided Control



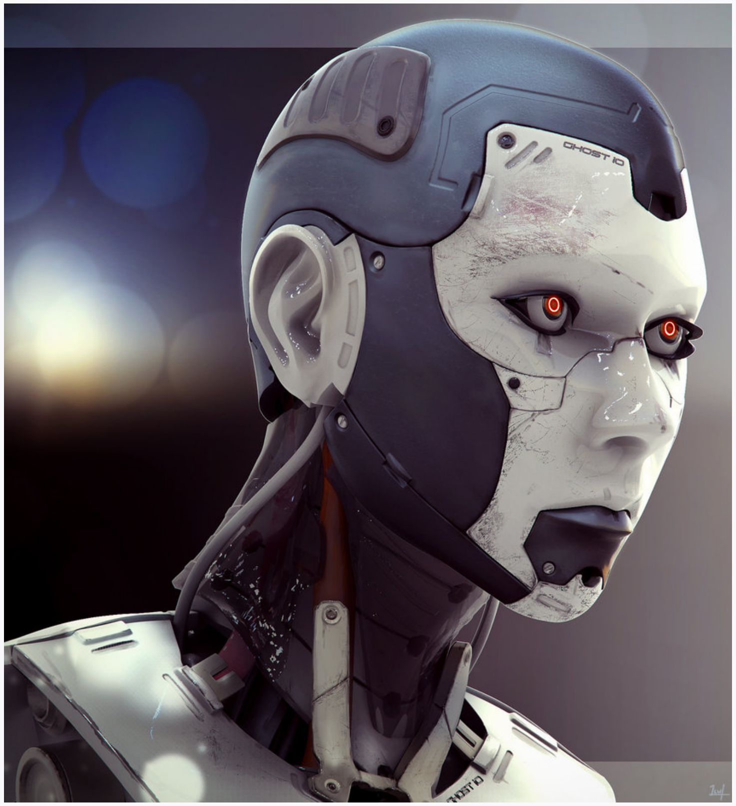


Autonomous navigation

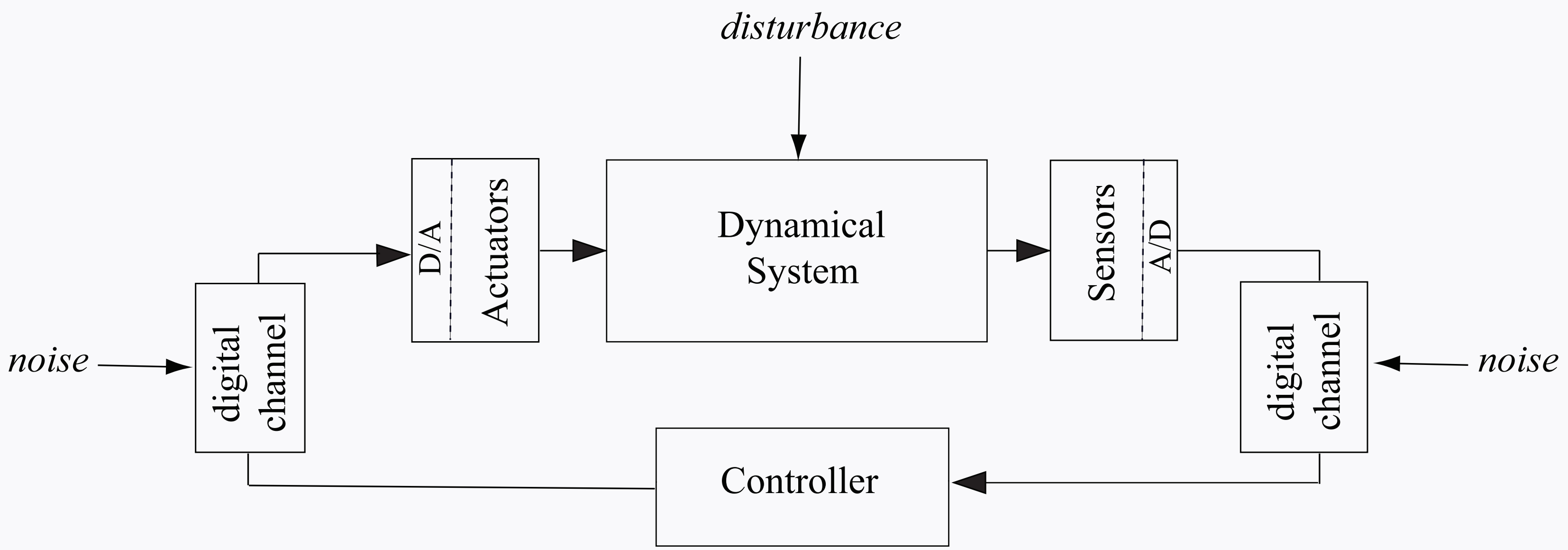
Exploratory missions

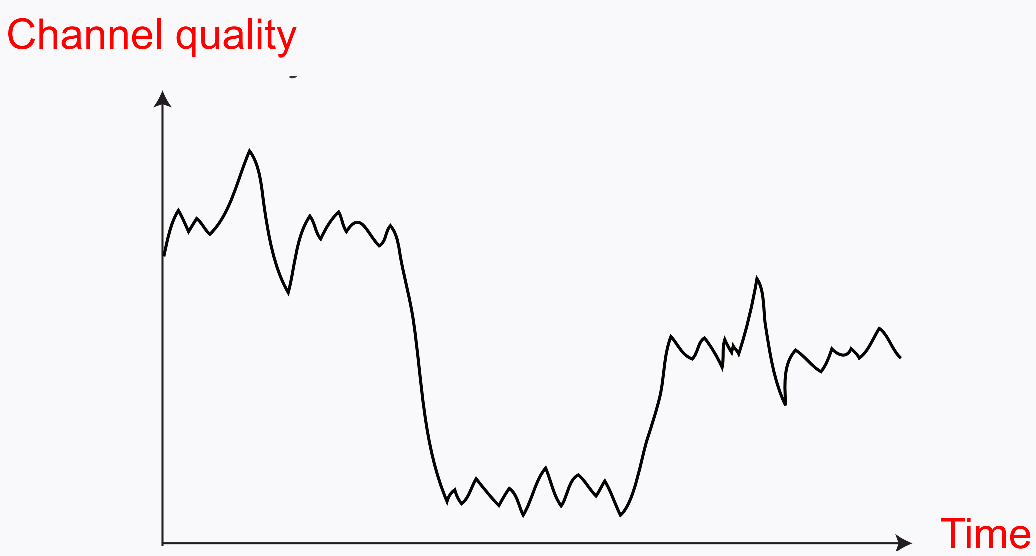


Cyborgs



Abstraction





Novel Challenges

- IT aspects: Account for quantization, rate-limitation, data losses, decoding errors. Control despite channel effects.
- AC aspects: How frequently actuate to ensure desired level of performance? Control only when needed.

- Objective of our project: Address both of these aspects.

Insufficiency of classical theory

Insufficiency of Shannon capacity (Sahai, Mitter 2006)

- Example: i.i.d. erasure channel

$$R_k \sim R = \begin{cases} r & \text{w.p. } 1-p \\ 0 & \text{w.p. } p \end{cases}$$

- Data rate theorem (Nair, Evans 2004)

$$|\lambda|^2 \mathbb{E}(2^{-2R}) < 1 \qquad \Longrightarrow \quad |\lambda|^2 (2^{-2r}(1-p) + p) < 1$$

$$\text{as } r \to \infty \;\; p < \frac{1}{|\lambda|^2}$$

- Shannon capacity

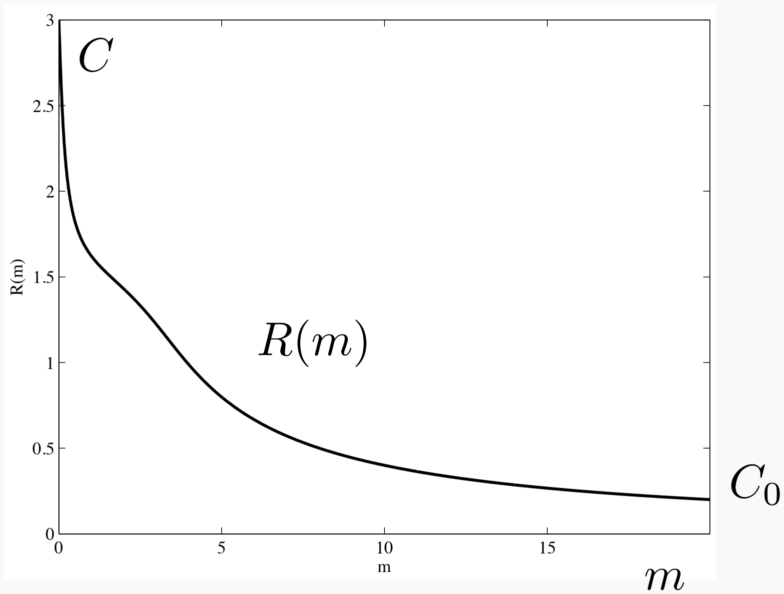
$$C = (1-p)r \to \infty$$

Some new results (Minero Franceschetti 2015)

- Studied threshold function for m-th moment stabilization of scalar systems over a large class of Markov channels
- Threshold capacity function interpolates smoothly between Shannon and zero-error capacities.
- Related to anytime capacity of Sahai and Mitter
- Allows to compute:
- Anytime capacity of r-bit, two-state Markov erasure channel
- Anytime capacity of arbitrary i.i.d. rate process, including explicit formulas for Poisson, Bernoulli, Geometric

Extremal properties

- The threshold function varies continuously between two extremal values of capacity: zero error and vanishing error



$$\lim_{m\to\infty} R(m) = C_0$$

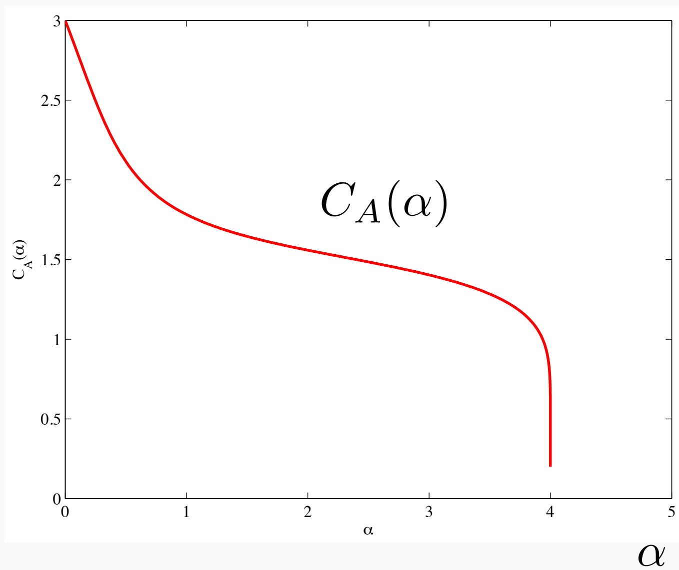
$$\lim_{m\to 0} R(m) = C$$

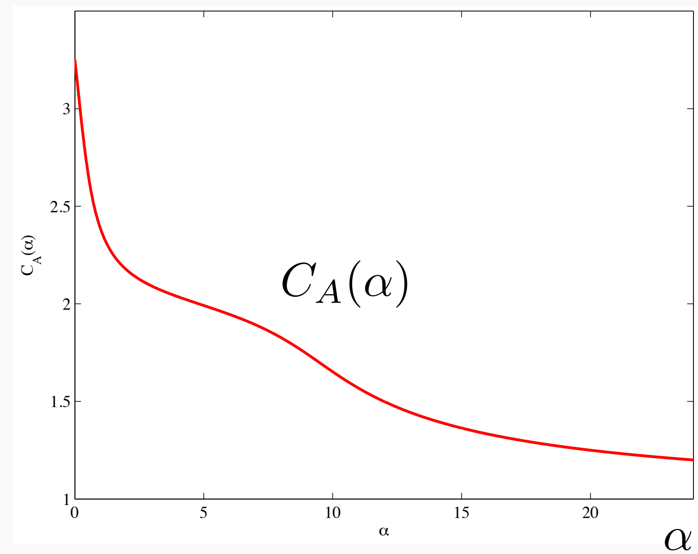
Relation with anytime capacity

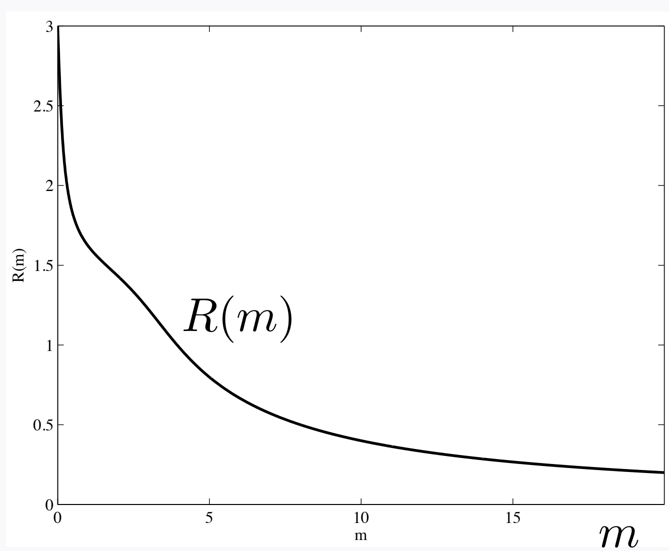
$$C_A(mR(m)) = R(m)$$

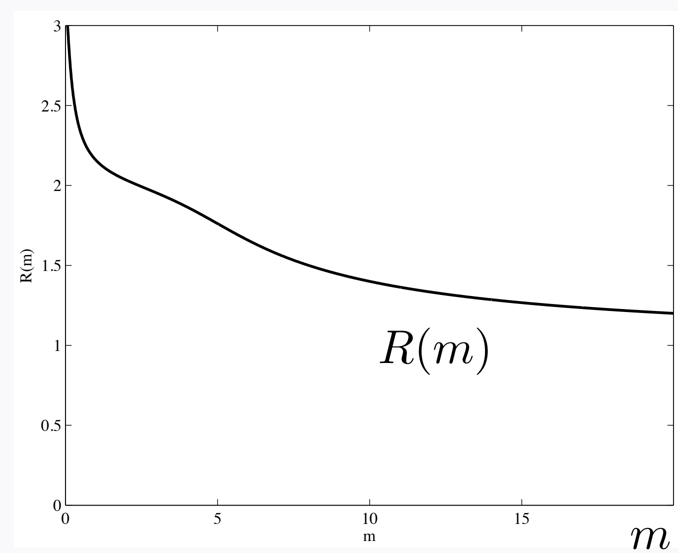
- Stability threshold function is a parametric representation of the anytime capacity
- $mR(m)$ corresponds to the anytime reliability exponent α
- If $R(m) \to C_0 > 0$ then anytime capacity has unbounded support
- If $R(m) \to C_0 = 0$ then anytime capacity has bounded support

Relation with anytime capacity





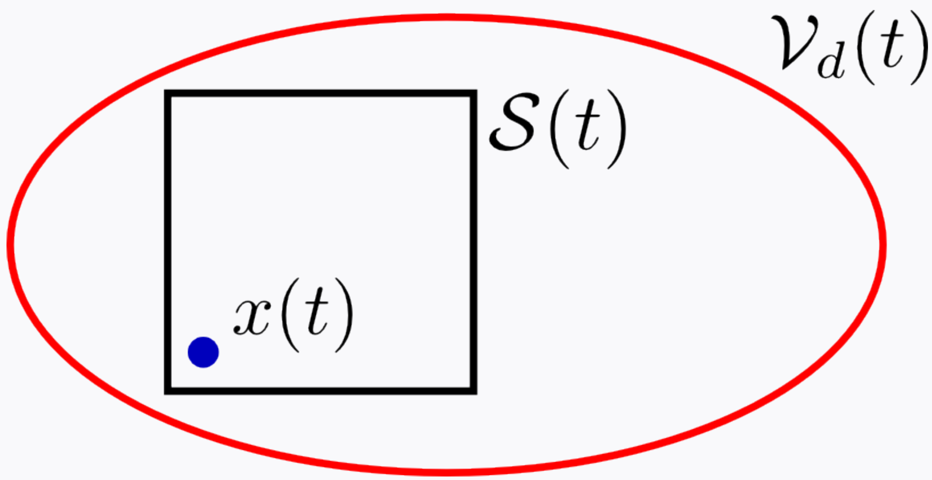




Event triggered control with rate constraints

Tallapragada, Cortes (2015)

- Continuous time
- Goal-driven transmission with performance guarantees

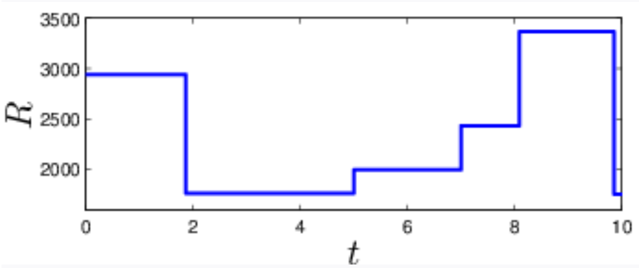


•Goal: state is contained in an exponentially shrinking sub-level set of a Lyapunov function.

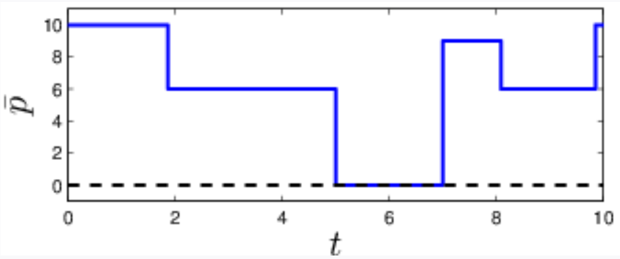
Event triggered control with time-varying rates

Tallapragada, Franceschetti, Cortes (2015)

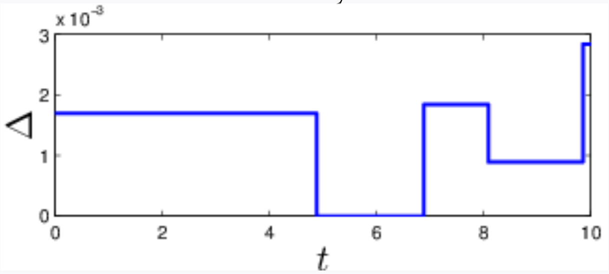
communication rate



max. packet size



delay



- Channel “Blackouts”
- Use knowledge of time evolution of the channel to decide when and what to transmit

Towards a theory of cyber-physical systems