Regional Science and Urban Economics 61 (2016) 26–37

Contents lists available at ScienceDirect

Regional Science and Urban Economics

journal homepage: www.elsevier.com/locate/regsciurbeco

Self-driving cars will change cities☆

Roman Zakharenko

National Research University Higher School of Economics, Office 3433, Shabolovka 26, Moscow 119049, Russia

ARTICLE INFO

JEL:
D61
O18
R41
R49

Keywords:
Self-driving cars
Autonomous vehicles
Commute
Parking
Urban forms

ABSTRACT

The effects of autonomous vehicles (AVs) on urban forms are modeled, calibrated, and analyzed. Vehicles are used for commute between peripheral home and central work, and require land for parking. An advantage of AVs is that they can optimize the location of day parking, relieving downtown land for other uses. They also reduce the per-kilometer cost of commute. Increased AV availability increases worker welfare, travel distances, and the city size. Land rents increase in the center but decrease in the periphery. Possible locations of AV daytime parking are analyzed. The effects of AV introduction on traffic and on mass transit coverage are discussed.

1. Introduction

The advent of self-driving cars, or autonomous vehicles (AVs henceforth) seems to be a matter of very near future. The rapidly growing literature, academic and non-academic, has discussed many virtues of AVs. They will reduce the cost of travel, especially for the disabled. They will allow minors to travel without adults present. They will relieve occupants from the burden of driving, enhancing travel experience. They will travel more safely, choose the route more optimally, and will increase highway throughput. They will be able to geographically separate themselves from their owners while not in use, in order to optimize parking costs.

The objective of this paper is to model and analyze the effects of AVs use on urban forms. In the analysis, we will focus on two relevant aspects of AVs: the lower cost of travel, relative to traditional vehicles (TVs henceforth), and the ability to optimize the location of parking.

Cars indeed require a lot of space, and TVs that cannot drive themselves typically require such space at every location their owner chooses to visit. As city centers typically accommodate a large proportion of the city jobs, currently they should also accommodate large amounts of parking capacity for those who travel to work by car. The use of land for parking crowds out other land uses, eventually leading to reduced density of economic activity. Shoup (2005, p. 130), and references therein, describe the extreme cases of such crowding out. Downtown Buffalo, New York, allocates half of its land to parking. Downtown Albuquerque, New Mexico, devotes to parking even more than half its land. And in downtown Topeka, Kansas, the share of land dedicated to non-parking uses is so small that, the author notes, there may be little reason to travel there and park.

When AVs appear on stage, cities will change dramatically. Downtowns, with parking space removed, will see an increase in the density of economic activity, causing productivity increase. Daytime parking will become more peripheral; it is possible that some locations of daytime and nighttime parking will coincide, allowing to take advantage of natural complementarity of the two types of parking and to reduce the total amount of urban land dedicated to parking.

At the same time, reduced costs of travel will make commuters accept longer travel distances in order to afford larger residences. This may increase the total amount of residential land.

This paper formulates the above ideas rigorously within a model, enabling an analysis of the effects of AVs on urban land use. The model gives answers to the following questions: when AVs emerge, where in the city will their owners work (relative to other workers) and live (relative to other residents)? How will other commute modes, such as travel by a TV or no travel at all, be affected by such innovation? What are the effects on the labor force welfare, land rents, commute distances, and traffic? Where exactly will AVs be parked during daytime? How will mass transit be affected by the new technology?

While some of the above questions can be answered unambiguously within the model, the answers to others depend on the values of model

☆ The article was prepared within the framework of the Basic Research Program at the National Research University Higher School of Economics (HSE) and supported within the framework of a subsidy granted to the HSE by the Government of the Russian Federation for the implementation of the Global Competitiveness Program.

E-mail addresses: rzakharenko@hse.ru.

http://dx.doi.org/10.1016/j.regsciurbeco.2016.09.003
Received 29 October 2015; Received in revised form 7 September 2016; Accepted 9 September 2016
Available online 15 September 2016
0166-0462/

parameters. For example, whether AVs expand or contract cities depends on whether the effect of AVs on city size through residential land demand (positive) dominates that through parking land demand (negative). To avoid the analysis of a large number of cases, some results are provided only for parameter values calibrated to a representative U.S. city.

For better analytical tractability, the main body of the paper models a city without traffic congestion and without mass transit. Section 5 discusses how each of these may be affected when AVs are introduced.

The existing literature that analyses the AV introduction from the urban economics perspective is scant. Fagnant and Kockelman (2015), in a survey of benefits and difficulties related to introduction of AVs, dedicate only one paragraph to land use issues. Zhang et al. (2015) simulate a city with random trip demand generation, to show a dramatic downward effect of shared AVs on parking demand. Their study however does not analyze how urban forms will be affected, by assuming exogenous location of travelers. Hayes (2011) points out that AVs will be able to park closer to each other, saving urban space.

Few recent policy and opinion papers also address the issue of AV effect on land use. In agreement with the model of this paper, Anderson et al. (2014) and Romem (2013) both predict that (i) by reducing travel costs, AVs will make their owners travel more, making cities larger and reducing residential density, and (ii) downtown economic activity will become more dense, due to relieved demand for central parking.

2. Model

A typical model that analyzes commuting by car and parking, e.g. Anderson and de Palma (2004), Arnott and Inci (2006), Brueckner and Franco (2015), assumes exogenous location(s) of a commuter work. Voith (1998) develops a model with endogenous size of the central business district (CBD), due to endogenous city population size; in his model however the CBD does not compete with other land uses, so the change of the CBD size (e.g. due to relieved parking demand) would not directly affect the rest of the city.

Thus, the existing models of commute are not satisfactory for our purposes, as the introduction of AVs is projected to increase the density of central economic activity and may reallocate all land uses in the city. For this reason, I develop a new model of location choice that allows to endogenize both the location of residence and the location of work.

2.1. Geography and population

Consider a monocentric two-dimensional city of a half-circular shape. At the center, there is a "port," the only communication with the rest of the world. Any other location is labeled by its distance a to the port. The measure of all available locations at radius a is, thus, πa. Throughout the paper, we refer to locations closer to the port (with smaller a) as more *central*, while the locations away from the port (with larger a) are *peripheral*. The city is illustrated in Fig. 1.

There are two goods in the economy, labeled as the "export" and the "import" good, that are exchanged one for another at the port.

The total labor force is exogenous and is equal to L.

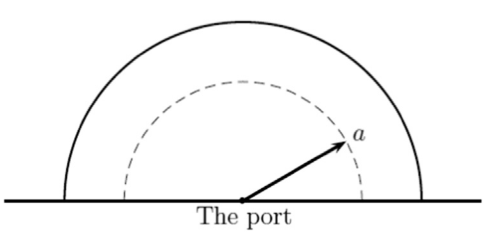

Fig. 1. The city geography.

2.2. Production

Production of the export good is done by perfectly competitive firms. Production of one unit of the export good requires one unit of labor and β units of land.

The output of the export industry is not demanded domestically and must be delivered to the port. The value of a unit of the export good at the port is ω. Transportation of the export good within the city is subject to transport cost τ per unit of distance, such that the value of the export good at a distance a from the port, net of transport costs, is $\omega - \tau a$. Thus, any production must be located within $a \in [0, \frac{\omega}{\tau})$. For simplicity, the parameter τ is exogenously given; a more sophisticated model could make τ dependent on traffic congestion.

The import good cannot be produced domestically and must be delivered from the port. It is available at the port at the normalized price of unity, and can be delivered to any location in the city at no additional cost.[1]

2.3. Preferences and consumption

The owners of labor and land are consumers who demand the import good.

Land owners demand nothing else and locate themselves at the land they own, though not occupying any of it and making all of it available for rent.

The workers also demand land for residence. The residential land is rented from the land owners. For each worker i, the preferences over import good consumption c_i and residential land s_i are determined by the Leontief utility function: $u_i = \min\left\{c_i, \frac{1}{\gamma}s_i\right\}$, with $\gamma > 0$. If the location of residence is different from the place of work, they have to commute, which incurs additional costs outlined below. Due to free mobility, worker utility must be the same for all workers, regardless of place of work and residence. The nature of the utility function also implies that both the amount of consumption and the amount of residential space are the same for all workers, too[2]:

$$u_i = c = \frac{s}{\gamma}, \quad \forall\, i, \tag{1}$$

for some $c > 0$, $s > 0$, which we use as equilibrium conditions.

2.4. Commute technology

The locations of work and residence are a matter of a workers' choice, and may or may not be the same. There are several possible commute modes available to the workers that we now outline.

2.4.1. No commute

One possible mode of commute is no commute at all, i.e. workers living and working at the same place. In this case, no additional costs are involved. I ignore the fact that some non-commuters may own a vehicle, incurring ownership costs and requiring space for parking. I assume that such costs are part of residential costs (if the vehicle is used for pleasure/family travel) or business costs (if used for business purposes). We label the workers using this mode of transportation as *non-commuters*.

[1] Adding the transport cost for the import good would further increase concentration of economic activity near the port. It would also reduce the consumer incentive to reside in the periphery.

[2] A different utility function, that allows some substitutability between consumption and residential space, would result in heterogenous workers' choices: those residing closer to the periphery would have more residential space and less consumption. While this outcome is empirically plausible, it makes the model much more cumbersome. Furthermore, it is not clear how it would contribute to understanding the impacts of autonomous vehicle technology.

2.4.2. Traditional vehicles

The TVs have to be operated by a human driver. They allow to separate the place of work from the place of residence. There are three types of associated costs:

(i) Variable cost δ_T per unit of distance of roundtrip commute. This includes both the vehicle operation cost and the opportunity cost of occupants' time in transit.
(ii) Parking cost: each car requires ν units of land, both at the location of work (for day parking) and at the location of residence (for night parking). We assume $\nu < s$ for all empirically plausible values of s. In other words, TV parking requires less space than any residence, which justifies the need to commute to/from peripheral home and to park downtown. There is a complementarity of residential (nighttime) and business (daytime) parking: the same parking spot can be used for both purposes.
(iii) Fixed cost $\rho_T > 0$ of a TV ownership, per commuter per working day.

The workers using this mode of transportation are labeled as *traditional commuters*.

2.4.3. Autonomous vehicles

While there may be many unique characteristics of autonomous vehicles, we focus on those that are likely to affect cities the most: their ability to free the user from the burden of driving, and to park themselves away from their owners. To simplify analysis, we assume that at night the AVs are always parked near their owners, while the location of daytime parking is endogenous and is optimally chosen.

Some studies, e.g. Godsmark et al. (2015), predict that most AVs will not be owned by the public but rather be used as a service, much like modern taxi. This argument, however, is less likely to apply to commuters, as most of them travel to/from work simultaneously, making multiple trips by the same vehicle less feasible. Many commuters may also choose to keep their vehicles personalized, particularly to keep some personal belongings in the vehicle, which also works against the transportation-as-a-service argument. In this paper, I assume that each vehicle is used only by one commuter. Possible sharing of AVs would reduce the fixed cost per user, as well as parking requirements per user.

The associated costs of an AV use are as follows:

(i) There are two variable costs per unit of travel distance. When an AV carries passengers, the cost is δ_A, which includes the opportunity cost of occupants' time in transit. Because the occupants no longer have to do the driving job, it is reasonable to assume $\delta_A < \delta_T$. When the car travels empty to/from a parking lot, the cost is δ_P which is even lower than δ_A, because no one's time is being spent in transit. Both δ_A and δ_P are measured per kilometer of roundtrip travel.
(ii) Parking requirement: we assume that an AV requires the same amount ν of space as a TV.[3] The location of daytime parking may be different from the location of work.
(iii) A per-capita fixed cost ρ_A of an AV ownership. Because AVs are likely to be more expensive than traditional vehicles, we assume $\rho_A > \rho_T$, and analyze various values of ρ_A ranging from infinity to ρ_T.

We label the workers using AVs as *new commuters*.

[3] It is possible that AVs will need less parking, as they can be parked closer to each other. Hayes (2011) provides a more detailed argument. At the same time, if AVs day-parking is shared with TV night-parking, space savings will not be realized, as each parking spot will have to be designed for the largest vehicle that uses it.

2.5. General considerations

Observe that the model satisfies the assumptions of the First Welfare theorem, thus no Pareto-improvement is possible in equilibrium. This property is useful for equilibrium characterization in the subsequent analysis.

3. Analysis: pre-AV equilibrium

First, we focus on the case when AVs are prohibitively expensive, so that only TVs can be used as a commute option.

3.1. Preliminaries

We now characterize several results that help to simplify the subsequent analysis.

Proposition 1. *In equilibrium, a commuter worker's place of residence a_2 is more peripheral than her place of work a_1:* $a_1 < a_2$.Intuitively, transportation costs of the export good make the workplace naturally gravitate towards the center, while no such gravity exists for the place of residence. The result holds for both traditional and new commuters. The proofs of this and all subsequent propositions are in Appendix A.

Proposition 2. *The same location cannot be used for both work and residence of traditional commuters.*

As a corollary, the potential complementarity of daytime and nighttime parking is never realized in equilibrium, and the commuters must bear the full costs of renting land for each type of parking.

3.2. Calculation of rents

We analyze an equilibrium in which there is a positive mass of traditional commuters. This implies that they occupy a continuum of locations for both work and residence.

3.2.1. Non-commuters

If a positive mass of non-commuters exist, a non-commuter that is located at a faces the budget constraint $\omega - \tau a \geq c + r(a)(\beta + s)$, held with equality in equilibrium. Thus, the rent at all locations of non-commuter presence is equal to

$$r_N\left(a\right) \equiv \frac{\omega - \tau a - c}{\beta + s}. \tag{2}$$

3.2.2. Traditional commuters

Consider a traditional commuter who works at a_1 and resides at a_2. By Proposition 1, $a_2 > a_1$. The commuter must park at a_1 during the day, and at a_2 at night, occupying ν units of space on both occasions. By Proposition 2, they must bear the full cost of land rent on both occasions. Given this information, the budget constraint of the commuter is

$$\omega - \tau a_1 - c - (\beta + \nu)r(a_1) - (s + \nu)r(a_2) - \delta_T(a_2 - a_1) - \rho_T \geq 0, \tag{3}$$

with equality in equilibrium.

Note that the left-hand side of (3) is an additively-separable function of a_1 and a_2, meaning that, in equilibrium, if a traditional commuter residing at some a_2 is indifferent between two work locations a_1, b_1, then so is a traditional commuter residing at $b_2 \neq a_2$. In other words, all locations of traditional commuter work must imply the same value of the left-hand side of (3), meaning that the rent at all work locations must be equal to

$$r_{TB}\left(a\right) \equiv \frac{\lambda_T - (\tau - \delta_T)a}{\beta + \nu}, \tag{4}$$

where λ_T is the land cost of a traditional commuter workplace (including parking) at the port. The equilibrium budget constraint (3) can now be rewritten as

$$\omega - c - \lambda_T - (s + \nu)r(a_2) - \delta_T a_2 - \rho_T = 0, \tag{5}$$

which results in the following equilibrium residential rent:

$$r_{TR}\left(a\right) \equiv \frac{\omega - c - \lambda_T - \rho_T - \delta_T a}{s + \nu}. \tag{6}$$

3.3. Allocation

The bid rent theory (Alonso et al. (1964) is a classical reference) implies that the actual type of land use at some location is the one that offers the highest rent for that location. This logic implies that the equilibrium rent at all locations satisfies

$$r(a) = \max\{r_N(a), r_{TB}(a), r_{TR}(a), 0\}. \tag{7}$$

Each element of the maximum in (7) is linear and non-increasing, therefore $r(a)$ is continuous, piecewise linear, convex, and non-increasing. Therefore, the relative location of various land uses can be determined by comparing the derivatives of land rent with respect to location: it should be increasing (becoming less negative) as we move from the center to periphery.

We assume that the model parameters are such that the slope of each element of the maximand in (7) is unique, which means that all types of land uses are located within compact zones; these zones are non-overlapping; each pair of these zones has at most one common point at their boundaries.

Prior to determining the actual allocation, we make the following observation: at point a where $r_{TB}(a) = r_{TR}(a) = r$, we have that $r_N(a) > r$. For proof, rewrite the equations $r_{TB}(a) = r$ and $r_{TR}(a) = r$ as follows:

$$\lambda_T - (\tau - \delta_T)a - (\beta + \nu)r = 0, \omega - c - \lambda_T - \rho_T - \delta_T a - (s + \nu)r = 0,$$

add them together, and solve for r to obtain $r < r_N(a)$. This result implies that commuter work and residence do not share a common boundary and must be separated by a non-commuter zone. Furthermore, because each commuter's work is more central than residence according to Proposition 1, we conclude that commuter work is the most central land use, non-commuter zone in the middle, and commuter residence is the most peripheral. Due to convexity of equilibrium rent, this conclusion also implies $|r'_{TB}| > |r'_N| > |r'_{TR}|$, which further implies

$$\frac{\tau}{\delta_T} > \frac{\beta + s}{s - \nu}, \tag{8}$$

for all equilibrium values of s, as a necessary condition of commute existence.

In the commuter work zone, since each worker requires $\beta + \nu$ units of land, the density of such workers must be equal to $\frac{1}{\beta+\nu}$. Likewise, the density of non-commuters in their zone is $\frac{1}{\beta+s}$, while the density of commuter residents in the appropriate zone is $\frac{1}{s+\nu}$.

3.4. Equilibrium

The characterization of the equilibrium consists of (i) amount of residential space s, (ii) dollar-valued variables c, λ_T, and (iii) vector $\mathbf{a} \equiv \{a_{TB}, a_N, a_{TR}\}'$ of location variables, representing the outer boundaries of the three zones. The values of the unknowns are captured by the following moments:

- Consumption – residential space balance

$$s - \gamma c = 0. \tag{9}$$

- Demographic constraint stating that the mass of all residents equals the population size,

$$\frac{\pi}{2}\frac{a_{TR}^2 - a_N^2}{s + \nu} + \frac{\pi}{2}\frac{a_N^2 - a_{TB}^2}{\beta + s} = L. \tag{10}$$

- The balance of commuter work and residence zones:

$$\frac{\pi}{2}\frac{a_{TR}^2 - a_N^2}{s + \nu} - \frac{\pi}{2}\frac{a_{TB}^2}{\beta + \nu} = 0 \tag{11}$$

- Rent continuity at zone boundaries: $r_N(a_{TB}) = r_{TB}(a_{TB})$, $r_{TR}(a_N) = r_N(a_N)$, $r_{TR}(a_{TR}) = 0$.

3.5. Calibration

We calibrate the exogenous model parameters to match a representative US city, as follows. The parking land requirement is borrowed from Manville and Shoup (2005, Table 5) who assume 325 spaces per hectare of land, or $\nu = 10,000/325 = 30.77$ square meters per vehicle.

To calibrate β, interpreted as a size of a workplace, we again refer to Manville and Shoup (2005, Table 5) who provide estimates of how much central-business-district (CBD) land is dedicated to parking in various cities around the world. We assume that all remaining CBD land is dedicated to production (including access roads and other infrastructure). Of 44 listed cities, 13 are in the United States; we take the median of those cities, Denver, as representative of the United States. The calculated share of parking land in Denver CBD is $\frac{\nu}{\beta+\nu} = 33\%$, thus $\beta = 61.54$ square meters.

The commute costs estimates are taken from USDoT 2016 National Transportation Statistics, Table 3–17. We use the 2009 data, to synchronize with other data sources described below. The average fixed cost of vehicle ownership is estimated at \$5976 per year. Note that a vehicle used for commute can also be used for other purposes, thus only a fraction of the above fixed cost can be attributed to commute. The precise fraction is unknown; we assume 50%, or \$2988, or $2988/251 = 11.90$ dollars per working day.[4] Next, we convert the per-vehicle measures into the per-person measures by employing the 2009 National Household Travel Survey estimate (Santos et al., 2011, Table 16) that commute vehicles had an average occupancy of 1.13 persons, thus the estimated fixed cost of commute is $\rho_T = 11.90/1.13 = \$10.53$.

USDoT 2016 National Transportation Statistics, Table 3–17 also estimates that the average variable cost of vehicle use is \$0.1674 per mile, or $0.1674/1.13 = \$0.1481$ per occupant per mile. We also add the opportunity cost of time spent in transit, assumed to be equal to the average wage of \$22.21 per hour in 2009 (from the Bureau of Labor Statistics). Santos et al. (2011, Table 27) estimate the average commute speed at 28.87 miles per hour, thus the time opportunity cost is 22.21/28.87=\$0.7690 per mile of travel, and the two costs combined are 0.1481+0.7690=\$0.9171 per mile of travel. Lastly, we convert miles into kilometers and account for the fact that there are usually two commute trips per day, to arrive at $\delta_T = 0.9171/1.6093 \times 2 = \$1.1397$ per kilometer of roundtrip commute.

The remaining four unknowns ω, τ, L, γ are calibrated by matching four moments generated by the model to data. The four moments are described below:

- Fraction of labor force commuting (by automobile). The value

[4] Santos et al. (2011, Table 24) estimate that 27.77% of all vehicle-miles traveled can be attributed to commute. At the same time, a large proportion of vehicles may never be used for commute and should not be included into our calculations; we focus only on vehicles that were purchased with the purpose of commute.

predicted by the model, $\frac{\pi}{2}\frac{a_{TB}^2}{L(\beta+\nu)}$, is matched to the 2009 empirical estimate of 0.861 (McKenzie and Rapino, 2011). We ignore other commute methods and assume the rest of population works from home.

- Average commute distance. The model prediction, $\frac{2}{3}\frac{a_{TR}^3-a_N^3}{a_{TR}^2-a_N^2}-\frac{2}{3}a_{TB}$, is matched to the 2009 national average of 12.09 miles, or 19.46 km (Santos et al., 2011, Table 27).
- Equilibrium size of residential lot. The 2009 American Housing Survey (U.S. Census Bureau, 2011, Tables 1–3) reports that the median residential lot size is 0.26 acres, or 1052 square meters. With 130.11 million housing units and 157.98 million workers, that is equivalent to 866.41 square meters of residential land per worker. This typically includes the residential parking spot, thus $s = 866.41 - 30.77 = 835.64$.
- Average consumer expenditure per worker. Besides consumption c itself, we assume that it includes commute and housing costs: $c + 0.861\left(\delta_T \times 19.46 + \rho_T\right) + \frac{1}{L}\int_{a_{TB}}^{a_N}\pi a\frac{s}{\beta+s}r_N\left(a\right)da + \frac{1}{L}\int_{a_N}^{a_{TR}}\pi a r_{TR}\left(a\right)da$. Its empirical counterpart is $9.847 \times 10^{12}/(157.98 \times 10^6)/251 = 248.33$ dollars per worker per workday, where the 9.847 trillion dollar figure is the Worldbank estimate of the US final household consumption expenditure in 2009.

These moments, as well as equilibrium conditions of Section 3.4 yield the following calibrated values of exogenous and endogenous model parameters: $\mathbf{a} = \{a_{TB}, a_N, a_{TR}\}' = \{9.84, 15.78, 34.04\}'$ kilometers, L=1.92 million workers, $\omega = 285.45$ dollars per worker per day, $\tau = 3.53$ dollars per kilometer, c=208.26 dollars per worker per day, $\lambda_T = 27.86$ dollars per central workplace per day, and γ=4.01 square meters of residential lot size per dollar of daily consumption.

4. Analysis: introduction of AVs

We now introduce the autonomous vehicle technology into the model.

4.1. Preliminaries

While Proposition 1 remains relevant for the new commute technology, Proposition 2 is not. The following two propositions are given instead.

Proposition 3. *If a commuter day - parks at her location of work, that parking space is not used at night.*Note that traditional commuters must day-park at their work location, thus the proposition is relevant for all of them.

Proposition 4. *The location of parking of a vehicle is never more central than the location of its owner.*

4.2. Location of day parking

Proposition 4 leaves three possibilities of where autonomous vehicles can be day-parked, which we explore separately.

4.2.1. Type I: Parking near owner's workplace

One possibility is that AVs are day-parked near owner's workplace, much like TVs. By Proposition 3, such parking lots are vacant at night, and the rent bid by new commuters of this type is analogous to that of traditional commuters (cf. (4))

$$r_{ABP}\left(a\right) \equiv \frac{\lambda_{AP} - (\tau - \delta_A)a}{\beta + \nu}, \tag{12}$$

for some λ_{AP}.

4.2.2. Type II: Dedicated parking belt

If the location of daytime parking a_3 is separated from the location of work a_1, with $a_3 > a_1$ by Proposition 4, there may be a special zone, “parking belt”, were AVs are stored during the day and which remains empty at night. The budget constraint of this type of commuter is

$$\omega - \tau a_1 - c - \beta r(a_1) - \nu r(a_3) - (s+\nu)r(a_2) - \delta_A(a_2 - a_1) - \delta_P(a_3 - a_1) - \rho_A \geq 0, \tag{13}$$

with equality in equilibrium. If there is a continuum of commuters of this type, they must be indifferent between multiple locations a_1 of work and of daytime parking a_3. Thus, the rent bid for locations of work must satisfy

$$r_{AB}\left(a\right) \equiv \frac{\lambda_A - (\tau - \delta_A - \delta_P)a}{\beta}, \tag{14}$$

for some λ_A. The rent bid for locations of parking is then

$$r_P\left(a\right) \equiv \frac{\lambda_P - \delta_P a}{\nu}, \tag{15}$$

for some λ_P.

4.2.3. Type III: Parking at residential lots

AVs may travel all the way back to commuter residential zones, to reap the benefits of daytime and nighttime parking complementarity. Because traditional commuters have higher costs of travel, we show below that they will live closer to the center, and thus new commuters will demand day parking near traditional commuter residences before demanding such parking near their own residences.

In this case, while the rent bid for AV daytime parking can still be described by (15), the costs of nighttime residential parking are reduced by the same amount, so the budget (5) of those traditional commuters who share nighttime parking with type-III daytime parkers must be rewritten as follows:

$$\omega - c - \lambda_T - sr(a_2) - \nu(r(a_2) - r_P(a_2)) - \delta_T a_2 - \rho_T = 0,$$

which, together with (15) yields the following rent bid (jointly by residents and by type-III parkers) at such locations:

$$r_{TRP}\left(a\right) \equiv \frac{\omega - c - \lambda_T + \lambda_P - \rho_T - (\delta_T + \delta_P)a}{s+\nu}. \tag{16}$$

If the population of traditional commuters is small, while the demand for type-III daytime parking is large, the latter may spill over into the new commuter residential zone. The rent bid, joint by residents and by daytime parkers, at such locations can be derived by analogy with (16):

$$r_{ARP}\left(a\right) \equiv \frac{\omega - c - \lambda_A - \rho_A - (\delta_A + \delta_P)a}{s+\nu}, \tag{17}$$

if the residents are of type-II or type-III. For residents of type-I, the parameter $-\lambda_A$ is replaced by $-\lambda_{AP} + \lambda_P$.

4.3. Residential rents

If a new commuter's residential parking is shared with someone's type-III daytime parking, their residential rent bid (joint with daytime occupants) is given by (17). Otherwise, if the residential parking remains empty during the day, the budget (13) of type-II,III new commuters residing at a can be rewritten as

$$\omega - c - \lambda_A - \lambda_P - (s+\nu)r(a) - \delta_A a - \rho_A = 0, \tag{18}$$

which implies the following rent bid for new commuter residence:

$$r_{AR}\left(a\right) \equiv \frac{\omega - c - \lambda_A - \lambda_P - \rho_A - \delta_A a}{s+\nu}. \tag{19}$$

For type-I commuters residing at a and not sharing their residential parking, the constant $\lambda_A + \lambda_P$ in (19) is replaced by λ_{AP}.

4.4. Allocation

This section explores where the new types of land use are located, relative to existing types and relative to each other. We exploit the same argument as in Section 3.3, that the equilibrium rent is piecewise-linear, decreasing, continuous, and convex.

Because $\delta_A < \delta_T$, it immediately follows that the slope of $r_{AR}(\cdot)$ is flatter (closer to zero) than that of $r_{TR}(\cdot)$ (cf. (6) and (19)), and therefore new commuter residence, without shared parking, is the most peripheral land use.

The same inequality $\delta_A < \delta_T$ also implies that the slope of $r_{ABP}(\cdot)$ is steeper (more negative) than that of $r_{TB}(\cdot)$ (cf. (4) and (12)), therefore type-I new commuters (those who day-park near work), if they exist, must work more centrally than traditional commuters.

Next, note that the residential rent bid r_{ARP} or r_{AR} (cf. (17) and (19)) paid by type-I new commuters differs from that of type-II,III only by the constant, λ_{AP} for type-I and $\lambda_A + \lambda_P$ for type-II,III. This means that a type with a larger constant makes a smaller residential rent bid and cannot exist in equilibrium. The proposition that follows characterizes the sufficient condition for exclusion of type-I new commuters.

Proposition 5. *Type-I new commuters cannot be present in equilibrium if*

$$\delta_P < \frac{\nu}{\beta + \nu}\left(\tau - \delta_A\right). \tag{20}$$

Note that the condition (20) is sufficient but not necessary for exclusion of type-I commuters: violation of (20) would exclude type-II commuters, but type-III could still dominate type-I. Also note that, if (20) is the case, we have that $|r'_{AB}(\cdot)| > |r'_{ABP}(\cdot)| > |r'_{TB}(\cdot)|$ and therefore the workplace of surviving type-II,III new commuters is more central than traditional commuter workplace. This is because new commuters have lower opportunity cost of travel and thus accept longer travel distances between residence and work.

4.5. Calibration, continued

The introduction of AVs greatly expands the number of possible land use types. In particular, different model parameters result in different locations of daytime AV parking. Instead of analyzing the relative position of all land uses, for all possible model parameters, we return to calibrated model of Section 3.5 to refine land use types to empirically plausible ones.

First, we calibrate the AV-specific parameters ρ_A, δ_A, δ_P. The cost of empty AV travel is assumed to be equal to the 2009 variable (running) cost of vehicle use (see Section 3.5) of 0.1481 dollars per user per mile of one-way empty travel, equivalent to $\delta_P = 0.1481/1.6093 \times 2 = 0.1841$ dollars per user per kilometer of roundtrip empty travel. The cost of travel with passengers is the vehicle running cost plus the opportunity cost of time, which we assume to be equal to 50% of the average wage. Thus, the cost of one-way mile is $0.1481 + 0.7690/2 = \$0.5326$, which is equivalent to $\delta_A = 0.5326/1.6093 \times 2 = \$0.6619$ per kilometer of roundtrip travel.

With this calibration, (20) is indeed verified so type-I new commuters (those day-parking near work) can be excluded.

The calibrated model implies $|r'_{TB}(\cdot)| > |r'_P(\cdot)| > |r'_N(\cdot)|$, therefore type-II commuters, if they exist, would day-park between the zones of work of traditional commuters and non-commuters. The calibration also implies $|r'_N(\cdot)| > |r'_{TRP}(\cdot)|$, therefore type-III commuters, if they exist, would day-park just outside of the non-commuter zone. Note that $|r'_N(\cdot)|$ and $|r'_{TRP}(\cdot)|$ depend on the endogenous parameter s. Introduction of AVs causes s to increase, which does not compromise the above inequalities. Therefore, the relative position of land uses is preserved as the city evolves.

The value of ρ_A, the fixed “standing” cost of AV ownership is equal to infinity at present time, as AVs are not yet on the market. In the analysis below, we study the effects of ρ_A decreasing from a prohibitively high value down to $\rho_A = \rho_T$, when TVs are fully replaced by AVs.

Define the *prohibitive cost* of an AV as the value of ρ_A that makes the new commuter budget (13) clear, assuming that the share of new commuters in the population is zero. The latter assumption implies that new commuter work is at the port, $a_1 = 0$, while new commuter residence is at the city edge, $a_2 = a_{TR}$. The location of parking depends on its type: for type-II parkers, it is at a_{TB} and costs $\nu r(a_{TB})$, while for type-III parkers, it is at a_N (further away) but is free. We calculate the total cost of daytime parking (empty travel from port to location, plus parking fees) to be \$3.27 for type-II parkers and \$2.90 for type-III, and thus conclude that type-III parking will be preferred. Then, the prohibitive cost of AV is $\rho_A = \omega - c - \beta r(0) - \delta_A a_{TR} - \delta_P a_N = 33.17$ dollars per worker per day, or 315% of the analogous TV cost, ρ_T.

4.6. Equilibrium

The above analysis has found that, in an equilibrium with new commuters, various land uses, if they exist, will be located in the following order, from most central to most peripheral:

1. new commuter work (rent $r_{AB}(\cdot)$, outer zone boundary b_{AB});
2. traditional commuter work ($r_{TB}(\cdot)$, b_{TB});
3. type-II parking belt ($r_P(\cdot)$, b_P);
4. non-commuter work and residence ($r_N(\cdot)$, b_N);
5. traditional commuter residence combined with type-III parking ($r_{TRP}(\cdot)$, b_{TRP});
6. other traditional commuter residence ($r_{TR}(\cdot)$, b_{TR});
7. new commuter residence ($r_{AR}(\cdot)$, b_{AR}).

Some of the above mentioned zones may be absent at some stages of the city evolution. When the population of traditional commuters becomes small enough, item 6 is replaced by new commuter residence with type-III parking and with rent $r_{ARP}(\cdot)$.

The characterization of equilibrium conditions is expanded from Section 3.4. For brevity, we consider only the case where all above enumerated land uses are present. The equilibrium consists of (i) amount of residential space s, (ii) dollar-valued variables c, λ_T, λ_A, λ_P, and (iii) outer zone boundaries enumerated above. These unknowns are found from the following moments:

- Consumption – residential space balance (9).
- Demographic constraint:

$$\frac{\pi}{2}\frac{b_{AR}^2 - b_N^2}{s + \nu} + \frac{\pi}{2}\frac{b_N^2 - b_P^2}{\beta + s} = L. \tag{21}$$

- The balance of traditional commuter residence and work zones:

$$\frac{\pi}{2}\frac{b_{TR}^2 - b_N^2}{s + \nu} - \frac{\pi}{2}\frac{b_{TB}^2 - b_{AB}^2}{\beta + \nu} = 0. \tag{22}$$

- The balance of new commuter residence and work zones:

$$\frac{\pi}{2}\frac{b_{AR}^2 - b_{TR}^2}{s + \nu} - \frac{\pi}{2}\frac{b_{AB}^2}{\beta} = 0. \tag{23}$$

- The balance of new commuter parking and work zones:

$$\frac{\pi}{2}\frac{b_{TRP}^2 - b_N^2}{s + \nu} + \frac{\pi}{2}\frac{b_P^2 - b_{TB}^2}{\nu} - \frac{\pi}{2}\frac{b_{AB}^2}{\beta} = 0. \tag{24}$$

- Rent continuity at zone boundaries: $r_{TB}(b_{AB}) = r_{AB}(b_{AB})$,

$r_P(b_{TB}) = r_{TB}(b_{TB})$, $r_N(b_P) = r_P(b_P)$, $r_{TRP}(b_N) = r_N(b_N)$, $r_{TR}(b_{TRP})= r_{TRP}(b_{TRP})$, $r_{AR}(b_{TR}) = r_{TR}(b_{TR})$, $r_{AR}(b_{AR}) = 0$.

4.6.1. Initial impact of AVs: central city only

When the cost ρ_A of AVs is prohibitively high, $\rho_A \geq 33.17$, we have that $b_{AB} = 0, b_{TB} = b_P = a_{TB}, b_N = b_{TRP} = a_N, b_{TR} = b_{AR} = a_{TR}$, parameters c, λ_T, s are equal to values calibrated in Section 3.5, $\lambda_A = \frac{\beta}{\beta+\nu}\lambda_T$. As Section 4.5 has found, type-III parking is initially strictly preferred to type-II, thus the first new commuter would park at $b_{TRP} = a_N$ for free, thus $\lambda_P = \delta_A a_N$.

What happens when ρ_A changes from the prohibitively high value by a marginal amount $-\Delta$? Clearly, as all moments describing the equilibrium are continuous in all endogenous model parameters, such parameters should change from their pre-AV levels by a marginal amount, too. Because initially we have $b_{AB} = 0$ (i.e. the amount of central land for new commuters is zero), the increase of this parameter by some infinitesimal db_{AB} would change the mass of new commuters by $\frac{\pi}{2}\frac{db_{AB}^2}{\beta} = \frac{\pi}{2}\frac{2b_{AB}db_{AB}}{\beta} = 0$. Given that, we can show that all other location variables would remain unchanged, and so would c, λ_T, λ_P. To preserve $r_{AR}(b_{AR}) = 0$, we have that $d\lambda_A = \Delta$; to satisfy $r_{AB}(db_{AB}) = r_{TB}(db_{AB})$, we have that $db_{AB} = \Delta\frac{\beta+\nu}{\beta(\delta_T-\delta_A-\delta_P)+\nu(\tau-\delta_A-\delta_P)}$.

4.6.2. Further increase in AV availability

Due to nonlinear nature of the system describing the equilibrium, obtaining fully analytical results is a tedious task, and we turn to numerical analysis. As ρ_A decreases down to ρ_T, four different patterns of daytime AV parking can be distinguished.

(i) As calibrated in the Section 4.5, type-III parking is initially strictly preferred, and the type-II parking belt is not present in equilibrium.

(ii) Type-III parking makes economic sense only if it is relatively proximate to the city center. Because the density of parking in the residential area is low, proximate parking spots fill up quickly and empty travel distances rapidly increase, increasing λ_P, the cost parameter of type-III parking. At $\rho_A = 29.89$, or 284% of ρ_T, the rent bid for type-II daytime parking at b_{TB} becomes competitive. As ρ_A continues to decrease, the two types of AV daytime parking coexist.

(iii) As the number of traditional commuters decreases, their residential area eventually becomes too small to accommodate all type-III AV parking. At $\rho_A = 14.04$, or 133% of ρ_T, all residence of traditional commuters is used for AV daytime parking, and further decrease in ρ_A results in such parking sprawling into the new commuter residences. By this time, however, only about 3.4% of all daytime parking is type-III.

(iv) At $\rho_A = 13.25$ (126% of ρ_T), traditional commuters become fully extinct. By this time, new commuters constitute 91.9% of the workforce. Only 3.1% of their daytime parking is type-III, the rest is type-II. Further decline in ρ_A increases the labor share of new commuters, city size, and the share of type-II daytime parking.

4.7. Results

This section illustrates how various city parameters are affected by introduction of AVs. Fig. 2 illustrates how zone boundaries will evolve following the decline of ρ_A. The four equilibrium patterns for different values of ρ_A, described in Section 4.6.2, are visible. The city radius increases by 3.5%, expanding the city land area by 7.1%. Total residential land increases proportionately with consumption, by 7.6%. For parking land, there are two effects: (i) overall increase in the number of commuters and vehicles increases demand for parking, and (ii) the fact that some parking land is used more efficiently, during both day and night, decreases the demand. I find that the overall effect is increase by 7.4%, due to small proportion of parking being shared by day and night parkers.

The evolution of commute distances is shown in Fig. 3. Initially, new commuters work at the port and live at the city edge, and thus have the maximal commute distance. As their community grows, their zones of work and residence expand towards each other, reducing the commute distance. The same force pushes traditional commuter zones of work and residence closer to each other, reducing their average commute distance as well. The third line shows average commute distance for all residents (including those who do not commute at all). The advent of AVs increases this distance by 13.9%, due to (i) increased city size and (ii) increased percentage of commuting labor force.

Fig. 4 compares land rents, as a function of distance to the port, under two scenarios: (i) before introduction of AVs, and (ii) when the cost ρ_A of AVs is equalized to ρ_T. At the port, the rent increases by 34% due to more productive land use. Outside of the center, the decline of traditional commute causes the rent decline: at a_{TB}, the initial outer edge of commuter work, it decreases by 39%; at a_N, the initial inner edge of commuter residence, the rent is down by 42%. Because the city expands, the rent at a_{TR}, the initial city edge, increases from zero to a small positive value.

5. Extensions

5.1. Traffic congestion

The model allows to calculate the amount of commuter traffic at every location, and to compare it across various equilibria. Traffic at a location of distance a from the port is calculated as the total number of commuters who live beyond a, but work closer than a, divided by the measure πa of all locations on the semicircle of radius a. We calculate the traffic, as a function of distance to port a, under two extreme scenarios: (i) prohibitive cost $\rho_A = \infty$ of AV, and (ii) AV cost equal to TV cost, $\rho_A = \rho_T$. For the latter scenario, we also show the traffic of empty AVs to/from their parking locations. The results are shown in Fig. 5. The maximum traffic occurs at the outer edge of commuter work zone, a_{TR} under the first scenario, when all commuters are traditional, and b_{AR} under the second, when all commuters are new. The maximum traffic under the second scenario is thus closer to the center; it is also higher by 28% due to larger number of commuters traveling to a more compact zone.

It is very possible that, while traffic increases, the same cannot be said about congestion. van den Berg and Verhoef (2015) survey the engineering literature on autonomous vehicles to conclude that automation of vehicles may increase road capacity anywhere from 7% to 200%, with a large positive effect of communication and cooperation between vehicles. Thus, the traffic increase will likely not be complemented with congestion increase, at least in American cities where most commute is already done by car, and where the existing road capacity is high.

Define the "main" traffic flow as that from residence to work in the morning, from work to residence in the evening. Define the "reverse" traffic flow as that of empty AVs from center to peripheral parking in the morning, vice versa in the evening. As long as traditional and new commute coexist, the main traffic flow will be higher than the reverse traffic flow. This is because the main traffic will consist of both AVs and TVs, while the reverse traffic will be made of empty AVs only.

This means that, if the government still opts for traffic control, it will suffice to control the main traffic only. Particularly, if control is done by means of congestion fees, the reverse (empty AV) traffic should optimally be allowed to travel for free. This may sound counter-intuitive, as modern policies tend to discourage travel of vehicles with few passengers. But charging the reverse traffic will lead to suboptimal (more central) AV parking, crowding out businesses and eventually

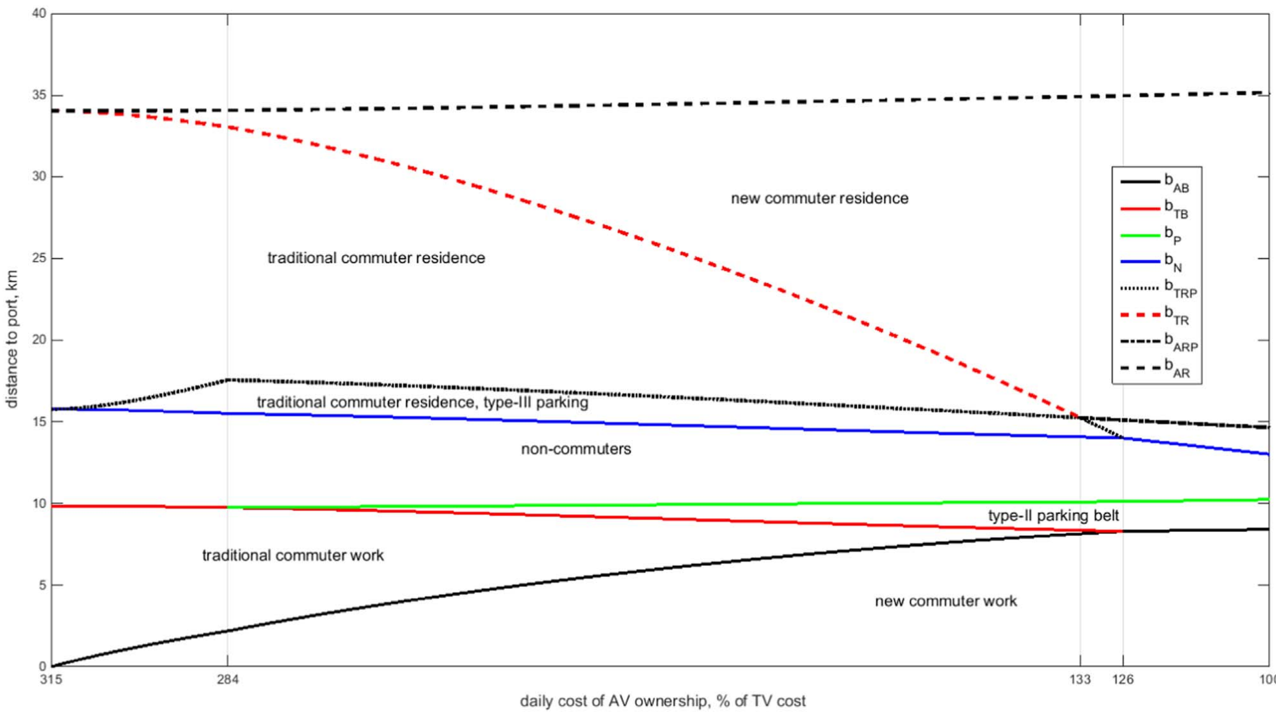

Fig. 2. Evolution of zone boundaries.

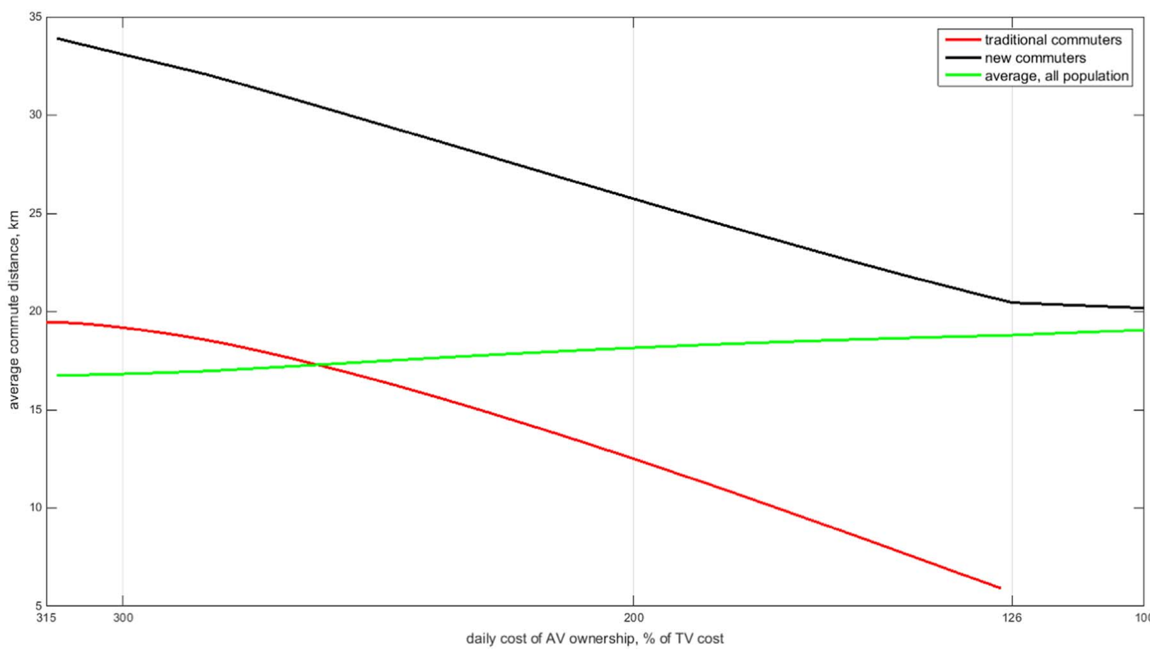

Fig. 3. Evolution of average commute distances.

reducing welfare.

5.2. Mass transit

We now investigate how cities with mass transit will be affected by the AV technology. For simplicity, we assume there are no traditional vehicles.

We assume that the existing mass transit allows anyone to travel at a cost δ_M per kilometer of roundtrip commute distance. By analogy with (8), we can show that, prior to AV introduction, the necessary condition of mass transit use is

$$\frac{\tau}{\delta_M} > \frac{\beta + s}{s}. \tag{25}$$

Moreover, because there are no fixed commute costs involved (vehicle ownership, parking), (25) is also the sufficient condition: if it is

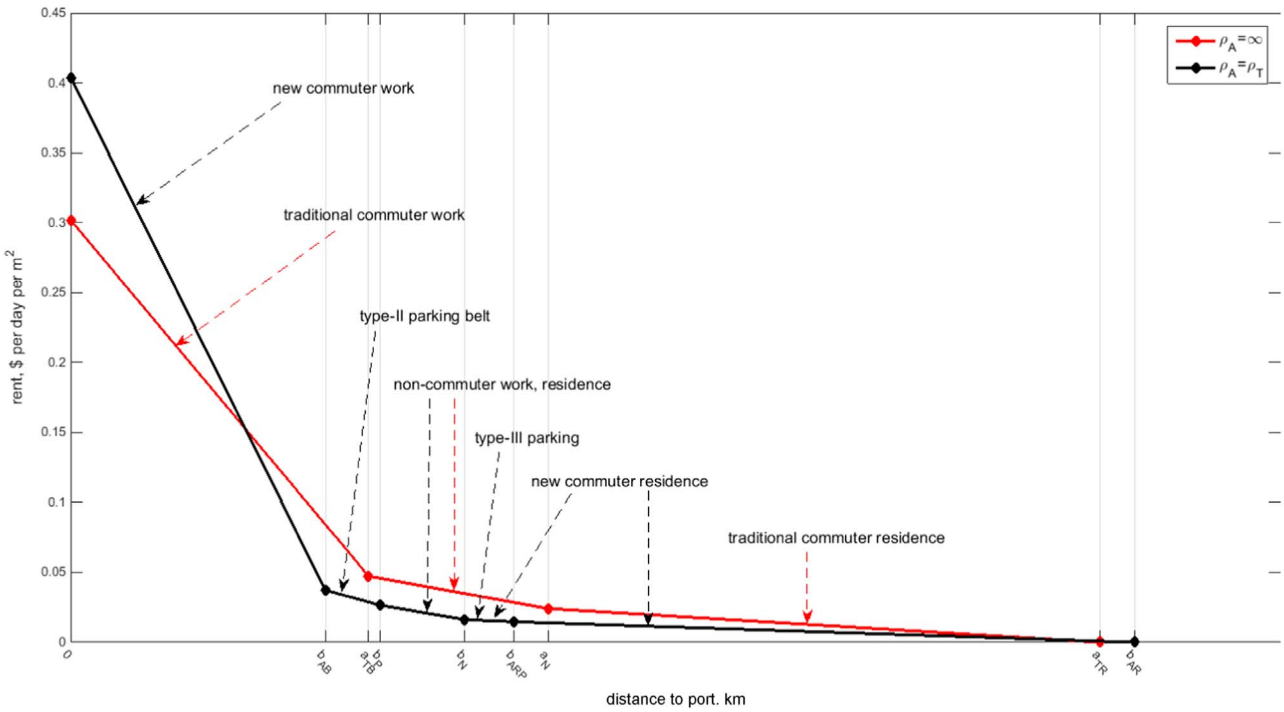

Fig. 4. Evolution of land rents.

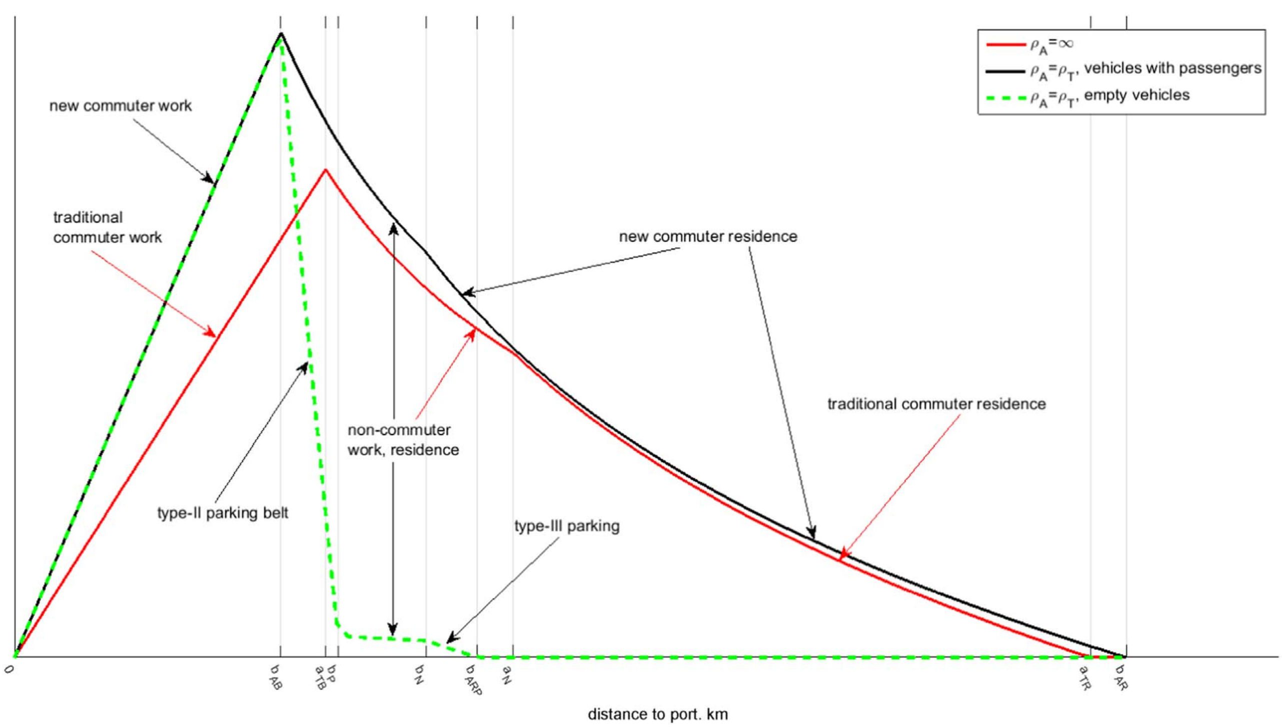

Fig. 5. Evolution of the city traffic.

satisfied, the entire pre-AV population uses mass transit, and the city is divided into a central work zone and a peripheral residential zone. We assume that (25) is indeed the case. Then, at work, the rent is characterized by (cf. (4) and (14))

$$r_{MB}\left(a\right) \equiv \frac{\lambda_M - (\tau - \delta_M)a}{\beta}, \tag{26}$$

for some λ_M, while at the residence it is

$$r_{MR}\left(a\right) \equiv \frac{\omega - c - \lambda_M - \delta_M a}{s}. \tag{27}$$

The effects of AV introduction then depend on the relationship between the mass transit parameter δ_M and AV travel parameters δ_A, δ_P, as follows.

5.2.1. High cost of AV travel

If $\delta_M \leq \delta_A$, both fixed (standing) and variable (running) costs of AV use are higher than those of mass transit, and the AVs are never used in equilibrium.

5.2.2. Intermediate cost of AV travel

If $\delta_A < \delta_M \leq \delta_A + \delta_P$, the variable cost of AV use is lower than that of mass transit as long as AVs do not have to travel empty after dropping off, or before picking up, the owner. In this case, a fraction of population may use AVs, but only for the peripheral leg of their trip; they will get off their AVs between the peripheral edge of the work zone and the central edge of the residential zone, and connect to mass transit to get to work. The rent at all work locations then satisfies (26).

Daytime parking will be of "park-and-ride" type, and will be located between work and residential zones. AVs will never travel more centrally than their location of daytime parking, and park immediately after their owners get off. In this sense, the AV use will be conceptually no different from that of modern park-and-ride programs. The rent at the park-and-ride zone satisfies (cf. (15)) $r_{MP}(a) \equiv \frac{\lambda_P - (\delta_M - \delta_A)a}{\nu}$, for some λ_P.

The residential rent satisfies (27) for those who use mass transit only, and (cf. (19)) $r_{AMR}\left(a\right) \equiv \frac{\omega - c - \lambda_M - \lambda_P - \rho_A - \delta_A a}{s + \nu}$ for those who combine AVs and mass transit. The latter has greater (closer to zero) slope, and is thus more peripheral than the former.

Under this scenario, lower cost ρ_A of AV, by increasing the share of population using it, will squeeze the mass transit coverage at the peripheral (residential) edge of the city. At the same time, near the center the entire population will still use mass transit to get to work.

5.2.3. Low cost of AV travel

If $\delta_A + \delta_P < \delta_M$, the variable cost of AV use, *including* return of the vehicle out of the city center, is lower than that of mass transit. In this case, those using AVs will never use mass transit. The rent at locations they use will be (14) at work, (15) at type-II parking, (17) at type-III parking and residence, and (19) at residence-only locations. New commuter work (residence) will be more central (more peripheral) than that of mass transit users. Cheaper AVs, by increasing the share of population using them, will squeeze the mass transit coverage on both central and peripheral ends.

6. Conclusion

The analysis of this paper demonstrates how a typical city may change following the introduction of a new commute technology. The change will be complex, with different parts of the city changing differently. The demand for daytime parking will be shifted to the periphery, allowing the increase of the density of economic activity and the rise of downtown land rents. At the same time, lower transportation costs will cause a city sprawl and rent decline outside of the city center. The calibration exercise shows that both central rent increase and peripheral rent decrease may be of the order of 30–40%.

The analysis also predicts the emergence of the dedicated "parking belt" where most commuter autonomous vehicles will be day-parked. It will be located just outside of the commuter work zone, and may accumulate as much as 97% of all commuter AVs. The remaining AVs will travel further, to the commuter residential zone, for day parking, to take advantage of natural complementarity of daytime and nighttime parking.

Although traffic is projected to increase, that will not necessarily cause a congestion increase, as AVs are expected to be operated more efficiently. If a regulator still chooses to limit traffic, it should focus on "main" traffic, towards the center in the morning, out of the center in the evening, and not limit the "return" traffic, consisting mainly of empty AVs that travel to/from their daytime parking locations.

In cities with mass transit, the impact of AVs will depend on the variable cost of their use, relative to that of the mass transit. Under some parameter values, AV users will connect to mass transit before entering the city center. Their vehicles will always stay outside of the center. With other parameter values, AV users will travel everywhere by car, reducing the mass transit coverage in city centers.

Appendix A. Proofs

Proof of Proposition 1. First, we rule out $a_1 = a_2$: that would enable the commuter to turn into a non-commuter, dropping all commute costs without sacrificing any output. Next, suppose $a_1 > a_2$ for some mass Δ of commuters; those commuters use Δs units of residential land at a_2 and $\Delta\beta$ units of land for production at a_1. Some land is also used for parking. Consider the following relocation: mass $\Delta\frac{s}{\beta + s}$ live and work at a_2, while the rest live and work at a_1. The total use of land for production and residence is unchanged, but the value of output has increased because some production is now closer to the port. Moreover, the entire mass Δ of commuters becomes non-commuters, thus the costs of commute, as well as parking land demand, are nullified. The savings can be used to increase consumption, e.g. that of landlords, resulting in a Pareto-improvement and contradicting the equilibrium status of the initial allocation.□

Proof of Proposition 2. Suppose the contrary, that some location a is both residence of some "group 1" of traditional commuters and workplace of another "group 2." Both groups have equal mass Δ of commuters.[5] By Proposition 1, the location of work a_1 of group 1 and the location of residence a_2 of group 2 satisfy $a_1 < a < a_2$. Group 1 uses $\Delta\beta$ units of land at a_1 and Δs units at a. Group 2 uses $\Delta\beta$ units at a and Δs units at a_2. Additional land is used for parking at all three locations. The variable commute costs of the two groups combined are $\delta_T\Delta(a_2 - a_1)$.

Consider the following relocation: the residence locations of the two groups are swapped. Group 2 is now entirely located at a, becoming a non-commuter group. Thus, demand for parking at a, as well as fixed commute costs of group 2, are nullified. Other costs, as well as the output, are unchanged. The savings can be used to achieve a Pareto-improvement, thus the initial allocation is not an equilibrium.□

Proof of Proposition 3. Suppose the contrary, that there is location a where there is a workplace and a parking space, the latter is used for both day and night parking, and those working at a are commuters who day-park also at a. Since night parking is always near a residence, there must be a residence at a, too. Without loss of generality, suppose the capacity of each of these (work, residence, parking) is Δ.[6] By Proposition 1, the commuter group that resides at a ("group 1") works at some $a_1 < a$. By the same proposition, the commuters who work at a ("group 2") reside at some $a_2 > a$.

[5] If the two groups have different sizes, we consider only a fraction of the larger group, such that their sizes are equalized.

[6] If the three uses have different capacities, we consider only a fraction of larger-capacity uses, such that the capacities are equalized.

Fraction f_i of commuters in group i are new, while the rest are traditional. Assume new commuters are a minority, i.e. $\frac{1}{2}(f_1 + f_2) \le \frac{1}{2}$; the proof is easily extended to the other case. The total cost of commute, aggregated across all groups, is equal to

$$\Delta(f_1\delta_A + (1 - f_1)\delta_T)(a - a_1) + \Delta(f_2\delta_A + (1 - f_2)\delta_T)(a_2 - a) + \Delta(f_1 + f_2)\rho_A + \Delta(2 - f_1 - f_2)\rho_T. \quad (A.1)$$

Consider the following relocation: all new commuters work at a_1 and live at a_2. Some traditional commuters fill up the workplace and the residence at a to capacity, becoming non-commuters; the remaining traditional commuters live at a_1 and work at a_2, just like the new ones.

Because mass Δ of traditional commuters become non-commuters, the fixed cost of vehicle ownership is reduced by $\Delta\rho_T$. The new total cost of commute is

$$\Delta(a_2 - a_1)[(f_1 + f_2)\delta_A + (1 - f_1 - f_2)\delta_T] + \Delta(f_1 + f_2)\rho_A + \Delta(1 - f_1 - f_2)\rho_T, \quad (A.2)$$

which is smaller than (A.1) by $\Delta((a_2 - a)f_1 + (a - a_1)f_2)(\delta_T - \delta_A) + \Delta\rho_T > 0$. Thus, the aggregate commute costs are reduced without reducing either aggregate production or aggregate residential space. The demand for parking at a is nullified, while the demand for parking at a_1, a_2 is unchanged. With proper redistribution, the reallocation results in a Pareto-improvement, which contradicts the equilibrium status of the initial allocation.□

Proof of Proposition 4. The proposition is trivial for TVs, as they are always parked near their owner. It is also trivial for night parking of AVs, which by assumption are also night-parked near owner. For AV day parking, consider the opposite scenario: there is a mass Δ of "group 1" of new commuters for whom the location of work a_1 is more peripheral than location of day parking a_2, i.e. $a_2 < a_1$. Then, the cost of an AV empty travel for parking is $\Delta\delta_P(a_1 - a_2)$. The parking space at a_2 can also be shared with mass $\Delta' \le \Delta$ of "group 2" commuters who reside and night-park at a_2. By Proposition 1, the residence of group 1 is more peripheral than a_1, while the workplace of group 2 is more central than a_2. The land use by group 1 at a_1 is $\Delta\beta$; the land use by both groups at a_2 is $\Delta\nu + \Delta' s$.

Consider the following relocation: the workplace of fraction $\frac{\Delta\nu + \Delta' s}{\Delta(\beta + \nu) + \Delta' s}$ of group 1 is moved to a_2; the space vacated at a_1 is filled by fraction $\frac{\Delta\beta}{\Delta(\beta + \nu) + \Delta' s}$ of group 2. In other words, we mix the two groups so that both are represented at each location in the same proportion. The AVs can now park near owners' work, the associated empty travel costs are nullified. The group 1 output gain net of additional commute costs, due to more central workplace of some group 1 members, is $\Delta\left(a_1 - a_2\right)\left(\tau - \delta_A\right)\frac{\Delta\nu + \Delta' s}{\Delta(\beta + \nu) + \Delta' s}$. The increased commute cost of group 2, due to more peripheral place of residence of some of them, is $\Delta'\left(a_1 - a_2\right)\delta_i\frac{\Delta\beta}{\Delta(\beta + \nu) + \Delta' s}$, where $\delta_i = \delta_A$ if group 2 are new commuters, and $\delta_i = \delta_T > \delta_A$ otherwise. By our earlier assumption, (8) is true, which can be used to show that the gain of group 1 is greater than the loss of group 2. A proper redistribution will lead to a Pareto-improvement, which contradicts the equilibrium status of the initial allocation.□

Proof of Proposition 5. Inequality (20) implies $|r'_{AB}(\cdot)| > |r'_{ABP}(\cdot)| > |r'_P(\cdot)|$, which further implies (i) if type-II new commuters are present, their place of work is more central than their place of daytime parking, which verifies Proposition 4 for them, and (ii) if type-I commuters are present and work at some location a, their corresponding rent bid should outbid all other rent bids at a, in particular those for work and day parking by type-II commuters:

$$r_{ABP}\left(a\right) = \frac{\lambda_{AP} - (\tau - \delta_A)a}{\beta + \nu} \ge \frac{\lambda_A - (\tau - \delta_A - \delta_P)a}{\beta} = r_{AB}\left(a\right),\ r_{ABP}\left(a\right) = \frac{\lambda_{AP} - (\tau - \delta_A)a}{\beta + \nu} \ge \frac{\lambda_P - \delta_P a}{\nu} = r_P\left(a\right).$$

Multiply the first inequality by β and the second by ν; add them together to arrive at $\lambda_{AP} \ge \lambda_A + \lambda_P$. From previous discussion, this inequality implies that type-II commuters outbid type-I commuters at residential locations, thus type-I should be excluded from equilibrium.□

Appendix B. Notational glossary

Var	Meaning	Unit
Exogenous parameters		
L	labor force	million workers
a	distance to port	km
β	land requirement per unit of output	m^2
γ	preference parameter	m^2 per \$ of consumption
δ_A	variable cost of operation of autonomous vehicle with passenger	\$ per roundtrip km
δ_P	variable cost of empty autonomous vehicle operation	\$ per roundtrip km
δ_T	variable cost of traditional vehicle operation	\$ per roundtrip km
ν	parking land requirement	m^2
ρ_A	fixed cost of autonomous vehicle operation	\$ per vehicle
ρ_T	fixed cost of traditional vehicle operation	\$ per vehicle
τ	output transport cost	\$ per km
ω	value of output at port	\$
Endogenous variables		
a_X	outer boundary of land use type X	km to port
c	worker consumption	\$ per worker
$r_X(a)$	land rent at distance a from port, under land use type X	\$ per m^2
s	worker residence size	m^2 per worker
λ_X	land rent at port, under land use type X	\$ per m^2

References

Alonso, W., et al., 1964. Location and Land Use: Toward a General Theory of Land Rent.

Anderson, J.M., Nidhi, K., Stanley, K.D., Sorensen, P., Samaras, C., Oluwatola, O.A., 2014. Autonomous Vehicle Technology: A Guide for Policymakers. RAND Corporation, Santa Monica, CA.

Anderson, S.P., de Palma, A., 2004. The economics of pricing parking. J. Urban Econ. 55 (1), 1–20.

Arnott, R., Inci, E., 2006. An integrated model of downtown parking and traffic congestion. J. Urban Econ. 60 (3), 418–442.

Brueckner, J.K., Franco, S.F., 2015. Parking and Urban Form.

Fagnant, D.J., Kockelman, K., 2015. Preparing a nation for autonomous vehicles: opportunities, barriers and policy recommendations. Transp. Res. Part A: Policy Pract. 77, 167–181.

Godsmark, P., Kirk, B., Gill, V., Flemming, B., 2015. Automated Vehicles: The Coming of the Next Disruptive Technology.

Hayes, B., 2011. Leave the driving to it. Am. Sci. 99, 362–366.

Manville, M., Shoup, D., 2005. Parking, people, and cities. J. Urban Plan. Dev. 131 (4), 233–245.

McKenzie, B., Rapino, M., 2011. Commuting in the United States: 2009. US Department of Commerce, Economics and Statistics Administration, US Census Bureau.

Romem, I., 2013. How will driverless cars affect our cities? ⟨http://www.Cityminded.org⟩.

Santos, A., McGuckin, N., Nakamoto, H.Y., Gray, D., Liss, S., 2011. Summary of travel trends: 2009 national household travel survey. Technical Report.

Shoup, D.C., 2005. The High Cost of Free Parking. Planners Press, Chicago.

U.S. Census Bureau, 2011. American Housing Survey for the United States: 2009. Current Housing Reports, Series H150/09. U.S. Government Printing Office, Washington, DC, 20401.

van den Berg, V.A., Verhoef, E.T., 2015. Robot Cars and Dynamic Bottleneck Congestion: The Effects on Capacity, Value of Time and Preference Heterogeneity.

Voith, R., 1998. Parking, transit, and employment in a central business district. J. Urban Econ. 44 (1), 43–58.

Zhang, W., Guhathakurta, S., Fang, J., Zhang, G., 2015. Exploring the impact of shared autonomous vehicles on urban parking demand: an agent-based simulation approach. Sustain. Cities Soc. 19, 34–45.